Faculty
Civil Engineering and GeoSciences

Exam	**CIE4190**
	Slender Structures
Total number of pages	**9 pages (excl cover)**
Date and time	**NOV-6-2019** from **13:30-16:30**
Responsible lecturer	**J.W. Welleman**

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of main topics with questions: 3

☒ **questions may differ in weight** *(the time mentioned is an indicator for the weight)*

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with red pen or with pencils.
- Calculators with CAS and/or wifi/BT and/or PDF capabilities
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ **books** ☐ **notes** ☐ **dictionaries** ☐ **syllabus**

☐ **formula sheets (see also below under 'additional information')** ☒ **calculators**

☐ **computer** ☐ **…**

☒ **scientific (graphical)calculator** ☒ **drawing material**

Additional information (if necessary to be filled in by the examiner)

- **Use for each problem a separate examination paper**
- **The question form contains fomula sheets which can be used.**
- **Students can take the question form home after the exam.**
- **No student leaves without delivering an exam paper with a name on it!**

Exam graded by: (the marking period is 15 working days at most)

Every suspicion of fraud is reported to the Board of Examiners.

Mobile Phone OFF.

Topic 1 : MatrixMethod
part 1.1 : Theory (25 min)

In fig. 1 a shear beam element with shear stiffness k is shown loaded with a linear distributed element load. This element has only two degrees of freedom which are the transverse displacements at both ends of the element.

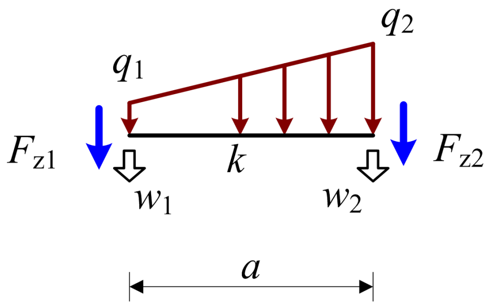


Figure 1 : Shear element with distributed load

Questions will deal with the discrete solving technique with the MatrixMethod.

Questions:

a) Derive, with a method of your choice, the element stiffness matrix.

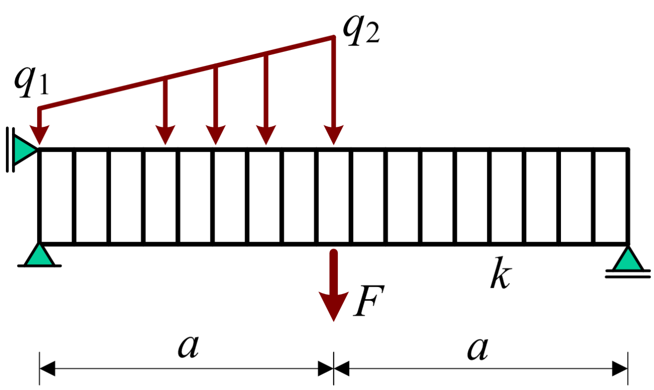
b) Derive, with a method of your choice, the required expressions to model the distributed load.

Topic 1 : MatrixMethod

part 1.2 : System **(30 min)**

A shear beam is loaded with a concentrated load F at mid span and a linear distributed load $q(x)$ as indicated in fig. 2. The shear beam is supported such that no rotation of the cross section can occur. The distributed load $q(x)$ increases from 15 to 30 kN/m and the concentrated load is 75 kN. The total span of the beam is 4 m.

This problem is modelled with the MatrixMethod and to reduce complexity, any axial deformation is neglected.



Given : $k = 1000$ kN; $a = 2$ m; $F = 75$ kN; $q_1 = 15$ kN/m; $q_2 = 30$ kN/m;

Figure 2 : Shear beam

Questions:

a) Setup the system of equations for this model. Clearly show the *unknowns* and *knowns* in this system.

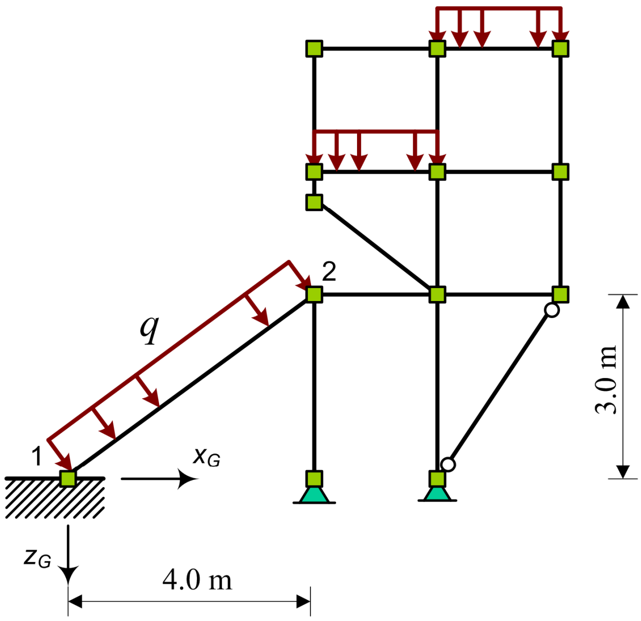
> Hint: In case you have difficulties with part 1.1 just use your sound engineering judgement for the element description.

b) Solve all unknowns and sketch the deformed beam with its maximum displacement.

c) Find the shear force directly to the left and to the right of the concentrated load F.

Topic 1 : MatrixMethod
part 1.3 : Element behavior (25 min)

In the frame structure of fig. 3 the nodal degree of freedoms are u, w and φ which represent the displacement in the global x and z direction and the rotation around the y-axis. The displacements of node 2 are given as (0.05; 0.10; 0.005). At node 1 the structure is fully clamped.



Given : units kN, m; q = 12 kN/m;

Figure 3 : Frame structure

The element between node 1 and 2 has an element stiffness matrix in the local coordinate system:

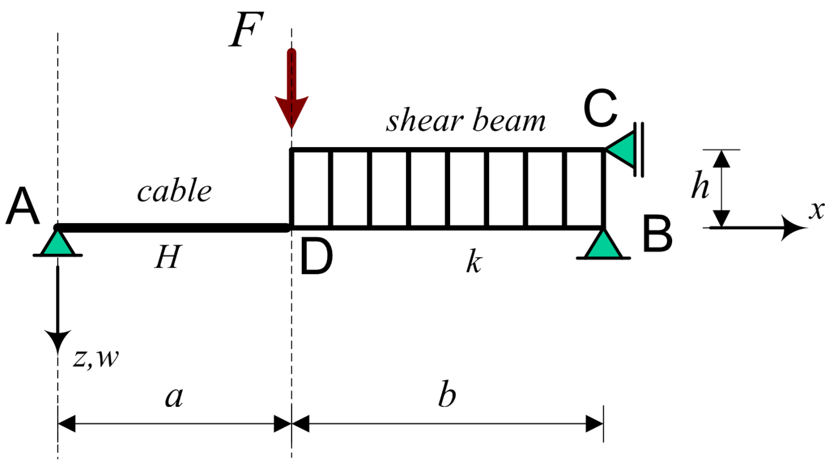
$$\begin{bmatrix} 1000 & 0 & 0 & -1000 & 0 & 0 \\ 0 & 240 & -600 & 0 & -240 & -600 \\ 0 & -600 & 2000 & 0 & 600 & 1000 \\ -1000 & 0 & 0 & 1000 & 0 & 0 \\ 0 & -240 & 600 & 0 & 240 & 600 \\ 0 & -600 & 1000 & 0 & 600 & 2000 \end{bmatrix}$$

Questions:

a) Find the *element force* vector for the element between node 1 and 2 and clearly show the direction of these element forces.
b) Sketch the moment distribution in kNm for this element and specify the values at characteristic points including deformation symbols or signs.

Topic 2 : Combining basic cases (50 min)

The shear beam in fig. 4 is linked to a cable at point D. The horizontal force in the cable is denoted with H and is given as 8000 kN. The shear beam with shear stiffness k is supported at B and C. The shear beam is constructed such that axial loads can be taken without axial deformation. The depth of the shear beam is h. The concentrated load is applied to the shear beam at D. In fig. 4 the structure is drawn in its original (unloaded) position. Assume a rigid cable which fits in the deformed position.



Given : a = 3 m; **:** b = 5 m; h = 0,8 m; F = 22 kN; k = 1000 kN; H = 8000 kN

Figure 4 : Shear beam with cable

Questions:

a) Describe in a few lines the load carrying capacity of this structure and define the unknowns in your strategy to solve the displacements and forces.

b) Derive all equations needed to obtain the unknowns.

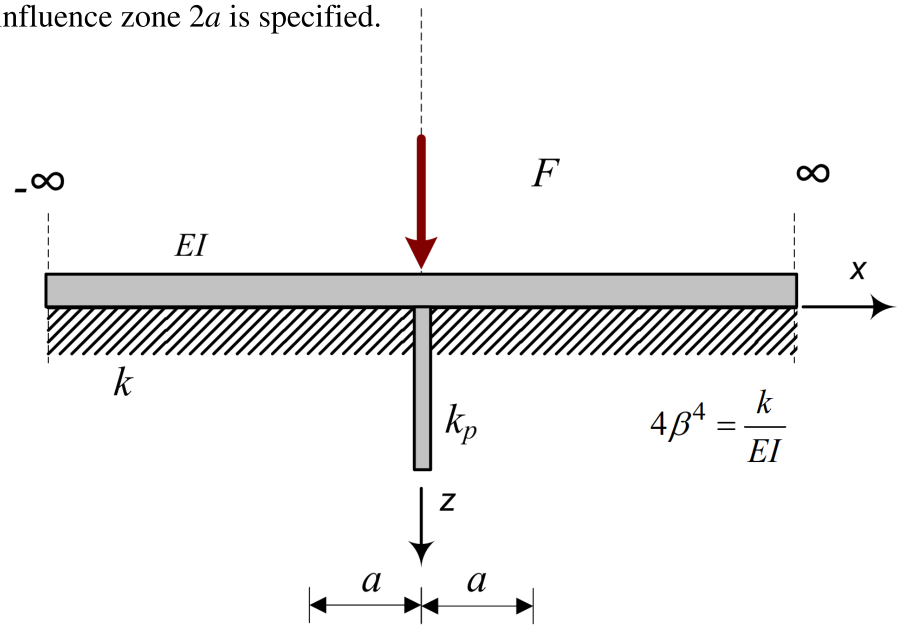
c) Solve the set of equations and present the displacement at D.

d) Sketch the distribution of the vertical force in the cross section along the x-axis.

e) Find all support reactions and clearly show the actual direction of these reactions.

Topic 3 : Beam on an elastic foundation (50 min)

A beam with bending stiffness *EI* on an elastic foundation *k* is sketched in fig. 5. The beam is loaded with a concentrated load. At the point of application of the load the beam is supported with a foundation pile. The pile-beam connection can be regarded as a hinge. The stiffness of the pile (including soil influence) is simplified as a spring with stiffness k_p. In fig. 5 an influence zone $2a$ is specified.



Given : $F = 100\ kN;\ EI = 2500\ \text{kNm}^2;\ k = 2\pi^4\ \text{kN/m}^2;\ k_p = 1000\ \text{kN/m};$

Figure 5 : Beam on elastic foundation, supported with foundation pile.

Questions:

a) Describe in a few lines the load carrying capacity of this system and how you can model this problem with the classical displacement method. You do not have to derive the ODE's for intermediate cases but you have to explain relevant data and equations.

b) The influence zone *a* most likely will need some attention during engineering. Explain briefly why and can you estimate the length of *a*?

c) Setup all necessary equations to solve this problem based on your outlined strategy. Strong advice : express all equations in terms of F, β, k and k_p.

d) Solve the unknowns and sketch the displacement field for the specified values. Put the values of the displacement for $x = 0$ and $x = a$ in the graph.

e) How would you judge the effectiveness of the foundation pile and quantify this with a number. Clearly explain your definition of "effectiveness".

FORMULAS

Temperature:

$$N = EA(\varepsilon - \varepsilon^T); \qquad M = EI(\kappa - \kappa^T)$$

$$\varepsilon^T = \frac{\alpha}{A}\int_A T(x,z)\mathrm{d}A; \quad \kappa^T = \frac{\alpha}{I}\int_A zT(x,z)\mathrm{d}A$$

Stress distributions in beams:

$$\sigma(z) = \frac{N}{A} + \frac{Mz}{I}$$

$$\tau(z) = \frac{6V\left(\frac{1}{4}h^2 - z^2\right)}{bh^3}$$ rectangular crosssections

Arch:

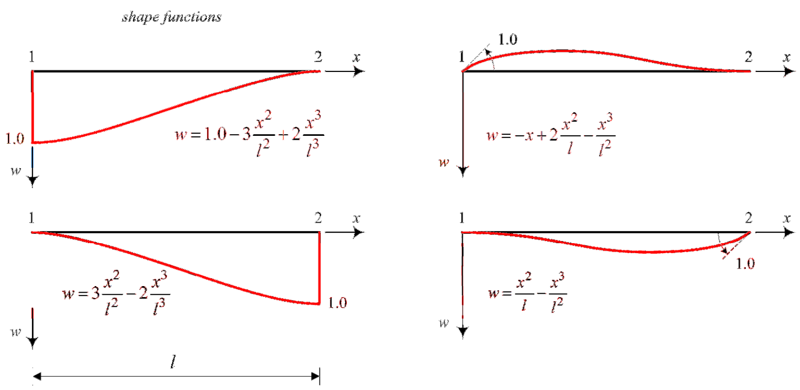
$$H = -\frac{\displaystyle\int_{arch} \frac{M^a z}{EI}\mathrm{d}x}{\displaystyle\int_{arch} \frac{z^2}{EI}\mathrm{d}x + \frac{l}{EA}}$$

Cable:

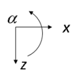
$$H^2 = \frac{q^2 l^3}{24\Delta};$$

$$f = \sqrt{\frac{3}{8} l \Delta} \text{ or } \Delta = \frac{8f^2}{3l}$$

Matrix Methods



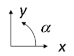
$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$



$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$\bar{f} = R\,f$ and $f = R^T \bar{f}$

$\bar{u} = R\,u$ and $u = R^T \bar{u}$



$$R = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Math tools:

$$y = Ae^{-\beta x}\sin(\beta x + \omega)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{2}\,\beta A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{1}{4}\pi\right)$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\beta^2 A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{1}{2}\pi\right)$$

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = -2\sqrt{2}\beta^3 A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{3}{4}\pi\right)$$

$$\int \sqrt{1 + x^2}\,\mathrm{d}x \approx \int \left[1 + \frac{1}{2}\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2\right]\mathrm{d}x$$

$$\int_0^a \sin^2\frac{\pi x}{a}\mathrm{d}x = \frac{a}{2} \qquad \int_0^a x\sin\frac{\pi x}{a}\mathrm{d}x = \frac{a^2}{\pi}$$

$$\int_0^a x^2\sin\frac{\pi x}{a}\mathrm{d}x = \frac{a^3\left(\pi^2 - 4\right)}{\pi^3}$$

catenary solution:

$$z = -\frac{H}{q}\cosh\left(-\frac{qx}{H} + C_1\right) + C_2$$

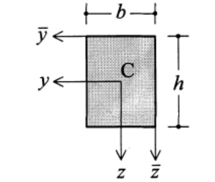
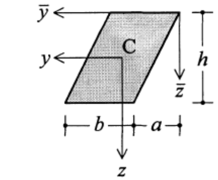
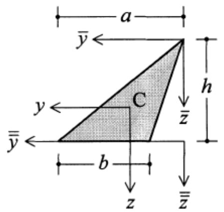
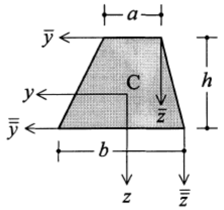
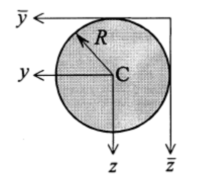
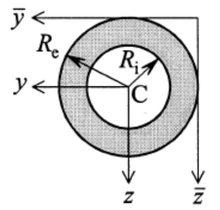
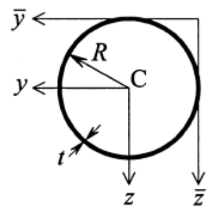
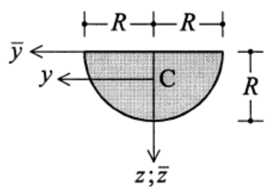
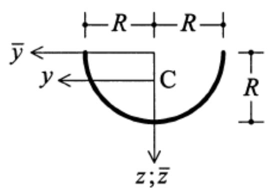
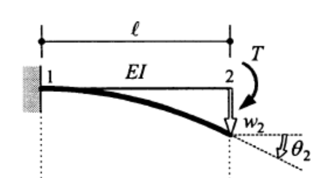
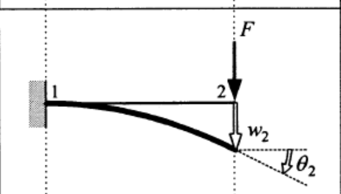
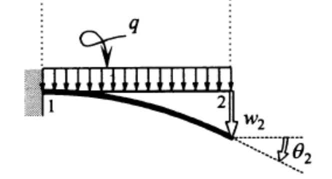
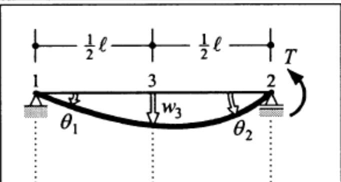
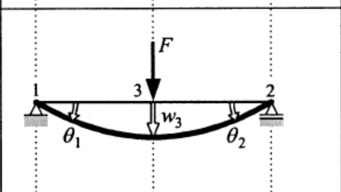
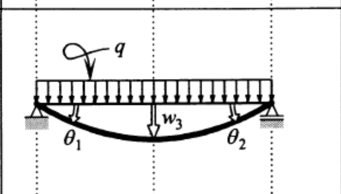
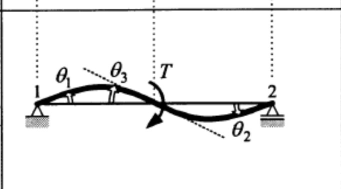
Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Rectangle $A = bh$ $\bar{y}_C = \frac{1}{2}b$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}b^3h$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = \frac{1}{3}b^3h$ $I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{4}b^2h^2$
	Parallelogram $A = bh$ $\bar{y}_C = \frac{1}{2}(a+b)$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}(a^2+b^2)bh$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = \frac{1}{12}abh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$
	Triangle $A = \frac{1}{2}bh$ $\bar{y}_C = \frac{1}{3}(2a-b)$ $\bar{z}_C = \frac{2}{3}h$	$I_{yy} = \frac{1}{36}(a^2-ab+b^2)bh$ $I_{zz} = \frac{1}{36}bh^3$ $I_{yz} = \frac{1}{72}(2a-b)bh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{4}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{8}(2a-b)bh^2$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}bh^3$
	Trapezium $A = \frac{1}{2}(a+b)h$ $\bar{z}_C = \frac{1}{3}\frac{a+2b}{a+b}h$	$I_{zz} = \frac{1}{36}\frac{a^2+4ab+b^2}{a+b}h^3$	$I_{\bar{z}\bar{z}} = \frac{1}{12}(a+3b)h^3$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}(3a+b)h^3$
	Circle $A = \pi R^2$	$I_{yy} = I_{zz} = \frac{1}{4}\pi R^4$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi R^4$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{5}{4}\pi R^4$ $I_{\bar{y}\bar{z}} = \pi R^4$

Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Thick-walled ring $A = \pi(R_e^2 - R_i^2)$	$I_{yy} = I_{zz} = \frac{1}{4}\pi(R_e^4 - R_i^4)$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi(R_e^4 - R_i^4)$	
	Thin-walled ring $A = 2\pi Rt$	$I_{yy} = I_{zz} = \pi R^3 t$ $I_{yz} = 0$ $I_p = 2\pi R^3 t$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = 3\pi R^3 t$
	Semicircle $A = \frac{1}{2}\pi R^2$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{4}{3\pi}R$ $= 0.424R$	$I_{yy} = \frac{1}{8}\pi R^4 = 0.393R^4$ $I_{zz} = (\frac{\pi}{8} - \frac{8}{9\pi})R^4$ $= 0.110R^4$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{8}\pi R^4$ $I_{\bar{y}\bar{z}} = 0$
	Semicircular ring $A = \pi Rt$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{2}{\pi}R$ $= 0.637R$	$I_{yy} = \frac{1}{2}\pi R^3 t$ $I_{zz} = (\frac{\pi}{2} - \frac{4}{\pi})R^3 t$ $= 0.298R^3 t$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{2}\pi R^3 t$ $I_{\bar{y}\bar{z}} = 0$

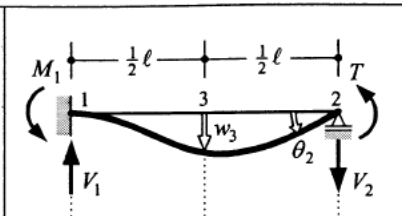
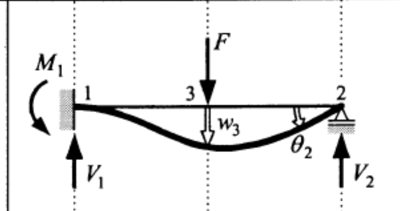
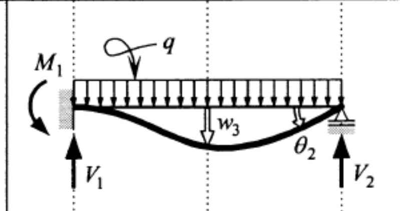
forget-me-nots

(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

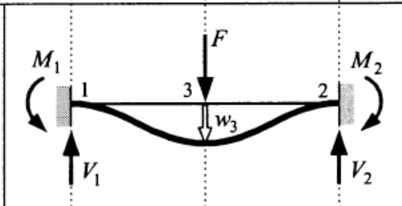
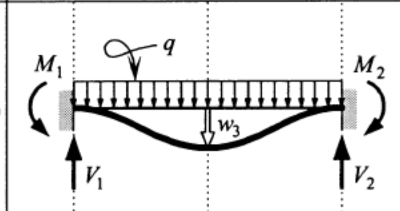
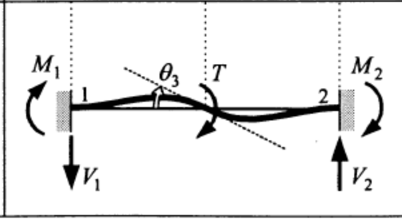
simply supported beam (statically determinate)

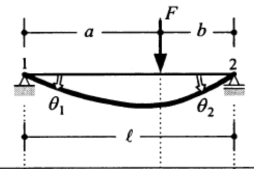
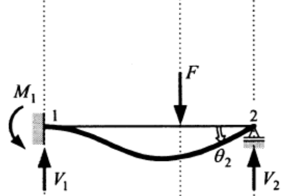
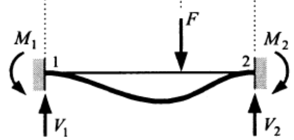
(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

statically indeterminate beam (one fixed end)

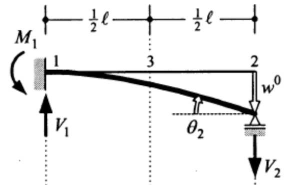
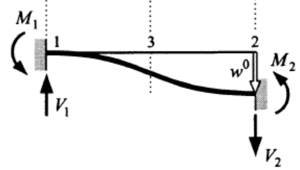
(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statically indeterminate beam (two fixed ends)

(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

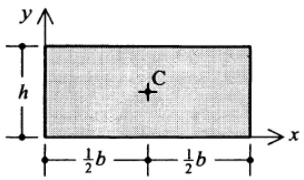
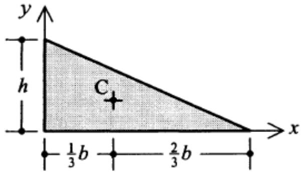
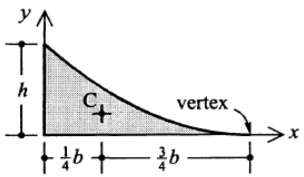
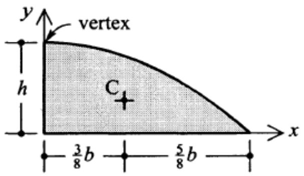
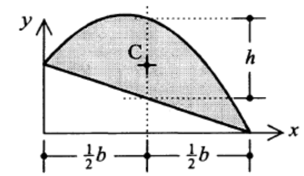
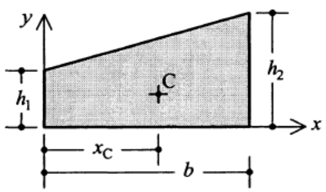
(c)		$\theta_1 = \dfrac{Fab(\ell+b)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(2\dfrac{a}{\ell} - 3\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fab(\ell+a)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(\dfrac{a}{\ell} - \dfrac{a^3}{\ell^3}\right)$
(d)		$M_1 = \dfrac{Fb(\ell^2-b^2)}{2\ell^2} = F\ell\left(\dfrac{a}{\ell} - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb(3\ell^2-b^2)}{2\ell^3} = F\left(1 - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(3\ell-a)}{2\ell^3} = F\left(\dfrac{3}{2}\dfrac{a^2}{\ell^2} - \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fa^2b}{4EI\ell} = \dfrac{F\ell^2}{4EI}\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$
(e)		$M_1 = \dfrac{Fab^2}{\ell^2} = F\ell\left(\dfrac{a}{\ell} - 2\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb^2(\ell+2a)}{\ell^3} = F\left(1 - 3\dfrac{a^2}{\ell^2} + 2\dfrac{a^3}{\ell^3}\right)$ $M_2 = \dfrac{Fa^2b}{\ell^2} = F\ell\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(\ell+2b)}{\ell^3} = F\ell\left(3\dfrac{a^2}{\ell^2} - 2\dfrac{a^3}{\ell^3}\right)$

support reactions and rotations at the beam ends

(f)		$M_1 = \dfrac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{3EI}{\ell^3}w^0$ $\theta_2 = \dfrac{3}{2}\dfrac{w^0}{\ell}$ $\theta_3 = \dfrac{9}{8}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{5}{16}w^0$
(g)		$M_1 = M_2 = \dfrac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{12EI}{\ell^3}w^0$ $\theta_3 = \dfrac{3}{2}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{1}{2}w^0$

settlements

properties of plane figures to be used for the moment-area theorems

(1)		rectangle: $y = h$ $A = bh$ $x_C = \frac{1}{2}b$
(2)		triangle: $y = h\left\{1 - \dfrac{x}{b}\right\}$ $A = \frac{1}{2}bh$ $x_C = \frac{1}{3}b$
(3)		parabola: $y = h\left\{1 - \dfrac{x}{b}\right\}^2$ $A = \frac{1}{3}bh$ $x_C = \frac{1}{4}b$
(4)		parabola: $y = h\left\{1 - \left(\dfrac{x}{b}\right)^2\right\}$ $A = \frac{2}{3}bh$ $x_C = \frac{3}{8}b$
(5)		parabola: $A = \frac{2}{3}bh$ $x_C = \frac{1}{2}b$
(6)		trapezium: $y = h_1 + (h_2 - h_1)\dfrac{x}{b}$ $A = \frac{1}{2}b(h_1 + h_2)$ $x_C = \frac{1}{3}b\dfrac{h_1 + 2h_2}{h_1 + h_2}$