Faculty
Civil Engineering and GeoSciences

Exam	**CIE4190**
	Slender Structures
Total number of pages	**8 pages (excl cover)**
Date and time	**NOV-08-2017** from **13:30-16:30**
Responsible lecturer	**J.W. Welleman**

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 4

☒ **questions may differ in weight** *(the time mentioned is an indicator for the weight)*

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with red pen or with pencils.
- Calculators with CAS and/or wifi/BT and/or PDF capabilities
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ **books** ☐ **notes** ☐ **dictionaries** ☐ **syllabus**

☐ **formula sheets (see also below under 'additional information')** ☒ **calculators**

☐ **computer** ☐ **...**

☒ **scientific (graphical)calculator** ☒ **drawing material**

Additional information (if necessary to be filled in by the examiner)

- **Use for each problem a separate examination paper**
- **The question form contains fomula sheets which can be used.**
- **Students can take the question form home after the exam.**
- **No student leaves without delivering an exam paper with a name on it!**

Exam graded by: (the marking period is 15 working days at most)

Every suspicion of fraud is reported to the Board of Examiners.

Mobile Phone OFF.

Problem 1 : Elementary cases and load carrying systems (45 min)

In figure 1 a structure is shown which consists of two linked beams. The beams are both fully fixed at the support A. The two beams are linked to each other by a rigid *two-force* member at B. The structure is loaded at B by a concentrated load F. The axial deformation is not taken into account.

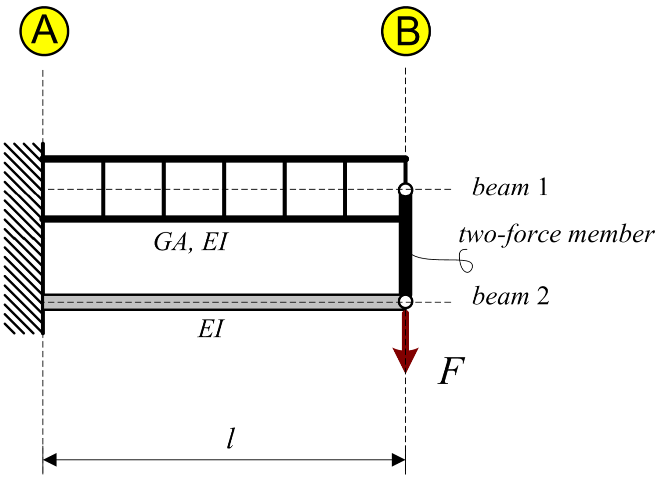


Figure 1 : Linked beams

NOTE:
Only describe the required relations to obtain the governing equations. You do not have to prove obvious prerequisite knowledge such as $A = \int_A dA;\ S = \int_A z dA;\ I = \int_A z^2 dA;$ etc.

Questions:

a) Which of the two beams 1 and or 2 is also referred to as a Navier beam?

b) Describe the behavior of each of the two beams and derive the required equations/relations to find the displacement field for each beam in its current situation. (introduce a coordinate system and the expressions used and explain them briefly)

c) Find, based upon your answer under b) an expression for the normal force in the rigid *two-force* link which connects the two beams and investigate the result for different values of the stiffness parameters of the beams.

d) Find the relation between the load F and the displacement at its point of application.

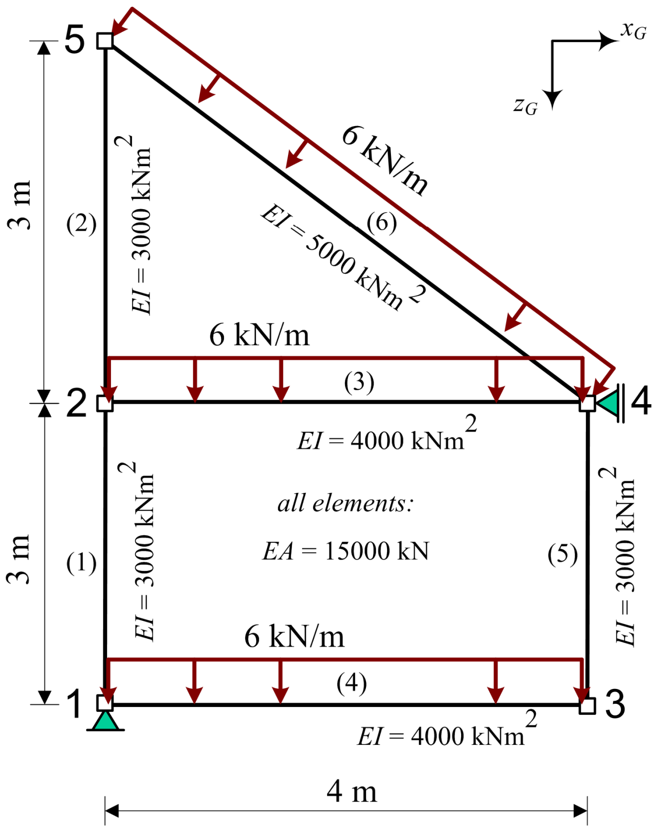
e) Derive this latter result also based upon a similar model with discrete springs and explain all steps involved.

Problem 2 : Matrix Method (45 min)

A frame structure as shown in figure 2 has been analysed with the Matrix Method. Use the specified numbers for the nodes and elements from figure 2. For educational reasons the structure is only supported at node 1 with a pin and at node 4 with a pinned roller.

The horizontal degree of freedom at node 4 is connected to the stiffness components of the system stiffness matrix row according to :

$$[0 \quad 0 \quad 0 \quad -3750 \quad 0 \quad 0 \quad -1333.333 \quad 0 \quad 2000 \quad 7176.133 \quad 1209.600 \quad 1280 \quad -2092.800 \quad -1209.600 \quad -720]$$



From the analysis the following results have been obtained:

DOF		DISPLACEMENTS
dof	1	0
dof	2	0
dof	3	-.00839
dof	4	.00455
dof	5	.00890
dof	6	-.00608
dof	7	-.00430
dof	8	.04951
dof	9	-.00604
dof	10	0
dof	11	.05021
dof	12	-.00489
dof	13	.02574
dof	14	.01250
dof	15	-.01096

NOTE:
All elements (e) have their local beam axis running from the lowest nodal number i to the highest nodal number j. Possible degrees of freedom are counted from node 1 in order of x, z and φ. If needed use the formula sheet.

Figure 2 : Frame structure

Questions:

a) Describe in a few lines the essence of the approach according to the Matrix Method. You can use this problem as an example to support your description. Clearly show how the unknowns in this method are handled.

b) Specify the input for the element loads (per element) and explain your answer briefly.

c) Set up the system load vector and show all steps required to obtain this vector.

d) Find the support reaction at node 4 with the specified data and check this result.

e) Show the origin of the stiffness components -3750 and 2000 from the specified data line of the system stiffness matrix and check its number.

Problem 3 : Cable (45 min)

A cable with a specific length is loaded with a single concentrated vertical load as shown in figure 3. The length of the cable is fixed to $(1+\beta)l$ in which β is a positive number. The deformation of the cable due to axial loading is neglected.

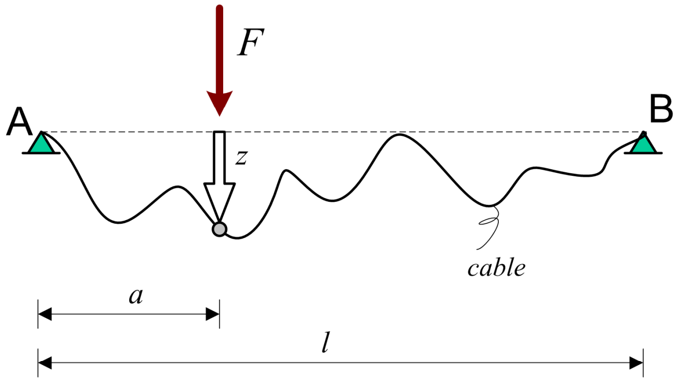


Figure 3 : Cable

An exact solution for this situation results in a "simple" expression for the horizontal component H of the force in the cable :

$$H = \frac{2(1+\beta)}{\sqrt{-8a^2\beta - 4a^2\beta^2 + 8al\beta + 4al\beta^2 + 4l^2\beta^2 + 4l^2\beta^3 + l^2\beta^4}} \frac{Fa(l-a)}{l}$$

Questions:

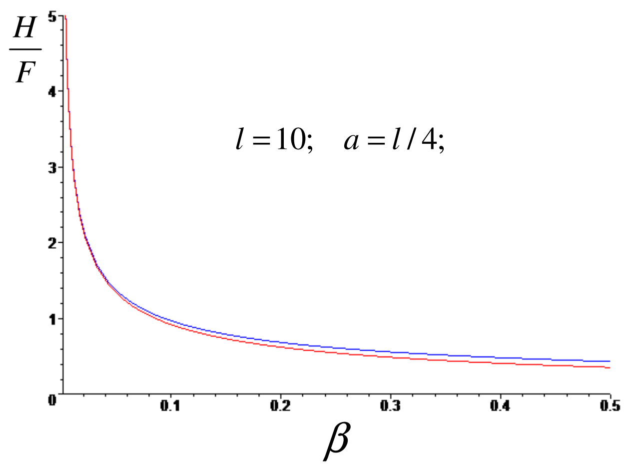
a) "In case of multiple concentrated loads on this cable, the expression given can be used to find the equilibrium position of the cable since in this case H is linear related to F. " What is your professional opinion on this statement, support your answer if needed with a sketch.

b) Find for the presented load case, the exact expression for z.

c) Explain how the expression shown for H can be found. Use the parameters shown, make sure all required equations are explained but do not solve these to obtain the presented expression (you can do that at home …)

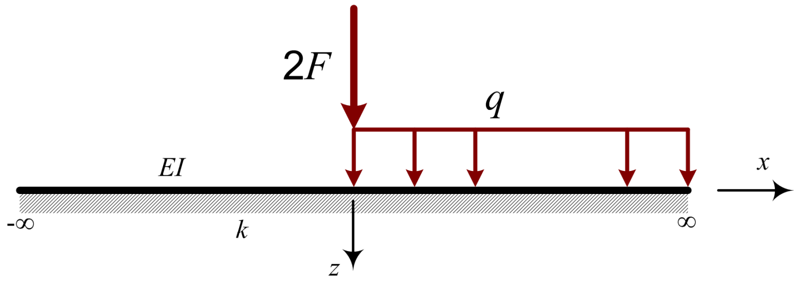
d) Find an alternative (approximate) expression for H by using simplifications which are common use in cable analysis.

e) With your expression a typical solution for H/F for various values of β can be presented as is shown in the graph. Which of the two graphs represents the approximate solution and for what reason?



Problem 4 : Continuous elastic supports (45 min)

A beam on an elastic foundation is loaded with a concentrated load $2F$ and a distributed load q for $x > 0$ only as can be depicted from figure 4.



Given : $EI = 5000\ \text{kNm}^2$; $k = \frac{625}{8}\pi^4$; $F = 100$ kN; $q = 5$ kN/m;

Figure 4 : Beam on an elastic foundation

NOTE:
Only describe the required relations to obtain the governing equations. You do not have to prove obvious prerequisite knowledge such as $A = \int_A dA;\ S = \int_A z dA;\ I = \int_A z^2 dA;$ etc.

Questions:

a) Derive the governing ODE for an Euler-beam on an elastic foundation. Show clearly the effect of the distributed load in all equations.

Let us assume that we all know the nature of the homogeneous solution for this ODE as: $w_h(x) = e^{-\beta x}\left(C_1 \cos \beta x + C_2 \sin \beta x\right) + e^{\beta x}\left(C_3 \cos \beta x + C_4 \sin \beta x\right)$

b) Find the “wave length” of this beam and explain its meaning.

c) Find the solution for the displacement w at $x = 0$. Explain your solution strategy and support this with sketches.

d) Find the solution for the moment M at $x = 0$. Explain your solution strategy and support this with sketches.

e) Find the bending moment M at $x = 4.0$ m. Explain your solution strategy and support this with sketches.

FORMULAS

Temperature:

$$N = EA(\varepsilon - \varepsilon^T); \qquad M = EI(\kappa - \kappa^T)$$

$$\varepsilon^T = \frac{\alpha}{A}\int_A T(x,z)\mathrm{d}A; \quad \kappa^T = \frac{\alpha}{I}\int_A zT(x,z)\mathrm{d}A$$

Stress distributions in beams:

$$\sigma(z) = \frac{N}{A} + \frac{Mz}{I}$$

$$\tau(z) = \frac{6V\left(\frac{1}{4}h^2 - z^2\right)}{bh^3} \quad \text{rectangular crosssections}$$

Arch:

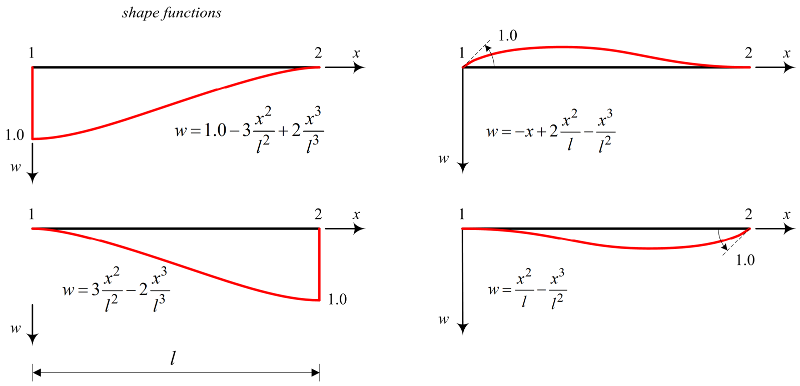
$$H = -\frac{\int_{arch} \frac{M^a z}{EI}\mathrm{dx}}{\int_{arch} \frac{z^2}{EI}\mathrm{dx} + \frac{l}{EA}}$$

Cable:

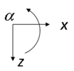
$$H^2 = \frac{q^2 l^3}{24\Delta};$$

$$f = \sqrt{\frac{3}{8} l\Delta} \text{ or } \Delta = \frac{8f^2}{3l}$$

Matrix Methods



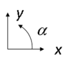
$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} & 0 & -\frac{12\,EI}{L^3} & -\frac{6\,EI}{L^2} \\ 0 & -\frac{6\,EI}{L^2} & \frac{4\,EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{2\,EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} & 0 & \frac{12\,EI}{L^3} & \frac{6\,EI}{L^2} \\ 0 & -\frac{6\,EI}{L^2} & \frac{2\,EI}{L} & 0 & \frac{6\,EI}{L^2} & \frac{4\,EI}{L} \end{bmatrix}$$



$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\bar{f} = R\,f \quad \text{and} \quad f = R^T\bar{f}$$

$$\bar{u} = R\,u \quad \text{and} \quad u = R^T\bar{u}$$



$$R = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Math tools:

$$y = Ae^{-\beta x}\sin(\beta x + \omega)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{2}\,\beta Ae^{-\beta x}\sin(\beta x + \omega - \tfrac{1}{4}\pi)$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\beta^2 Ae^{-\beta x}\sin(\beta x + \omega - \tfrac{1}{2}\pi)$$

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = -2\sqrt{2}\,\beta^3 Ae^{-\beta x}\sin(\beta x + \omega - \tfrac{3}{4}\pi)$$

$$\int\sqrt{1+x^2}\,\mathrm{d}x \approx \int\left[1 + \frac{1}{2}\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2\right]\mathrm{d}x$$

catenary solution:

$$z = -\frac{H}{q}\cosh\left(-\frac{qx}{H} + C_1\right) + C_2$$

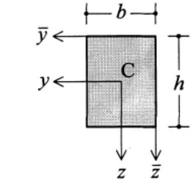
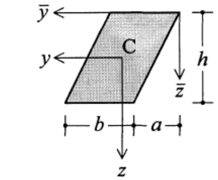
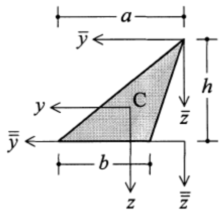
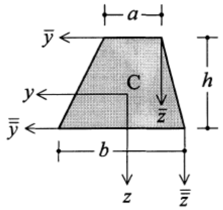
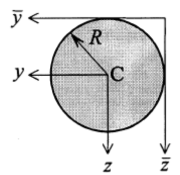
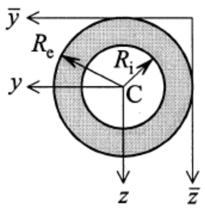
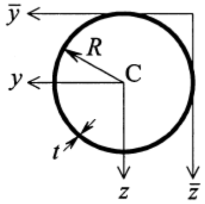
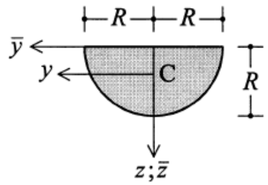
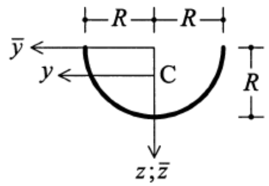
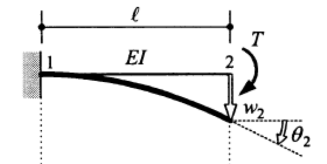
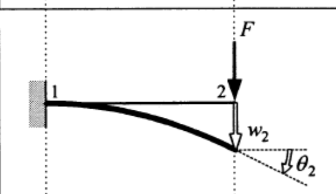
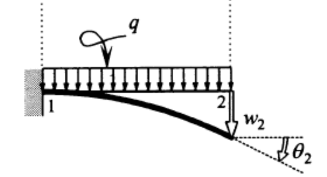
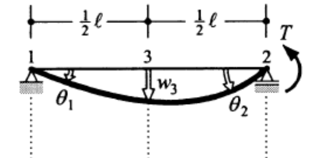
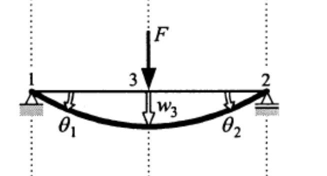
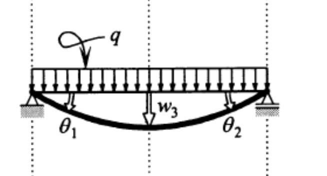
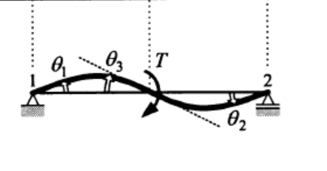
Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Rectangle $A = bh$ $\bar{y}_C = \frac{1}{2}b$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}b^3h$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = \frac{1}{3}b^3h$ $I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{4}b^2h^2$
	Parallelogram $A = bh$ $\bar{y}_C = \frac{1}{2}(a+b)$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}(a^2+b^2)bh$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = \frac{1}{12}abh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$
	Triangle $A = \frac{1}{2}bh$ $\bar{y}_C = \frac{1}{3}(2a-b)$ $\bar{z}_C = \frac{2}{3}h$	$I_{yy} = \frac{1}{36}(a^2-ab+b^2)bh$ $I_{zz} = \frac{1}{36}bh^3$ $I_{yz} = \frac{1}{72}(2a-b)bh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{4}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{8}(2a-b)bh^2$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}bh^3$
	Trapezium $A = \frac{1}{2}(a+b)h$ $\bar{z}_C = \frac{1}{3}\frac{a+2b}{a+b}h$	$I_{zz} = \frac{1}{36}\frac{a^2+4ab+b^2}{a+b}h^3$	$I_{\bar{z}\bar{z}} = \frac{1}{12}(a+3b)h^3$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}(3a+b)h^3$
	Circle $A = \pi R^2$	$I_{yy} = I_{zz} = \frac{1}{4}\pi R^4$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi R^4$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{5}{4}\pi R^4$ $I_{\bar{y}\bar{z}} = \pi R^4$

Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Thick-walled ring $A = \pi(R_e^2 - R_i^2)$	$I_{yy} = I_{zz} = \frac{1}{4}\pi(R_e^4 - R_i^4)$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi(R_e^4 - R_i^4)$	
	Thin-walled ring $A = 2\pi Rt$	$I_{yy} = I_{zz} = \pi R^3 t$ $I_{yz} = 0$ $I_p = 2\pi R^3 t$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = 3\pi R^3 t$
	Semicircle $A = \frac{1}{2}\pi R^2$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{4}{3\pi}R$ $= 0.424R$	$I_{yy} = \frac{1}{8}\pi R^4 = 0.393R^4$ $I_{zz} = (\frac{\pi}{8} - \frac{8}{9\pi})R^4$ $= 0.110R^4$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{8}\pi R^4$ $I_{\bar{y}\bar{z}} = 0$
	Semicircular ring $A = \pi Rt$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{2}{\pi}R$ $= 0.637R$	$I_{yy} = \frac{1}{2}\pi R^3 t$ $I_{zz} = (\frac{\pi}{2} - \frac{4}{\pi})R^3 t$ $= 0.298R^3 t$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{2}\pi R^3 t$ $I_{\bar{y}\bar{z}} = 0$

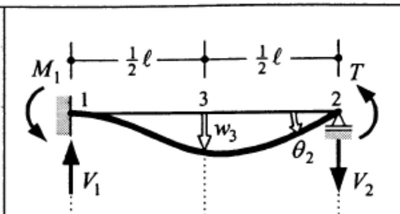
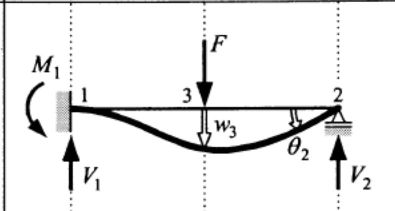
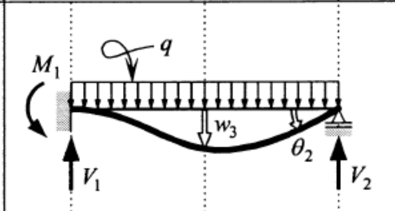
(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

forget-me-nots

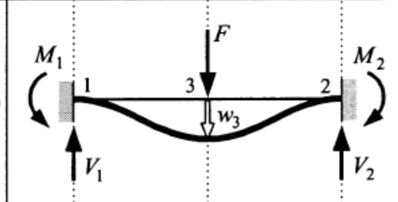
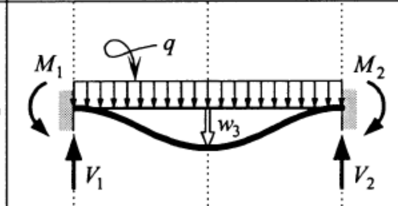
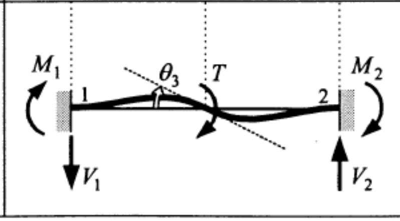
(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

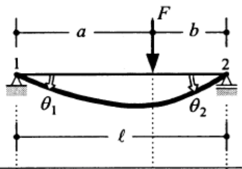
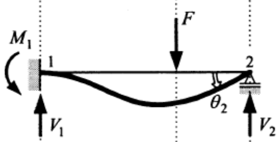
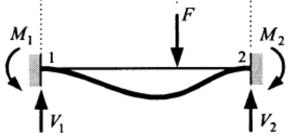
simply supported beam (statically determinate)

statically indeterminate beam (one fixed end)

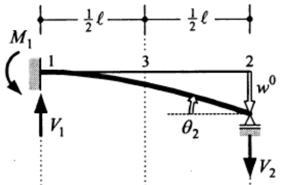
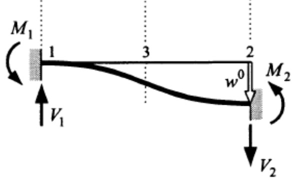
(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statically indeterminate beam (two fixed ends)

(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

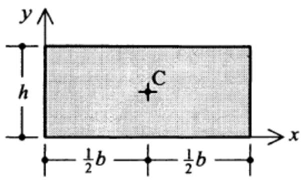
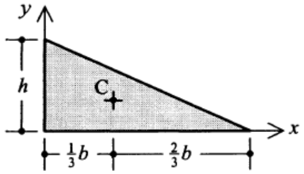
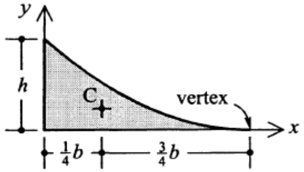
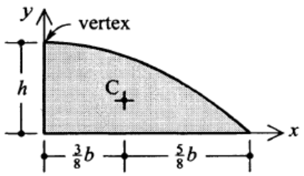
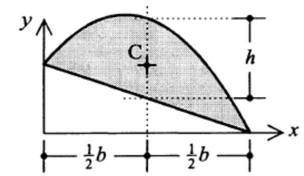
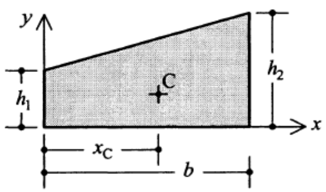
(c)		$\theta_1 = \dfrac{Fab(\ell+b)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(2\dfrac{a}{\ell} - 3\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fab(\ell+a)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(\dfrac{a}{\ell} - \dfrac{a^3}{\ell^3}\right)$
(d)		$M_1 = \dfrac{Fb(\ell^2-b^2)}{2\ell^2} = F\ell\left(\dfrac{a}{\ell} - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb(3\ell^2-b^2)}{2\ell^3} = F\left(1 - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(3\ell-a)}{2\ell^3} = F\left(\dfrac{3}{2}\dfrac{a^2}{\ell^2} - \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fa^2b}{4EI\ell} = \dfrac{F\ell^2}{4EI}\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$
(e)		$M_1 = \dfrac{Fab^2}{\ell^2} = F\ell\left(\dfrac{a}{\ell} - 2\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb^2(\ell+2a)}{\ell^3} = F\left(1 - 3\dfrac{a^2}{\ell^2} + 2\dfrac{a^3}{\ell^3}\right)$ $M_2 = \dfrac{Fa^2b}{\ell^2} = F\ell\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(\ell+2b)}{\ell^3} = F\ell\left(3\dfrac{a^2}{\ell^2} - 2\dfrac{a^3}{\ell^3}\right)$

support reactions and rotations at the beam ends

(f)		$M_1 = \dfrac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{3EI}{\ell^3}w^0$ $\theta_2 = \dfrac{3}{2}\dfrac{w^0}{\ell}$ $\theta_3 = \dfrac{9}{8}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{5}{16}w^0$
(g)		$M_1 = M_2 = \dfrac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{12EI}{\ell^3}w^0$ $\theta_3 = \dfrac{3}{2}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{1}{2}w^0$

settlements

properties of plane figures to be used for the moment-area theorems

(1)		rectangle: $y = h$ $A = bh$ $x_C = \frac{1}{2}b$
(2)		triangle: $y = h\left\{1 - \dfrac{x}{b}\right\}$ $A = \frac{1}{2}bh$ $x_C = \frac{1}{3}b$
(3)		parabola: $y = h\left\{1 - \dfrac{x}{b}\right\}^2$ $A = \frac{1}{3}bh$ $x_C = \frac{1}{4}b$
(4)		parabola: $y = h\left\{1 - \left(\dfrac{x}{b}\right)^2\right\}$ $A = \frac{2}{3}bh$ $x_C = \frac{3}{8}b$
(5)		parabola: $A = \frac{2}{3}bh$ $x_C = \frac{1}{2}b$
(6)		trapezium: $y = h_1 + (h_2 - h_1)\dfrac{x}{b}$ $A = \frac{1}{2}b(h_1 + h_2)$ $x_C = \frac{1}{3}b\dfrac{h_1 + 2h_2}{h_1 + h_2}$