BENDING

- ELASTICITY VERSUS PLASTICITY
  - FULLY PLASTIC MOMENT
  - SHAPE FACTOR
  - EXAMPLES

- BEHAVIOUR OF THE CROSS SECTION
  - MOMENT-CURVATURE
  - PLASTIC ZONES
  - IDEAL PLASTIC HINGE

- STRUCTURAL BEHAVIOUR (LIMIT ANALYSIS)
  - BEAMS
  - FRAMES
MODEL FOR THE LIMIT STATE ANALYSIS

- Concentrate all plastic deformation in one cross section, the plastic hinge

- Failure whenever a mechanism occurs: failure mechanism
APPROACH SO FAR

- INCREMENTAL METHOD
  - result: Load – Deformation Diagram

- DIRECT (EQUILIBRIUM) METHOD
  - result: Collapse or Limit Load

NEW APPROACH

- STATIC CHECK FOR ALL ASSUMED MECHANISMS USING VIRTUAL WORK
  - result: Collapse or Limit Load
INCREMENTAL METHOD WITH PROPORTIONAL LOADING

- Increase the load until the bending moment at a cross section reaches the full plastic moment. This is the location of the first plastic hinge.

- Change the structural model by adding a hinge and increase the load. Compute the additional moment distribution with the new structural model and determine the location of the next plastic hinge.

- Repeat step 2 until the occurrence of a mechanism. The load is the collapse load or limit load and the mechanism is the collapse mechanism.
EXAMPLE of the INCREMENTAL METHOD

Statically Indeterminate \( n=2 \)

Bending stiffness \( EI \)

Kinematically Indeterminate \( n=-1 \)

Kinematically Determinate \( n=0 \)

\[ q_p = q_1 + q_\| \]

\[ M_{\text{line}} \]

\[ \frac{1}{12} q_1 l^2 = M_p \]

\[ \frac{1}{24} q_1 l^2 \]

\[ \frac{1}{8} q_\| l^2 = \frac{1}{2} M_p \]

\[ q_p = q_1 + q_\| = \frac{16M_p}{l^2} \]
RESULTS OF THE INCREMENTAL METHOD

Load – Deflection Diagram

\[ q \times \frac{M_p}{l^2} \]

\[ w_C \times \frac{M_p l^2}{96EI} \]
EQUILIBRIUM METHOD  (lower bound, chpt 7)

$M$-distribution at collapse

SAFE ($M < M_p$) AND STATICALLY ADMISSIBLE (equilibrium) COLLAPSE LOAD IS EQUAL OR LARGER THAN THE OBTAINED LOAD

kinematically indeterminate (mechanism)
n=-1

$$2M_p = \frac{1}{8} ql^2$$

$$q_p = \frac{16M_p}{l^2}$$

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NEW APPROACH BASED ON COLLAPSE MECHANISMS

1. Determine the max number of required plastic hinges to obtain a mechanism

2. Determine the number of possible positions for a plastic hinge to occur

3. Find the number of possible mechanisms

4. Compute for each mechanism the collapse load using the principle of Virtual Work
COLLAPSE LOAD (upper bound method)

- Of all possible mechanisms the mechanism with the smallest load will be the collapse mechanism.

- The actual collapse load will be smaller or equal to the found collapse load, never larger! (Prager’s upper-bound-theorema)

- For all cross sections the moment distribution at collapse must be bounded by the strength of the section. In other words, at no cross section the moments may exceed the plastic limit. If so, the mechanism can not be the collapse mechanism! (uniqueness theorem)
ASSUMPTIONS

- LIMIT STATE AT COLLAPSE
- NO INFO ON DEFORMATION
- IDEAL RIGID-PLASTIC MATERIAL BEHAVIOUR

- ALL PLASTIC DEFORMATION CONCENTRATED AT A CROSS SECTION, **PLASTIC HINGE**

\[ M \]

\[ M_p \]

\[ \kappa \]

NO ADDITIONAL LOAD CAPACITY DUE TO THE SHAPE FACTOR !!

\[ \alpha = 1 \]
EXAMPLE

- Nr of required hinges for a mechanism: 2
  \[(1 - 2) = -1 = \text{mechanisme}\]
- Nr of possible position for a hinge: A and C
- Nr of mechanisms: 1

Statically Indeterminate n=1
MECHANISM

Principle of Virtual Work:
Equilibrium of a rigid body for any kinematically admissible displacement only for zero virtual work. (CTB1110 and CTB2210)
VIRTUAL WORK

\[ \delta A = -M_p \delta \theta_1 - M_p \delta \theta_1 - M_p \delta \theta_2 + F \times \delta w = 0 \]

\[ \delta A = -2M_p \frac{\delta w}{a} - M_p \frac{\delta w}{b} + F \delta w = 0 \]

\[ F_p = M_p \left( \frac{2}{a} + \frac{1}{b} \right) \]

\[ \theta_1 = \frac{w}{a} \]

\[ \theta_2 = \frac{w}{b} \]
POSSIBLE POSITIONS FOR PLASTIC HINGES

- At the bar ends
- At the supports (fixed supports)
- Under concentrated loads and distributed loads
- At changes in strength ($M_p$ changes in magnitude)

Example: 9 possible positions
Required: $4 + 1 = 5$ hinges
NUMBER OF MECHANISM

- DETERMINE THE DEGREE OF STATICALLY INDETERMINANCY \( (n) \)
- DETERMINE THE MAXIMUM REQUIRED NUMBER OF HINGES FOR A MECHANISM \( (n+1) \)
- DETERMINE THE POSSIBLE POSITIONS OF HINGES
- COMPUTE THE MAXIMUM NUMBER OF COMBINATIONS

NUMBER OF POSSIBLE MECHANISMS ?

STATICALLY INDETERMINATE \( n > 0 \)
STATICALLY DETERMINATE \( n = 0 \)
MECHANISM \( n = -1 \)
PERMUTATIONS AND COMBINATIONS
(STATISTICS)

THE NUMBER OF COMBINATIONS OF \( r \) FROM \( n \) ELEMENTS
\((r \leq n)\) IF THE ORDER DOES NOT MATTER:

\[
C_n^r = \binom{n}{r} = \frac{n!}{(n - r)! \cdot r!}
\]

“\( n \) over \( r \)”
PRACTICAL SOLUTIONS

USE BASIC or DEFAULT MECHANISMS (see notes)

- Partial beam mechanism
- Global sway mechanism
- Partial mechanism
- Partial rotational node mechanism
EXAMPLE

DETERMINE THE COLLAPSE LOAD $F_p$?

given:

$a = 1.0$ m

$$M_p = W_p \times f_y = 919,8 \times 10^3 \times 235 \times 10^{-6} = 216 \text{ kNm}$$
SYSTEMATIC APPROACH  (upper bound method)

- Determine the maximum required number of hinges for a mechanism \( s \)
- Determine the number of possible positions for hinges \( p \)

- Compute the number of possible mechanisms
  \[ C_n^r = \binom{p}{s} = \frac{p!}{(p-s)!s!} \]

- Determine for each mechanism the ultimate load with virtual work

- The mechanism with the smallest load is the collapse mechanism

- CHECK : The moment distribution at collapse must remain between the plastic boundaries of each cross section (uniqueness)