I N T R O D U C T I O N

Structural Mechanics theme

STABILITY of EQUILIBRIUM

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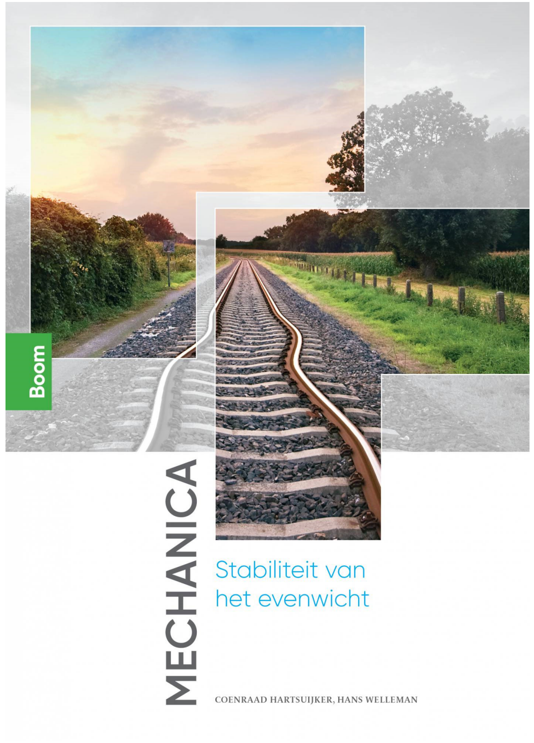
TUD portal : **Brightspace**

website : https://icozct.tudelft.nl/TUD_CT/CM3bridge/collegestof/stabiliteit/

LEARNING MATERIALS (portal)



Book: "Mechanica - Stabiliteit van het evenwicht", C. Hartsuijker en J.W. Welleman, 2023, Boom

Notes: "Knik en Eurocode 3" en "Kip", J.W. Welleman

ANS: Homework assignments (weekly)

Lectures: Slides and example files from the portal/website

LEARNING TRAJECTORY

- Order of topics based on increasing complexity of the governing equations and required math
- Focus on a systematic approach to understand the phenomena
- Recap of math partly in class but primarily DIY !
- Some examples presented in Python using SymPy

STABILITY OF EQUILIBRIUM

OBJECTIVES OF TODAY

- 1 Introduction
  - Familiar with stability phenomena
  - Familiar with some definitions and idiom

- 2 Rigid rods with one degree of freedom
  - Understanding of the assessment of equilibrium in the deformed state
  - Understanding the difference between a GL and GNL approach
  - Familiar with free body diagram and finding equilibrium condition in deformed state
  - Examples

BASIC KNOWLEDGE AND DEFINITIONS

- Rigidity, braced / unbraced
- Place retaining
- Shape retaining

→ DETERMINED BY GEOMETRY AND SUPPORT CONDITIONS

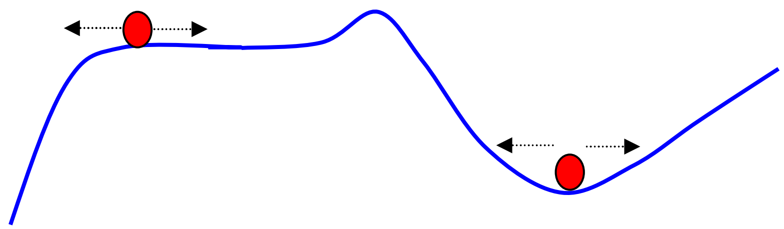
- Stability

→ DETERMINED BY STRUCTURE AND LOADING

STABILITY

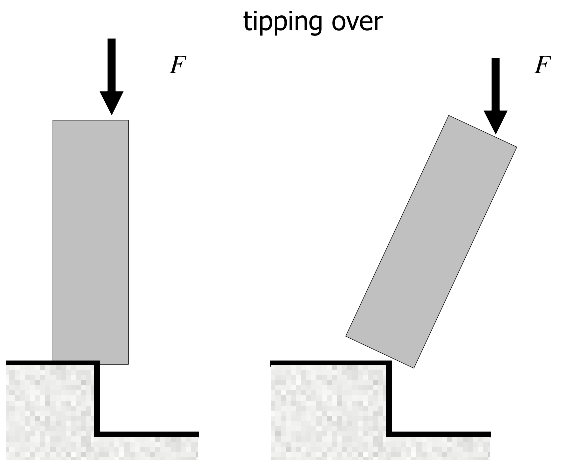
Determine the nature of the equilibrium in the deformed situation

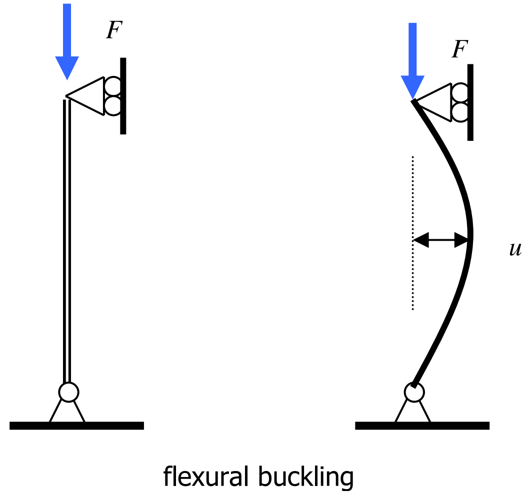
Stable equilibrium if the system in all neighbouring kinematic admissible perturbations returns to the equilibrium position.

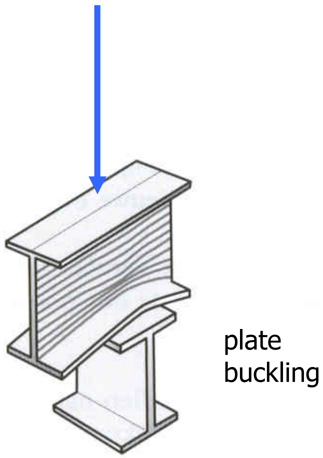


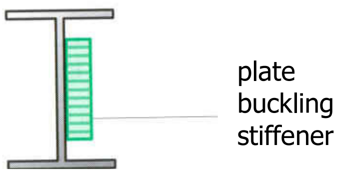
(also known as a reliable equilibrium)

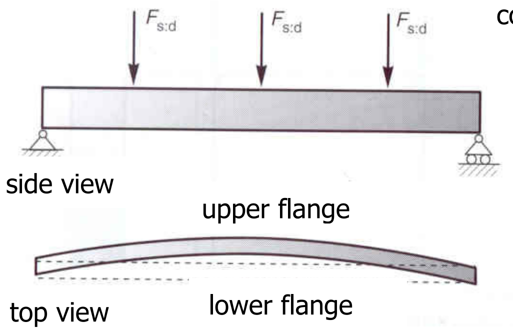
STABILITY PHENOMENA

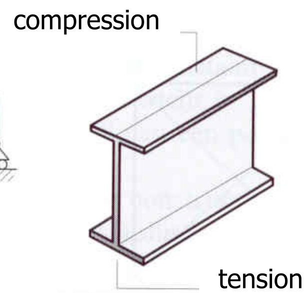


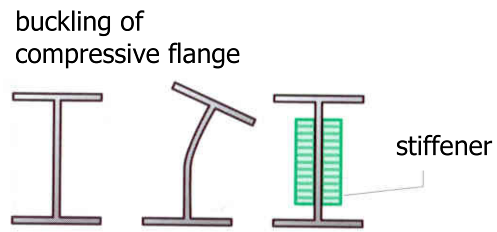




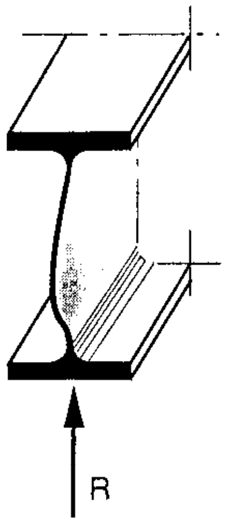








EXAMPLES



buckling at
support due to
concentrated loads

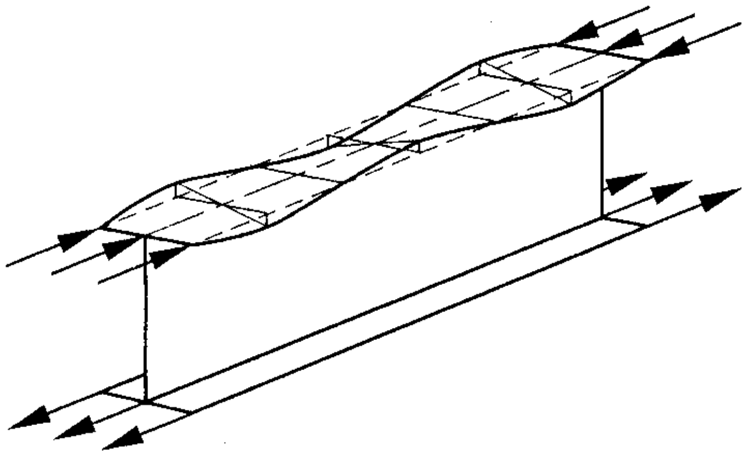
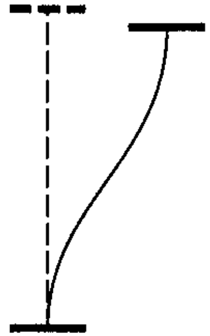
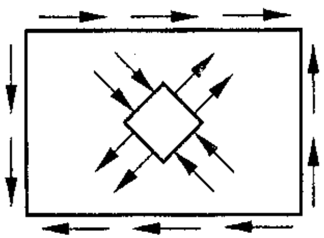


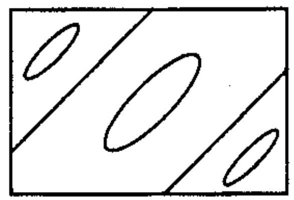
plate buckling



flexural buckling

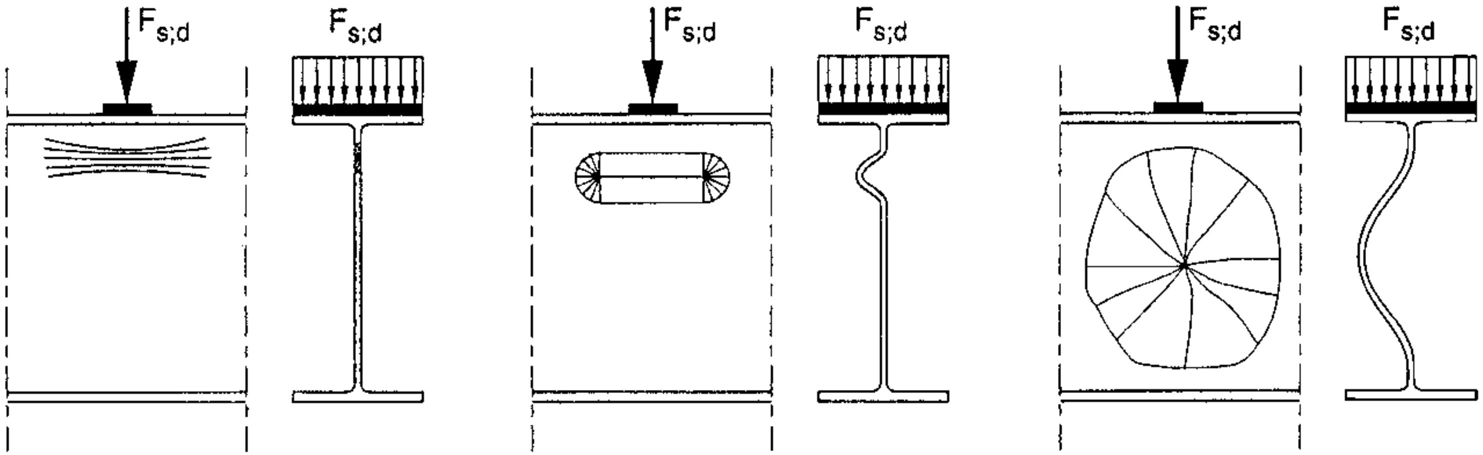


in plane stresses

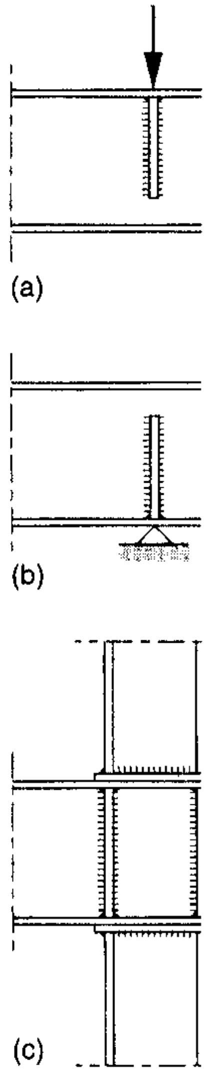


elastic buckling

EXAMPLES

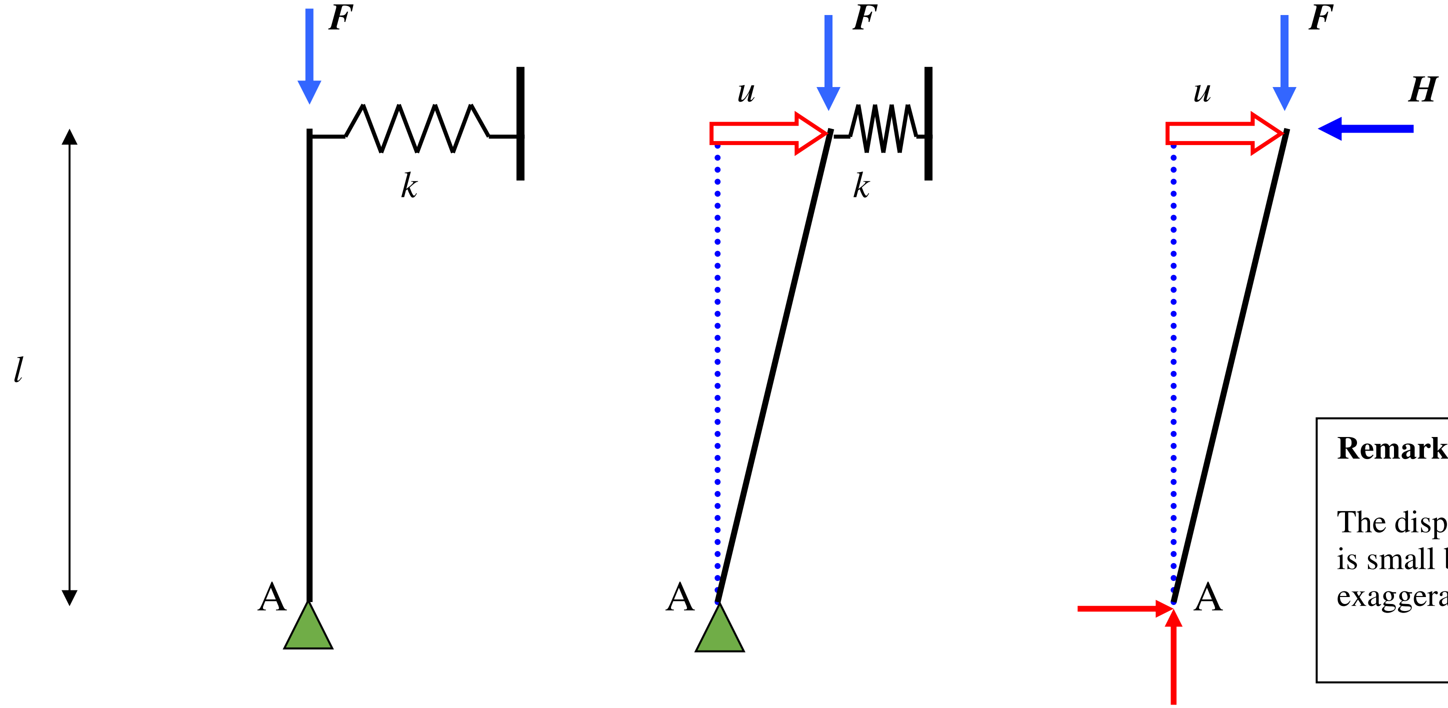


yielding **local buckling** **global buckling**



Also possible in horizontal plane …

RIGID BARS WITH ONE DEGREE OF FREEDOM

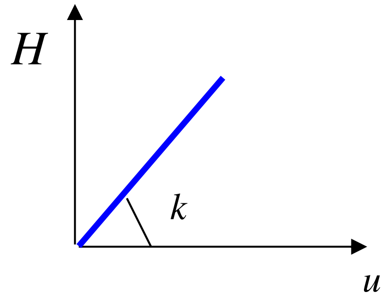


Structure with loads *Deformed situation* *Free body diagram*

Remark :

The displacement u is small but shown exaggerated

NATURE OF THE EQUILIBRIUM?



Check sum of moments around A

Action: Fu

Reaction: $Hl = k\,u\,l$

Equilibrium condition : $F_c u = k\,u\,l \quad \Leftrightarrow$

critical load

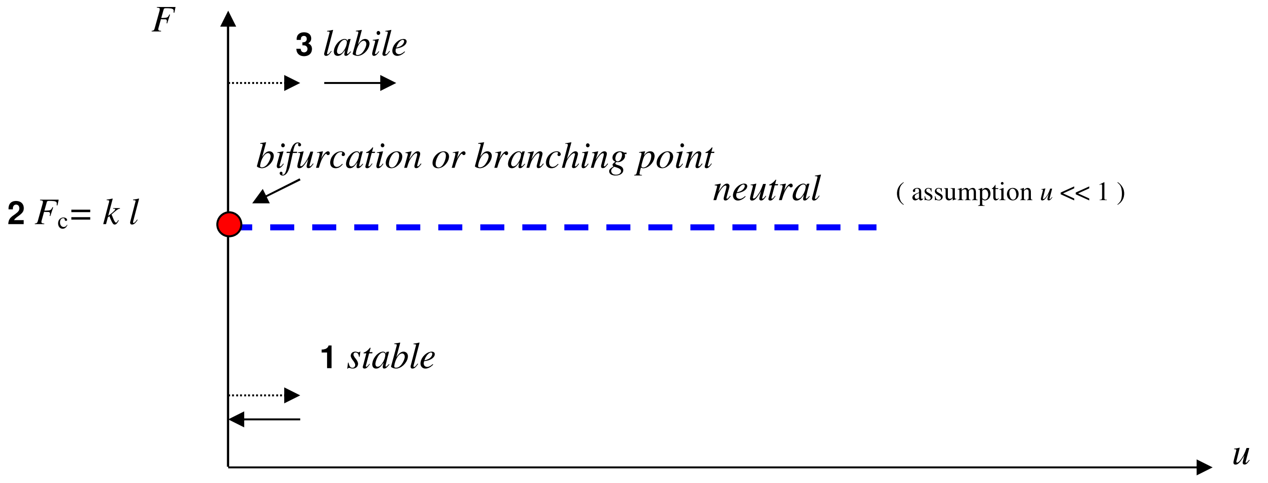
$$u\left(F_c - kl\right) = 0 \quad \Rightarrow \quad F_c = kl$$

$$or:$$

$$u = 0\,???$$

POSSIBILITIES

1. Reaction is larger than the action
2. Reaction equals action
3. Action exceeds reaction



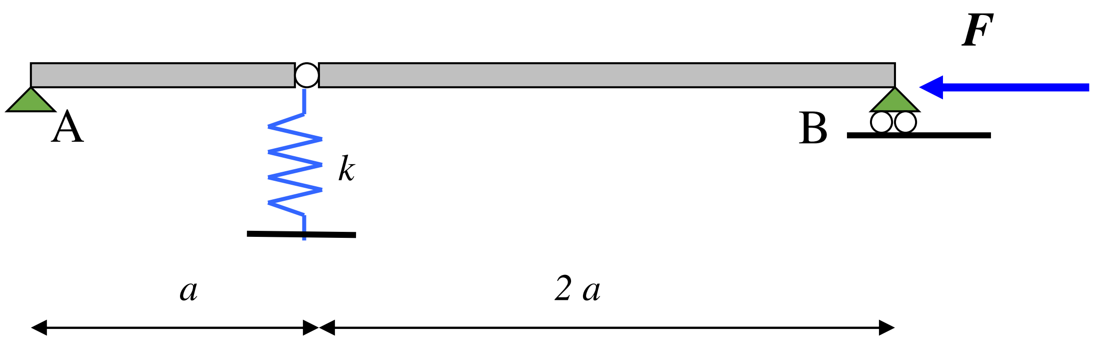
LINEAR MECHANICS

- Set up the equilibrium conditions in the undeformed state
- NO displacement terms in the equilibrium conditions
- Geometrical LINEAIR (or 1^{st} order) calculation

STABILITY

- Set up the equilibrium conditions in the deformed state
- Displacements components in the equilibrium condition
- Geometrical NON-LINEAR (or 2^{nd} order) calculation

EXAMPLE 1 : RIGID BAR, supported by a spring



Given:

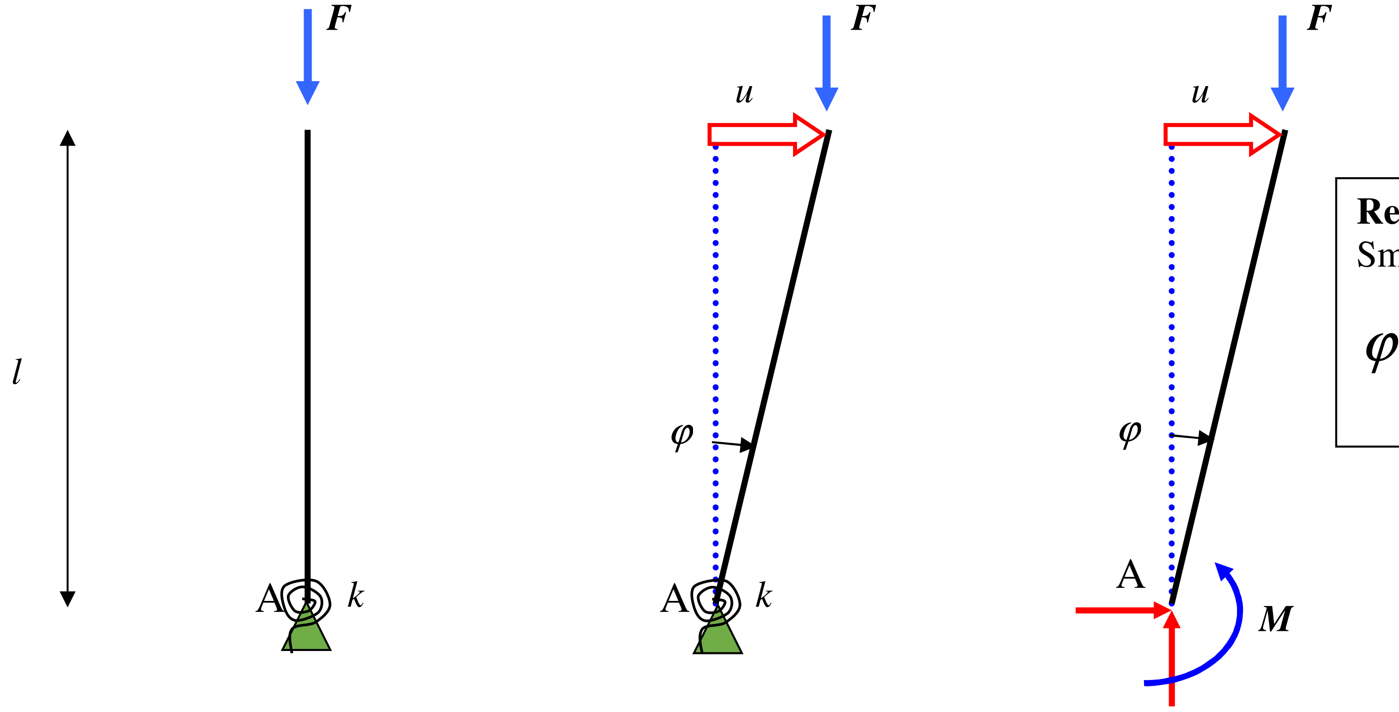
$k = 300$ kN/m

$a = 2{,}0$ m

Question :

Determine the critical load $\boldsymbol{F_c}$

RIGID BAR WITH ROTATIONAL SPRING

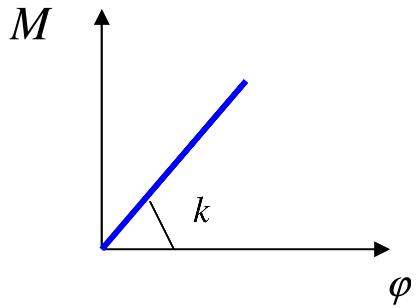


Structure with loads *Deformed state* *Free body diagram*

Remark :
Small displacements:

$$\varphi = \frac{u}{l}$$

NATURE OF THE EQUILIBRIUM ?



Check sum of moments around A

Action: $F u$

Reaction: $M = k\,\varphi = k\,\dfrac{u}{l}$

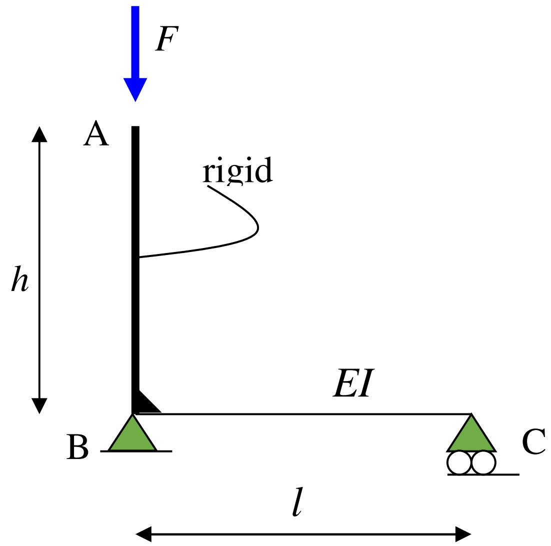
Equilibrium condition :

$$F_c\, u = \frac{k\, u}{l} \quad \Leftrightarrow$$

$$u\left(F_c - \frac{k}{l}\right) = 0 \quad \Rightarrow \quad F_c = \frac{k}{l}$$

critical load

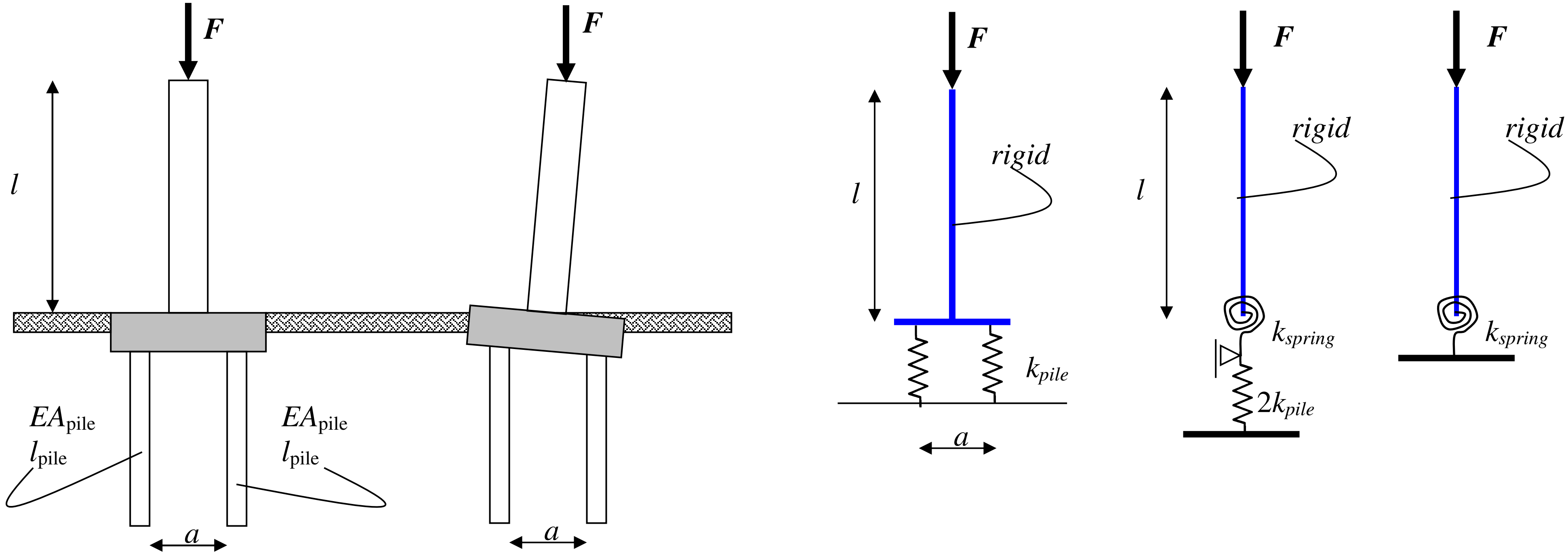
EXAMPLE 2 : Rigid rod, supported by flexible beam



Question:

Determine the critical load F_c

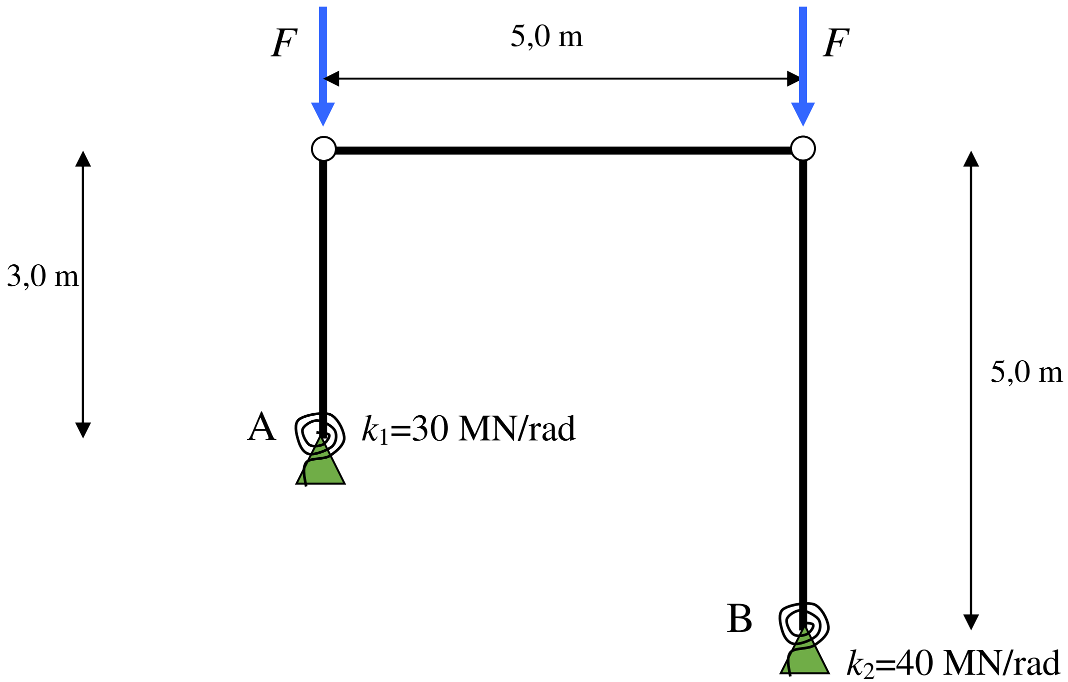
EXAMPLE 3 : Rigid rod with spring support



Question:

Determine the critical load F_c

EXAMPLE 4 : Coupled rigid rods



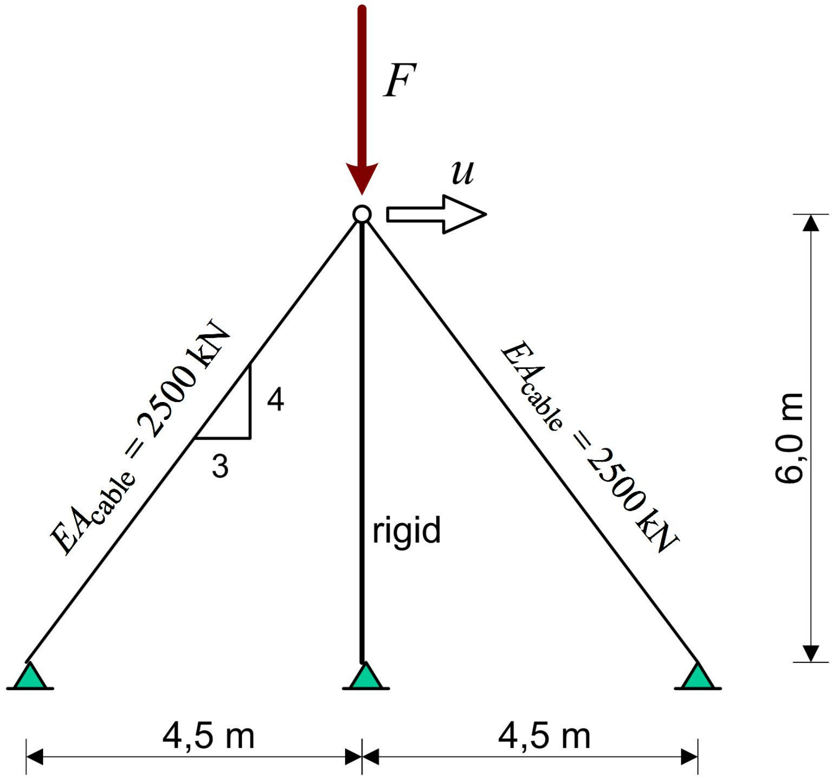
Question:

The critical load $\boldsymbol{F}_c$

EXAMPLE 5

assume :

$u \ll 1$



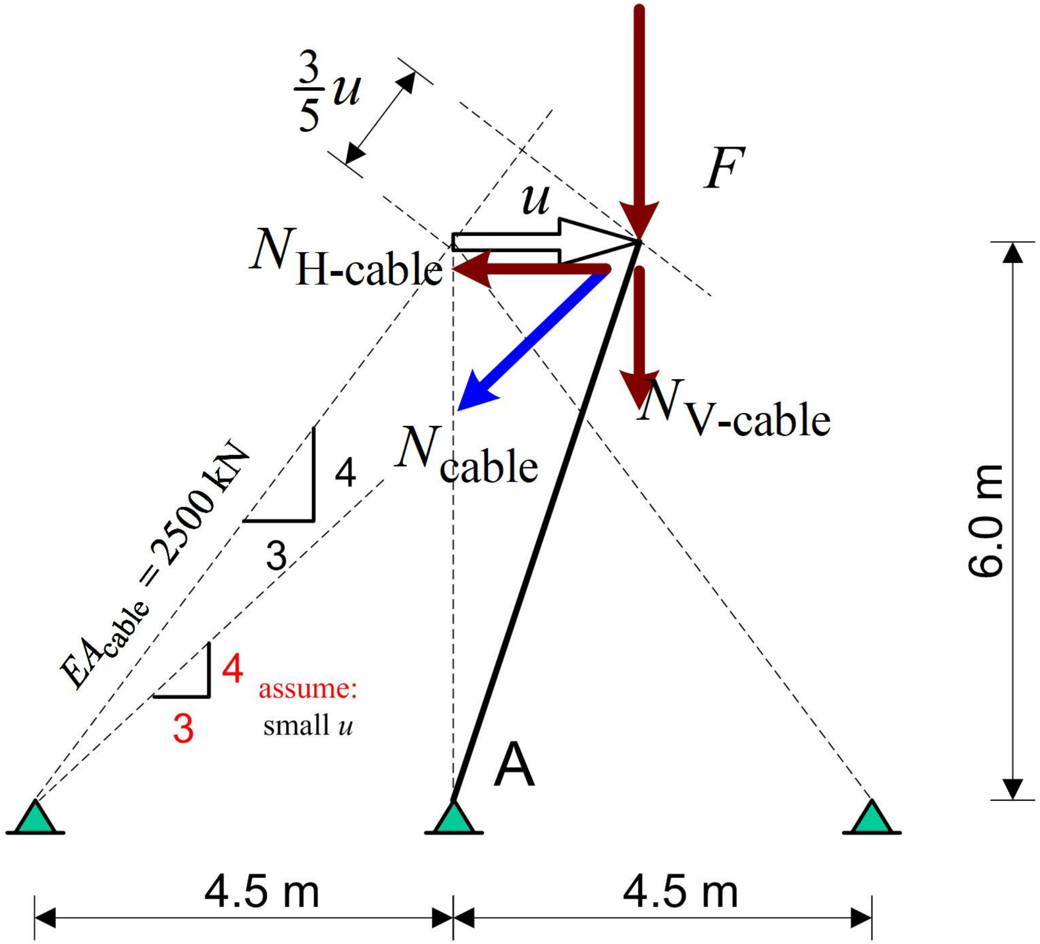
Question:

Find the critical load $\boldsymbol{F}_c$

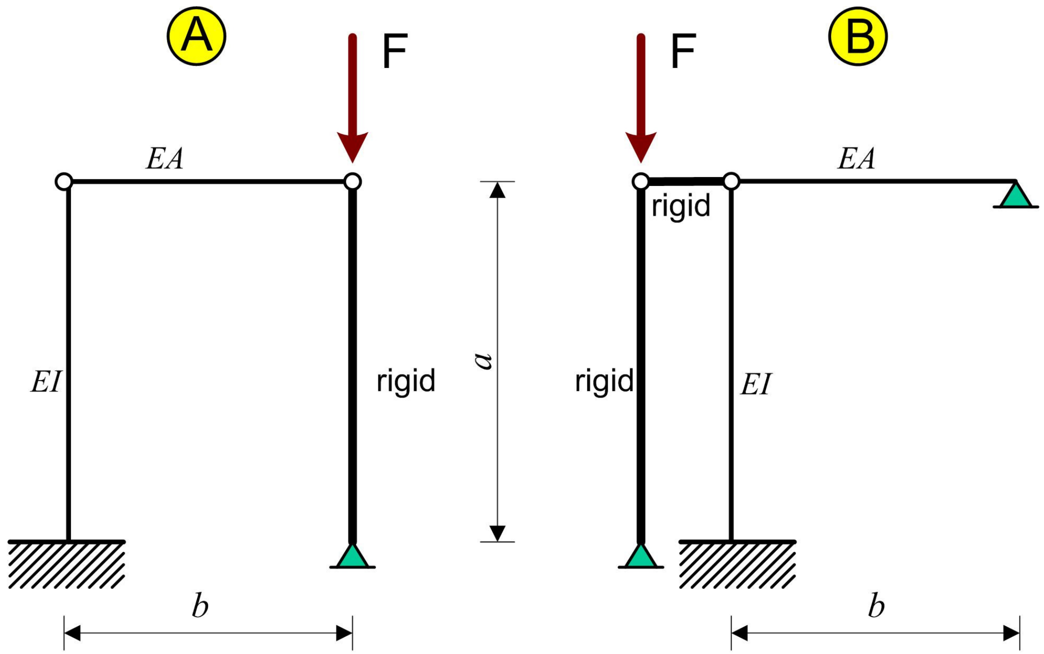
SYSTEMATIC APPROACH

$assume:$

$u \ll 1$



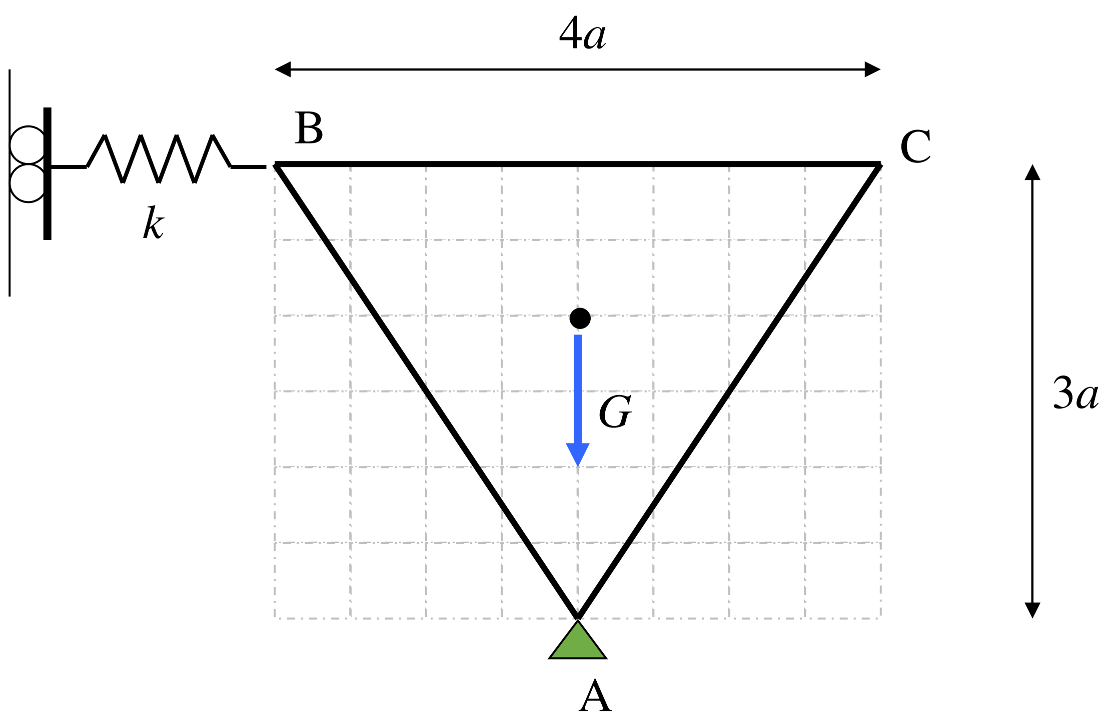
ASSIGNMENT 1



Question:

Find the critical load F_c

ASSIGNMENT 2



Question:

The minimum required spring stiffness k for a stable equilibrium?