BENDING

- ELASTICITY VERSUS PLASTICITY
  - FULLY PLASTIC MOMENT
  - SHAPE FACTOR
  - EXAMPLES

- BEHAVIOUR OF THE CROSS SECTION
  - MOMENT-CURVATURE
  - PLASTIC ZONES
  - IDEAL PLASTIC HINGE

- STRUCTURAL BEHAVIOUR (LIMIT ANALYSIS)
  - BEAMS
  - FRAMES
M-κ BEHAVIOUR OF THE CROSS SECTION

moment

\[ M_p \]

\[ M_e \]

curvature \( \kappa \)

elastic

elasto-plastic

plastic
MOMENT-CURVATURE

ELASTIC:

\[
\frac{1}{2} h \quad \text{y-as} \quad \frac{1}{2} h
\]

cross section

\[
\kappa \quad \varepsilon_y = \kappa_e \frac{1}{2} h
\]

strain-diagram

\[
\sigma = f_y
\]

stress-diagram

\[
\frac{M}{W} \Rightarrow M = W\sigma
\]

start with:

\[
M_e = Wf_y
\]

thus:
MOMENT-CURVATURE

ELASTO PLASTIC:

\[ \frac{1}{2} \left( \frac{1}{2} h - \frac{1}{2} a \right) + \frac{1}{2} a = \frac{1}{4} (h + a) \]

\[ \frac{1}{2} f_y b (h - a) \]

\[ \frac{1}{2} f_y b \frac{1}{2} a = \frac{1}{4} f_y ab \]

\[ M = \frac{1}{6} f_y b a^2 + \frac{1}{2} f_y b (h - a) \times 2 \times \frac{1}{4} (h + a) = \frac{1}{4} b h^2 \left( 1 - \frac{a^2}{3h^2} \right) \times f_y \]
MOMENT – CURVATURE

\[ M = \frac{1}{4} bh^2 \left( 1 - \frac{a^2}{3h^2} \right) \times f_y \]

\[ M_e = \frac{1}{6} bh^2 \times f_y \]

\[ \frac{M}{M_e} = \left( 1,5 - \frac{a^2}{2h^2} \right) = 1,5 - \frac{1}{2} \left( \frac{\kappa_e}{\kappa} \right)^2 \]
MOMENT - CURVATURE
For rectangular cross sections

Elasto-plastic:
\[
\frac{M}{M_e} = 1,5 - \frac{1}{2} \left( \frac{\kappa_e}{\kappa} \right)^2
\]
PLASTIC ZONE IN THE BEAM

stress in critical cross section due to increasing load

$1 = \text{elastic}$
$2 = \text{elastic limit } = M_e$
$3 = \text{elasto plastic}$
$4 = \text{elasto plastic}$
$5 = \text{fully plastic } = M_p$

\[ \frac{1}{4} F_{\text{max}} l = M_p \quad \Rightarrow \quad F_{\text{max}} = F_p = \frac{4M_p}{l} \]
PLASTIC ZONE

\[ p = \frac{\alpha - 1}{\alpha} l \]

\[ \alpha = \frac{M_p}{M_e} \]
DEVELOPMENT OF CROSS SECTION IN FULL PLASTICITY, STATICALLY DETERMINATE SYSTEM CHANGES INTO A MECHANISM DUE TO THE PRESENCE OF THE PLASTIC HINGE

FAILURE MECHANISM
MODEL FOR THE LIMIT STATE ANALYSIS

- All deformation concentrated in plastic hinges
- Failure whenever a mechanism occurs: failure mechanism
EXAMPLES

Statically determinate
\( n = 0 \)

Statically indeterminate
\( n = 2 \)

Kinematically indeterminate (mechanism)
\( n = -1 \)
INCREMENTAL ANALYSIS

\[
\frac{q}{\left(\frac{M_p}{l^2}\right)}
\]

\[
\frac{w_c}{\left(\frac{M_p l^2}{96EI}\right)}
\]

clamp beam

beam