Faculty
Civil Engineering and GeoSciences

Exam	**CIE4190**
	Slender Structures
Total number of pages	**8 pages (excl cover)**
Date and time	**JAN-26-2018** from **13:30-16:30**
Responsible lecturer	**J.W. Welleman**

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 3

☒ **questions may differ in weight** *(the time mentioned is an indicator for the weight)*

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with red pen or with pencils.
- Calculators with CAS and/or wifi/BT and/or PDF capabilities
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ **books** ☐ **notes** ☐ **dictionaries** ☐ **syllabus**

☐ **formula sheets (see also below under 'additional information')** ☒ **calculators**

☐ **computer** ☐ **...**

☒ **scientific (graphical)calculator** ☒ **drawing material**

Additional information (if necessary to be filled in by the examiner)

- **Use for each problem a separate examination paper**
- **The question form contains fomula sheets which can be used.**
- **Students can take the question form home after the exam.**
- **No student leaves without delivering an exam paper with a name on it!**

Exam graded by: (the marking period is 15 working days at most)

Every suspicion of fraud is reported to the Board of Examiners.

Mobile Phone OFF.

Problem 1 : Elementary cases and load carrying systems (45 min)

In figure 1 a cantilever shear-beam with span l is shown. The beam is supported at A and loaded with a concentrated load F at the free end in B. The effective shear stiffness is a function of x with constants b and k_o as is also indicated in figure 1 (with b and $k_o \geq 0$) .

Questions will deal with the continuous modelling of this beam but also with the derivation of an equivalent (discrete) element which can be used in the matrix method. A symbolic representation of an element between nodes i and j is also shown in the figure.

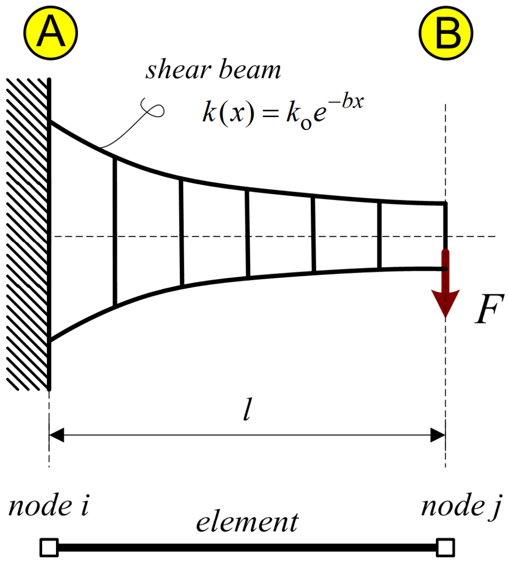


Figure 1 : Shear-beam

NOTE:
Only describe the required relations to obtain the governing equations. You do not have to prove obvious prerequisite knowledge.

Questions:

a) Describe the (continuous) behavior of this beam in your own coordinate system and specify all required equations/relations to find its displacement field for the presented load case.
(introduce all the expressions and explain them briefly, do not solve the problem)

b) Find, based upon your answer under a) an expression for the displacement field.

c) Find the relation between the load F and the displacement at its point of application.

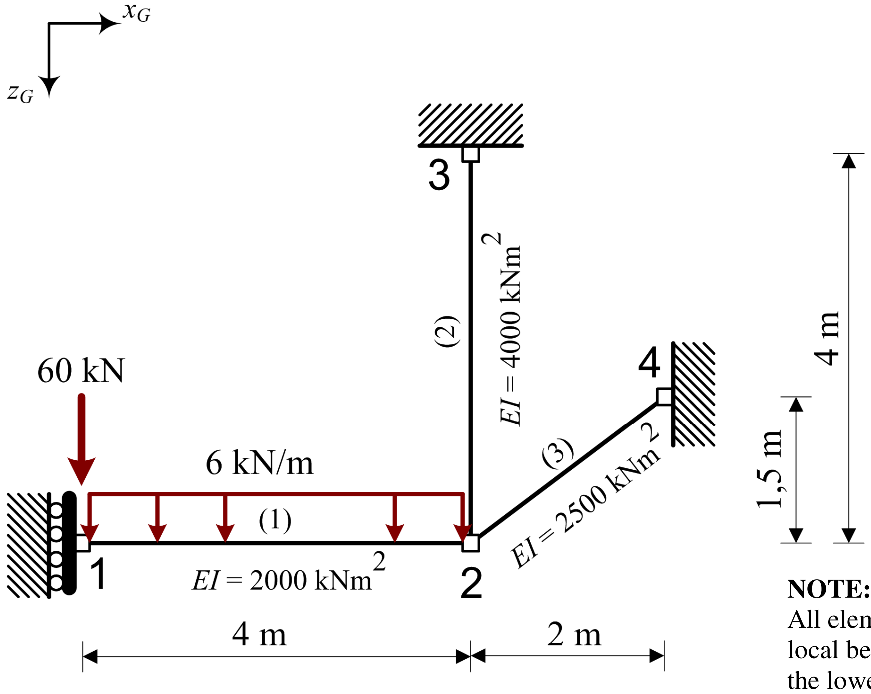
d) Describe the element model for the Matrix Method for this particular beam. (so what do you need to model this problem with the Matrix Method)

e) Derive the element stiffness matrix based upon your element description. Explain all steps involved.

Problem 2 : Matrix Method (45 min)

A planar frame structure is shown in figure 2. Only the possible rotations and displacements at the nodes as well as the moment distribution are of interest in an analysis based on the Matrix Method (hint: use this to reduce your element definitions).

The frame consists of three elements which are rigidly connected at node 2. Node 1 is fixed for rotation but is allowed to move in the global z_G-direction. At node 3 and 4 all displacements and rotations are constrained. A concentrated load of 60 kN is applied at node 1 and element (1) is loaded with a constant distributed load of 6 kN/m as indicated in the figure. For each element the bending stiffness is shown in the figure. Any axial and shear deformation of the elements is neglected.



NOTE:
All elements (e) have their local beam axis running from the lowest nodal number i to the highest nodal number j. Possible degrees of freedom are counted from node 1 in order of x, z and φ. If needed use the formula sheet.

Figure 2 : Frame structure

Questions:

a) Describe in a few lines the essence of the Matrix Method. You can use this problem as an example to support your description. Clearly show how the unknowns in this method are handled.

b) Model this problem with the *minimum number of required degrees of freedom* per element and specify for each element the connected degrees of freedom. Specify also the complete element description (stiffness matrix and the equivalent element load vector). Pay attention to the possible movements of the nodes and make use of this!

c) Setup the complete system of equations for this model. Clearly show the unknowns in this system.

d) Solve the system displacement vector and find the support reaction(s) at node 1.

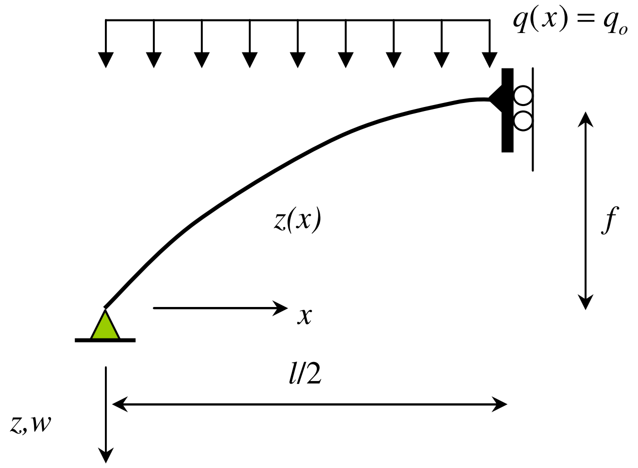
e) Describe in a few lines the limitation(s) of your reduced model.

Problem 3 : Arch (45 min)

An arch structure loaded with a constant distributed load q_o is shown in figure 3. The geometry of this arch can be described with a sine function:

$$z(x) = -f \sin\left(\frac{\pi x}{l}\right)$$

The arch is supported at the origin with a hinged support and at the other end with a vertical clamped roller which prevents a rotation and horizontal displacement of the arch. The axial deformation of the arch is neglected.



Given parameters: $f = 15$ m q = 5 kN/m
$l = 80$ m EI-arch = 150000 kNm^2

Figure 3 : Arch structure

Questions:

a) Describe in a few lines the two solution techniques for arches which were addressed in this course. Clearly show the limitations and possible assumptions. Also demonstrate which of the two methods is most favourable for the given problem if the moment distribution in the arch is asked for.

b) Find for the presented load case, the horizontal reaction at the roller. Use if needed, the math tools from the formula sheet on page 4 of this exam paper.

c) Find for the presented load case, the moment distribution in the arch. Use if needed, the math tools from the formula sheet on page 4 of this exam paper.

d) Sketch the moment distribution and specify the value of the moment in the arch at the end with the roller. Also show with deformation symbols the "sign" of the bending moment.

FORMULAS

Temperature:

$$N = EA(\varepsilon - \varepsilon^T); \qquad M = EI(\kappa - \kappa^T)$$

$$\varepsilon^T = \frac{\alpha}{A}\int_A T(x,z)\mathrm{d}A; \quad \kappa^T = \frac{\alpha}{I}\int_A zT(x,z)\mathrm{d}A$$

Stress distributions in beams:

$$\sigma(z) = \frac{N}{A} + \frac{Mz}{I}$$

$$\tau(z) = \frac{6V\left(\frac{1}{4}h^2 - z^2\right)}{bh^3}$$ rectangular crosssections

Arch:

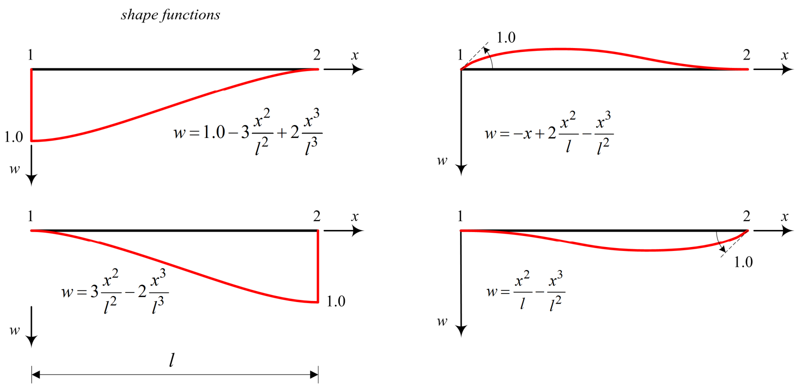
$$H = -\frac{\int_{arch} \frac{M^a z}{EI}\mathrm{d}x}{\int_{arch}\frac{z^2}{EI}\mathrm{d}x + \frac{l}{EA}}$$

Cable:

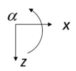
$$H^2 = \frac{q^2 l^3}{24\Delta};$$

$$f = \sqrt{\frac{3}{8} l \Delta} \text{ or } \Delta = \frac{8f^2}{3l}$$

Matrix Methods



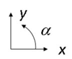
$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$



$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$\bar{f} = R\, f$ and $f = R^T \bar{f}$

$\bar{u} = R\, u$ and $u = R^T \bar{u}$



$$R = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Math tools:

$$y = Ae^{-\beta x}\sin(\beta x + \omega)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{2}\,\beta A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{1}{4}\pi\right)$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\beta^2 A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{1}{2}\pi\right)$$

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = -2\sqrt{2}\beta^3 A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{3}{4}\pi\right)$$

$$\int\sqrt{1+x^2}\,\mathrm{d}x \approx \int\left[1 + \frac{1}{2}\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2\right]\mathrm{d}x$$

$$\int_0^a \sin^2\frac{\pi x}{a}\mathrm{d}x = \frac{a}{2} \qquad \int_0^a x\sin\frac{\pi x}{a}\mathrm{d}x = \frac{a^2}{\pi}$$

$$\int_0^a x^2 \sin\frac{\pi x}{a}\mathrm{d}x = \frac{a^3\left(\pi^2 - 4\right)}{\pi^3}$$

catenary solution:

$$z = -\frac{H}{q}\cosh\left(-\frac{qx}{H} + C_1\right) + C_2$$

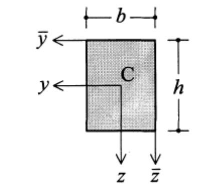
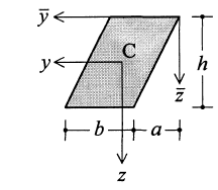
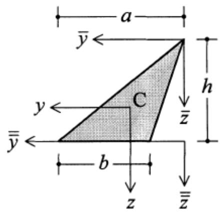
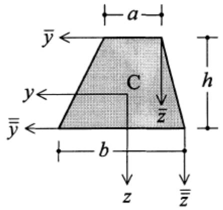
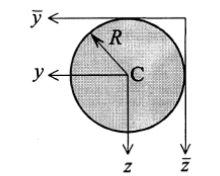
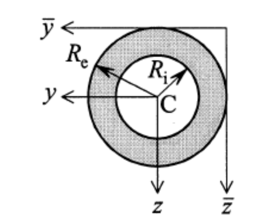
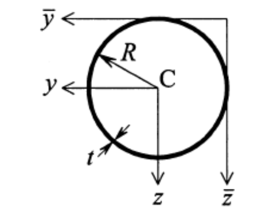
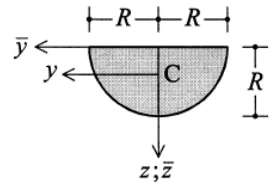
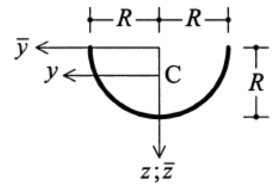
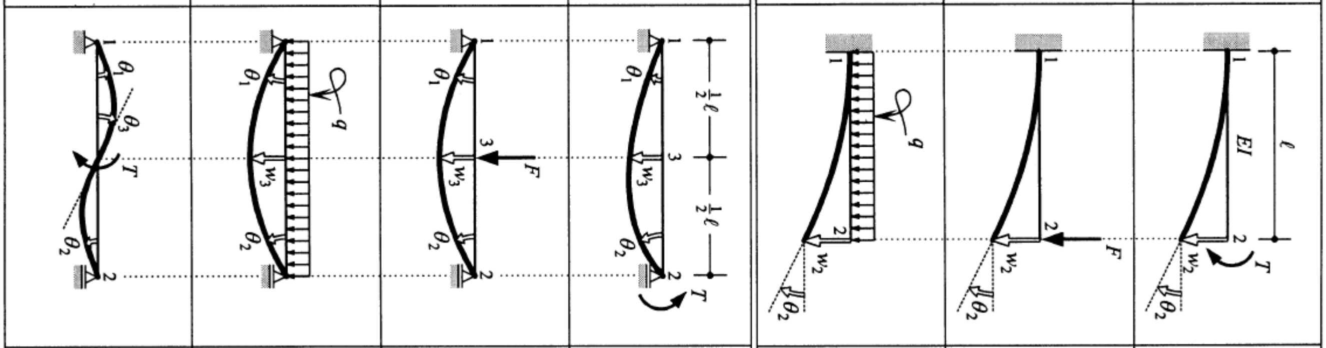
Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Rectangle $A = bh$ $\bar{y}_C = \frac{1}{2}b$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}b^3h$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = \frac{1}{3}b^3h$ $I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{4}b^2h^2$
	Parallelogram $A = bh$ $\bar{y}_C = \frac{1}{2}(a+b)$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}(a^2+b^2)bh$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = \frac{1}{12}abh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$
	Triangle $A = \frac{1}{2}bh$ $\bar{y}_C = \frac{1}{3}(2a-b)$ $\bar{z}_C = \frac{2}{3}h$	$I_{yy} = \frac{1}{36}(a^2-ab+b^2)bh$ $I_{zz} = \frac{1}{36}bh^3$ $I_{yz} = \frac{1}{72}(2a-b)bh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{4}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{8}(2a-b)bh^2$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}bh^3$
	Trapezium $A = \frac{1}{2}(a+b)h$ $\bar{z}_C = \frac{1}{3}\frac{a+2b}{a+b}h$	$I_{zz} = \frac{1}{36}\frac{a^2+4ab+b^2}{a+b}h^3$	$I_{\bar{z}\bar{z}} = \frac{1}{12}(a+3b)h^3$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}(3a+b)h^3$
	Circle $A = \pi R^2$	$I_{yy} = I_{zz} = \frac{1}{4}\pi R^4$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi R^4$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{5}{4}\pi R^4$ $I_{\bar{y}\bar{z}} = \pi R^4$

Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Thick-walled ring $A = \pi(R_e^2 - R_i^2)$	$I_{yy} = I_{zz} = \frac{1}{4}\pi(R_e^4 - R_i^4)$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi(R_e^4 - R_i^4)$	
	Thin-walled ring $A = 2\pi Rt$	$I_{yy} = I_{zz} = \pi R^3 t$ $I_{yz} = 0$ $I_p = 2\pi R^3 t$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = 3\pi R^3 t$
	Semicircle $A = \frac{1}{2}\pi R^2$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{4}{3\pi}R$ $= 0.424R$	$I_{yy} = \frac{1}{8}\pi R^4 = 0.393R^4$ $I_{zz} = (\frac{\pi}{8} - \frac{8}{9\pi})R^4$ $= 0.110R^4$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{8}\pi R^4$ $I_{\bar{y}\bar{z}} = 0$
	Semicircular ring $A = \pi Rt$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{2}{\pi}R$ $= 0.637R$	$I_{yy} = \frac{1}{2}\pi R^3 t$ $I_{zz} = (\frac{\pi}{2} - \frac{4}{\pi})R^3 t$ $= 0.298R^3 t$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{2}\pi R^3 t$ $I_{\bar{y}\bar{z}} = 0$

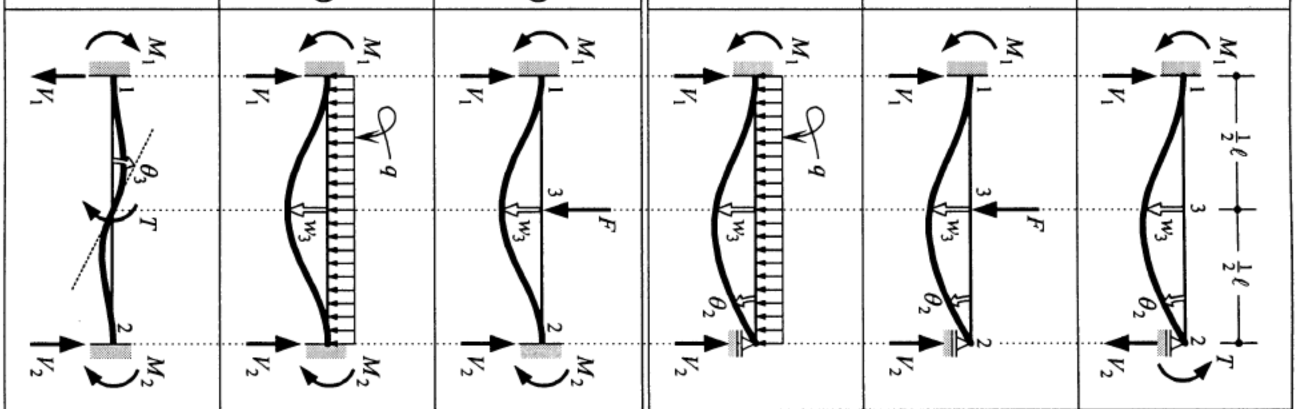


forget-me-nots

(1)	$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)	$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)	$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

simply supported beam (statically determinate)

(4)	$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)	$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

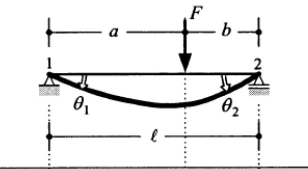
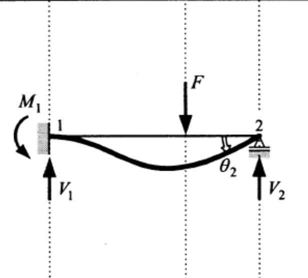
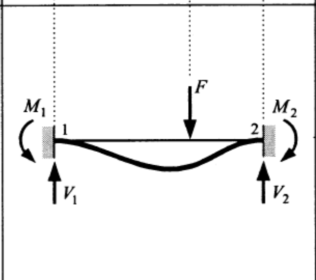
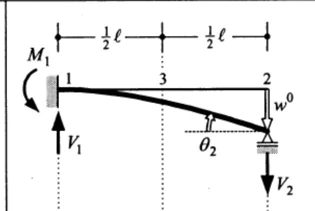
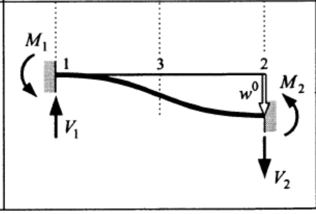


statically indeterminate beam (one fixed end)

(7)	$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$; $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)	$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$; $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)	$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$; $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statically indeterminate beam (two fixed ends)

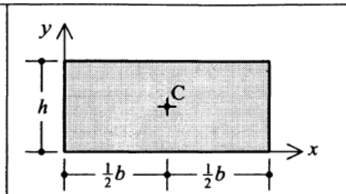
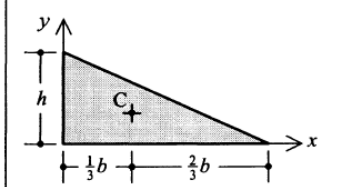
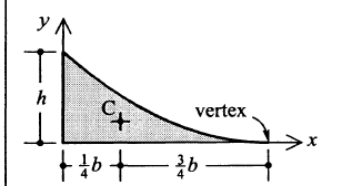
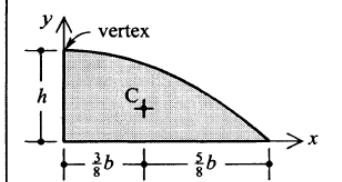
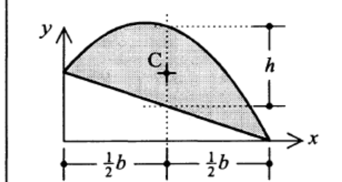
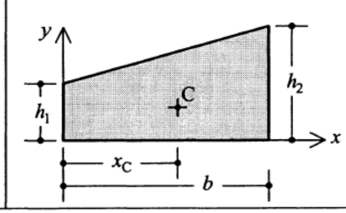
(10)	$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$; $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)	$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$; $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)	$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$; $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

(c)		$\theta_1 = \dfrac{Fab(\ell+b)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(2\dfrac{a}{\ell} - 3\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fab(\ell+a)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(\dfrac{a}{\ell} - \dfrac{a^3}{\ell^3}\right)$
(d)		$M_1 = \dfrac{Fb(\ell^2-b^2)}{2\ell^2} = F\ell\left(\dfrac{a}{\ell} - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb(3\ell^2-b^2)}{2\ell^3} = F\left(1 - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(3\ell-a)}{2\ell^3} = F\left(\dfrac{3}{2}\dfrac{a^2}{\ell^2} - \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fa^2b}{4EI\ell} = \dfrac{F\ell^2}{4EI}\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$
(e)		$M_1 = \dfrac{Fab^2}{\ell^2} = F\ell\left(\dfrac{a}{\ell} - 2\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb^2(\ell+2a)}{\ell^3} = F\left(1 - 3\dfrac{a^2}{\ell^2} + 2\dfrac{a^3}{\ell^3}\right)$ $M_2 = \dfrac{Fa^2b}{\ell^2} = F\ell\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(\ell+2b)}{\ell^3} = F\ell\left(3\dfrac{a^2}{\ell^2} - 2\dfrac{a^3}{\ell^3}\right)$
(f)		$M_1 = \dfrac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{3EI}{\ell^3}w^0$ $\theta_2 = \dfrac{3}{2}\dfrac{w^0}{\ell}$ $\theta_3 = \dfrac{9}{8}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{5}{16}w^0$
(g)		$M_1 = M_2 = \dfrac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{12EI}{\ell^3}w^0$ $\theta_3 = \dfrac{3}{2}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{1}{2}w^0$

support reactions and rotations at the beam ends (c)–(e)

settlements (f)–(g)

properties of plane figures to be used for the moment-area theorems

(1)		rectangle: $y = h$ $A = bh$ $x_C = \frac{1}{2}b$
(2)		triangle: $y = h\left\{1 - \dfrac{x}{b}\right\}$ $A = \frac{1}{2}bh$ $x_C = \frac{1}{3}b$
(3)		parabola: $y = h\left\{1 - \dfrac{x}{b}\right\}^2$ $A = \frac{1}{3}bh$ $x_C = \frac{1}{4}b$
(4)		parabola: $y = h\left\{1 - \left(\dfrac{x}{b}\right)^2\right\}$ $A = \frac{2}{3}bh$ $x_C = \frac{3}{8}b$
(5)		parabola: $A = \frac{2}{3}bh$ $x_C = \frac{1}{2}b$
(6)		trapezium: $y = h_1 + (h_2 - h_1)\dfrac{x}{b}$ $A = \frac{1}{2}b(h_1 + h_2)$ $x_C = \frac{1}{3}b\dfrac{h_1 + 2h_2}{h_1 + h_2}$