SHORT ANSWERS and hints

This version only contains answers and hints how to find these answers. Also some important remarks are made related to the theory. Your exam paper must contain full explanation and derivation, this paper is NOT. This is on purpose to challenge you to find the correct derivation yourself and to link to the theory.

The amount of computational effort is very limited. In this exam no system of equations needed to be solved nor were matrix multiplications needed. Only very basic calculations were required based upon your own derived solution strategy.

Problem 1.1

a) Use either the ODE method or shape functions, see theory and examples in the slides and the Maple files.

The element stiffness matrix becomes:

$$K^{(e)} = \frac{k}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

b) The equivalent nodal forces can be found with shape functions:

$$f_{eq}^{(e)} = \begin{bmatrix} \frac{1}{3}q_1 l + \frac{1}{6}q_2 l \\ \frac{1}{6}q_1 l + \frac{1}{3}q_2 l \end{bmatrix}$$

The total element description (local coordinate system) therefor is:

$$\begin{bmatrix} F_{z-i} \\ F_{z-j} \end{bmatrix} = \frac{k}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} w_i \\ w_j \end{bmatrix} - \begin{bmatrix} \frac{1}{3}q_1 l + \frac{1}{6}q_2 l \\ \frac{1}{6}q_1 l + \frac{1}{3}q_2 l \end{bmatrix}$$

Remark:
Pay some attention to the expressions derived and also check on units.
This can be very helpful to avoid errors.

Problem 1.2

a) Describe the model and assemble the system. Clearly show the unknowns and the knowns.

NOTE : In this formulation all unknowns are positive in the positive z-direction.

Assembling the two elements results in a basic system of equations:

$$500\begin{bmatrix} 1 & -1 & \\ -1 & 2 & -1 \\ & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ w_2 \\ 0 \end{bmatrix}=\begin{bmatrix} A_V+20 \\ 75+25 \\ C_V \end{bmatrix}$$

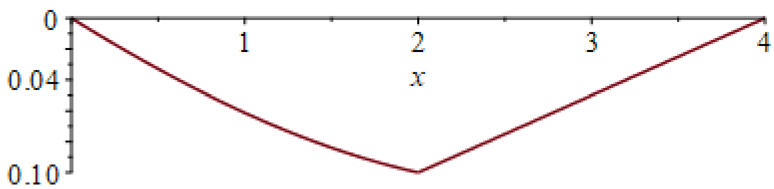
b) Solving the unknown displacement at node 2 results in:

$$w_2=\frac{100}{1000}=0.1\text{ m}$$

Solving the support reactions from equation 1 and 3 results in:

$$A_V=-70\text{ kN }(so\uparrow);\quad C_V=-50\text{ kN }(so\uparrow);$$

A sketch of the deformed structure is given. The left element will not be straight although this does not follow from the discrete results.



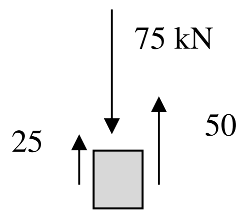
c) Return to the element definition and compute the element forces at the end. Subsequently translate these element forces to the internal cross sectional force V.

$$\begin{bmatrix} F_{z-i} \\ F_{z-j} \end{bmatrix}=\frac{k}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} w_i \\ w_j \end{bmatrix}-\begin{bmatrix} \frac{1}{3}q_1 l+\frac{1}{6}q_2 l \\ \frac{1}{6}q_1 l+\frac{1}{3}q_2 l \end{bmatrix}$$

For element (1) and (2) this results in:

$$F_{2z}^{(1)}=500\times 0.1-25=25\text{ kN}\quad\Rightarrow\quad V_{left}=25\text{ kN}$$

$$F_{2z}^{(2)}=500\times 0.1=50\text{ kN}\quad\Rightarrow\quad V_{right}=-50\text{ kN}$$



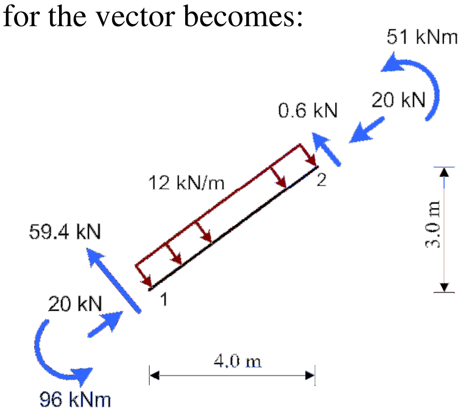
Problem 1.3

a) The *element force vector* is not the same as the equivalent element load vector so understand what is asked for. The transpose answer for the vector becomes:

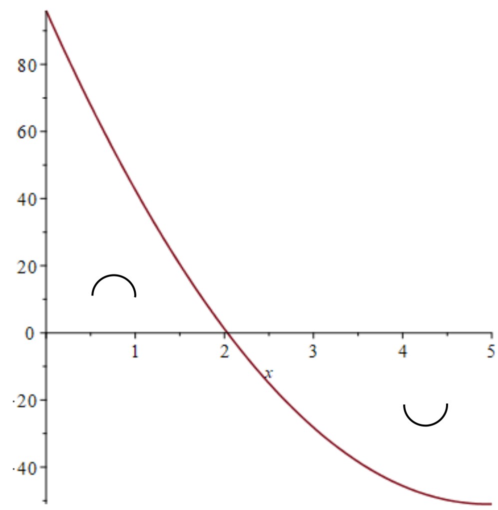
$$f^{(e)T} = [20 \quad -59.4 \quad 96 \quad -20 \quad -0.60 \quad 51]$$

These are the forces at the element ends and the actual directions are shown in the sketch of the element (apply theory).

Clearly understand the need for local or global description during the process.



b) The moment distribution is sketched below. Essential is the parabolic distribution and the correct sign information. You must understand, from the theory, the difference between T and M.



Problem 2

a) Have a close look at the supports and derive your model from this.

b) You can simplify your solution technique when you realise that the structural response will show a linear displacement field at both sides of the concentrated load. Derive all required expressions to solve the problem. This can be very compact!

c) The displacement at the point of application of the load F results in:

$$w_F = \frac{ab}{(a+b)H + ak} F = \frac{33}{6700} = 0,004925 \text{ m}$$

d) The total vertical forces in the structure results in:

$$V_1(x) = H\frac{w_F}{a} = \frac{880}{67} = 13.13 \text{ kN} \quad 0 < x \leq a$$

$$V_2(x) = -H\frac{w_F}{b} - F_{shear} = -\frac{594}{67} = -8.866 \text{ kN} \quad a < x \leq a+b$$

No need to mention the jump of 22.0 kN in the shear force diagram at the point of application of the concentrated load of 22 kN.

e) The support reactions follow from equilibrium:

$C_{\text{H}} = 6.16 \text{ kN } (\rightarrow)$

$A_{\text{V}} = 13.13 \text{ kN } (\uparrow)$; $A_{\text{H}} = 8000 \text{ kN } (\leftarrow)$

$B_{\text{V}} = 8.86 \text{ kN } (\uparrow)$; $B_{\text{H}} = 7993,84 \text{ kN } (\rightarrow)$

It is obvious that this question had a specific meaning with respect to the model. From the horizontal reactions you should have reflected on your own model. So actually this question was a clear hint to check your model as described under a).

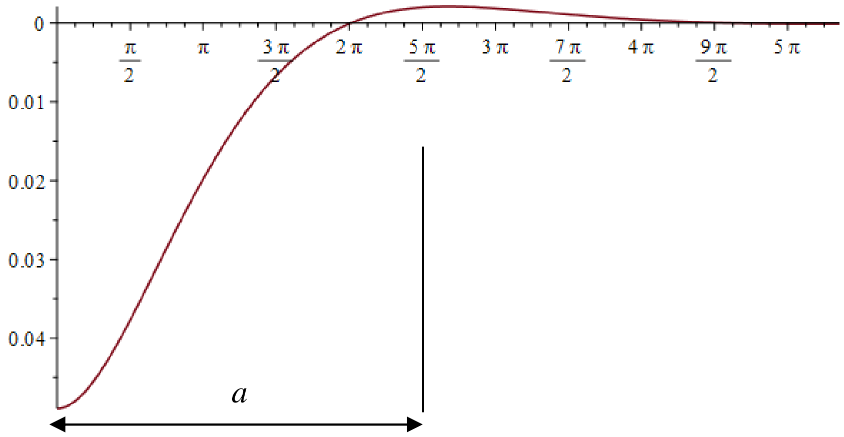
Problem 3

a) See theory. It is obvious that the discrete foundation pile will not be part of a continuous model in terms of an ODE.

b) The maximum influence zone a can be estimated as approx. 8 m. Explain why, you remember from the theory the Hetényi example.

c) Show the general solution and the boundary conditions used to solve the problem. Clearly show how the foundation pile is handled.

d) Solve your unknowns and sketch the result:



e) The shear force in the beam at $x = 0$ is approx. 25.5 kN. So the pile takes:

$$pile = \left(\frac{100 - 2 \times 25.5}{100} \right) \times 100\% = 49\%$$

Here the effectiveness is taken as the percentage of the load taken by the pile. Other options are also possible:

- judge the reduction of the bending moment due to the effect of the pile,
- judge the reduction of the displacement due to the effect of the pile.

The argument used and the quantification with own results matter here.