Faculty
Civil Engineering and GeoSciences

Remote Open book Exam	**CIE4190**
	Slender Structures
Total number of pages	**8 pages (excl cover)**
Date and time	**JAN-21-2022** from **13:30-16:30**
Responsible lecturer	**J.W. Welleman**

Only handwritten papers which have been uploaded (in pdf) in time will be assessed, Read also 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of main topics with questions: **3** (each in separate block with time constraint)

☒ **questions may differ in weight** *(the time mentioned is an indicator for the weight)*

Use of tools and sources of information (to be filled in by course examiner)

Deliver your own work according to integrity statement:

- No collaboration or help "from others",
- No communication through any kind of medium,
- No distribution of files or answers to others,
- Etc.

Allowed:

☒ **books** ☒ **notes** ☒ **dictionaries** ☒ **syllabus**

☒ **formula sheets (see also below under 'additional information')** ☒ **calculators**

☒ **computer** ☐ **...**

☒ **scientific (graphical)calculator** ☒ **drawing material**

Additional information (if necessary to be filled in by the examiner)

- **Use your own examination paper with name and study number on all pages**
- **The question form contains fomula sheets which can be used.**
- **Papers will only be graded if the integrity statement has been uploaded.**
- **Upload in time, every part as ONE PDF file, with standard name format:**

  surname_student number_exam part example BOND_007_1

Exam graded by: (the marking period is 15 working days at most)

Every suspicion of fraud is reported to the Board of Examiners.

Mobile Phone OFF.

FORMULAS

Temperature:

$$N = EA(\varepsilon - \varepsilon^T); \qquad M = EI(\kappa - \kappa^T)$$

$$\varepsilon^T = \frac{\alpha}{A}\int_A T(x,z)\mathrm{d}A; \quad \kappa^T = \frac{\alpha}{I}\int_A zT(x,z)\mathrm{d}A$$

Stress distributions in beams:

$$\sigma(z) = \frac{N}{A} + \frac{Mz}{I} \qquad G = \frac{E}{2(1+\nu)}\ [\text{N/mm}^2]$$

$$\tau(z) = \frac{6V\left(\frac{1}{4}h^2 - z^2\right)}{bh^3}$$ rectangular crosssections

Arch:

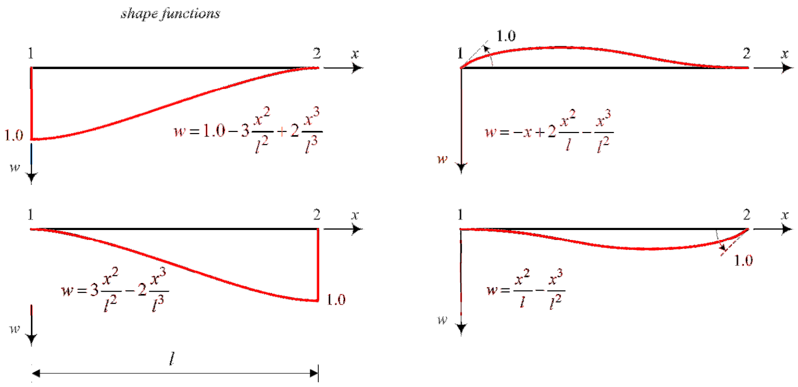
$$H = -\frac{\int_{arch} \frac{M^a z}{EI}\mathrm{d}x}{\int_{arch} \frac{z^2}{EI}\mathrm{d}x + \frac{l}{EA}}$$

Cable:

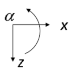
$$H^2 = \frac{q^2 l^3}{24\Delta};$$

$$f = \sqrt{\frac{3}{8} l \Delta} \text{ or } \Delta = \frac{8f^2}{3l}$$

Matrix Methods



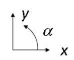
$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$



$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$\bar{f} = R\ f$ and $f = R^T \bar{f}$

$\bar{u} = R\ u$ and $u = R^T \bar{u}$



$$R = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Math tools:

$$y = Ae^{-\beta x}\sin(\beta x + \omega)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{2}\,\beta A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{1}{4}\pi\right)$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 2\beta^2 A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{1}{2}\pi\right)$$

$$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = -2\sqrt{2}\beta^3 A e^{-\beta x}\sin\left(\beta x + \omega - \tfrac{3}{4}\pi\right)$$

$$\int\sqrt{1+x^2}\,\mathrm{d}x \approx \int\left[1 + \frac{1}{2}\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2\right]\mathrm{d}x$$

$$\int_0^a \sin^2\frac{\pi x}{a}\mathrm{d}x = \frac{a}{2} \qquad \int_0^a x\sin\frac{\pi x}{a}\mathrm{d}x = \frac{a^2}{\pi}$$

$$\int_0^a x^2\sin\frac{\pi x}{a}\mathrm{d}x = \frac{a^3\left(\pi^2 - 4\right)}{\pi^3}$$

catenary solution:

$$z = -\frac{H}{q}\cosh\left(-\frac{qx}{H} + C_1\right) + C_2$$

basic math:

$$e^{\ln(a)} = a; \quad e^{-\ln(a)} = \tfrac{1}{a}; \quad z(x) = \frac{4fx(l-x)}{l^2}\ (parabola)$$

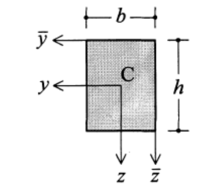
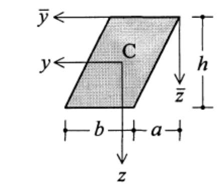
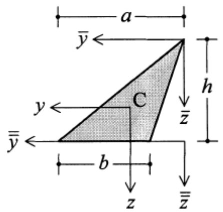
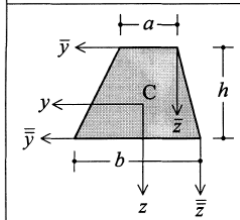
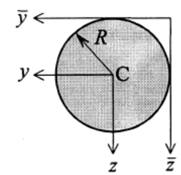
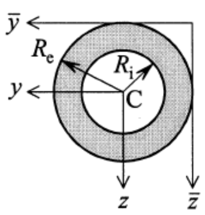
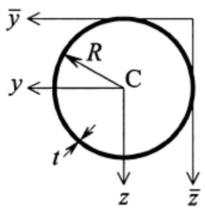
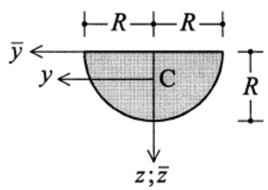
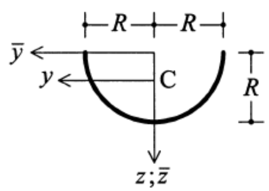
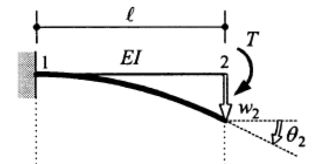
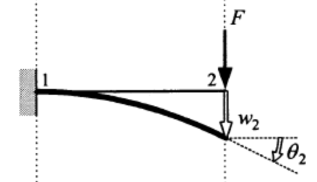
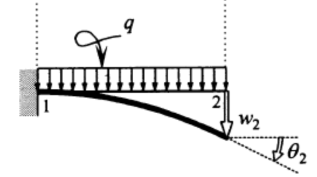
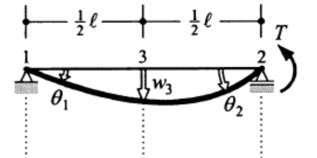
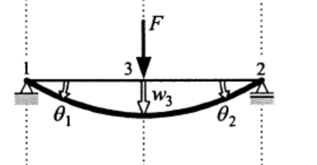
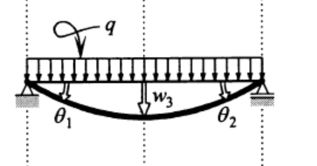
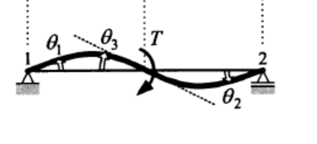
Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Rectangle $A = bh$ $\bar{y}_C = \frac{1}{2}b$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}b^3h$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = \frac{1}{3}b^3h$ $I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{4}b^2h^2$
	Parallelogram $A = bh$ $\bar{y}_C = \frac{1}{2}(a+b)$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}(a^2+b^2)bh$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = \frac{1}{12}abh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$
	Triangle $A = \frac{1}{2}bh$ $\bar{y}_C = \frac{1}{3}(2a-b)$ $\bar{z}_C = \frac{2}{3}h$	$I_{yy} = \frac{1}{36}(a^2-ab+b^2)bh$ $I_{zz} = \frac{1}{36}bh^3$ $I_{yz} = \frac{1}{72}(2a-b)bh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{4}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{8}(2a-b)bh^2$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}bh^3$
	Trapezium $A = \frac{1}{2}(a+b)h$ $\bar{z}_C = \frac{1}{3}\frac{a+2b}{a+b}h$	$I_{zz} = \frac{1}{36}\frac{a^2+4ab+b^2}{a+b}h^3$	$I_{\bar{z}\bar{z}} = \frac{1}{12}(a+3b)h^3$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}(3a+b)h^3$
	Circle $A = \pi R^2$	$I_{yy} = I_{zz} = \frac{1}{4}\pi R^4$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi R^4$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{5}{4}\pi R^4$ $I_{\bar{y}\bar{z}} = \pi R^4$

Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Thick-walled ring $A = \pi(R_e^2 - R_i^2)$	$I_{yy} = I_{zz} = \frac{1}{4}\pi(R_e^4 - R_i^4)$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi(R_e^4 - R_i^4)$	
	Thin-walled ring $A = 2\pi Rt$	$I_{yy} = I_{zz} = \pi R^3 t$ $I_{yz} = 0$ $I_p = 2\pi R^3 t$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = 3\pi R^3 t$
	Semicircle $A = \frac{1}{2}\pi R^2$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{4}{3\pi}R$ $= 0.424R$	$I_{yy} = \frac{1}{8}\pi R^4 = 0.393R^4$ $I_{zz} = (\frac{\pi}{8} - \frac{8}{9\pi})R^4$ $= 0.110R^4$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{8}\pi R^4$ $I_{\bar{y}\bar{z}} = 0$
	Semicircular ring $A = \pi Rt$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{2}{\pi}R$ $= 0.637R$	$I_{yy} = \frac{1}{2}\pi R^3 t$ $I_{zz} = (\frac{\pi}{2} - \frac{4}{\pi})R^3 t$ $= 0.298R^3 t$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{2}\pi R^3 t$ $I_{\bar{y}\bar{z}} = 0$

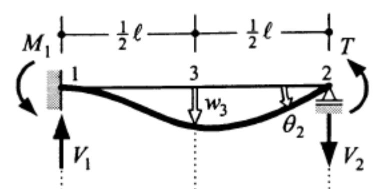
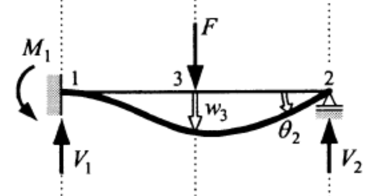
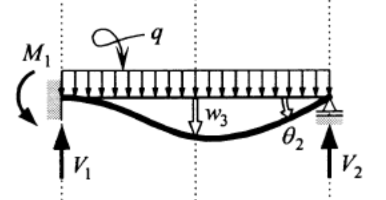
forget-me-nots

(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

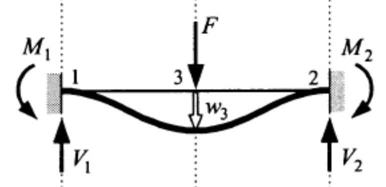
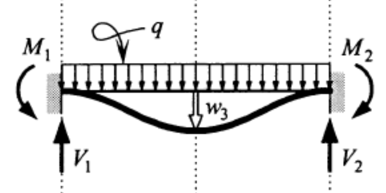
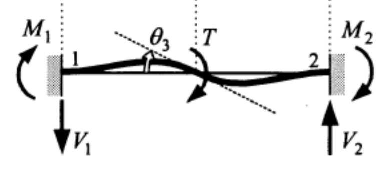
simply supported beam (statically determinate)

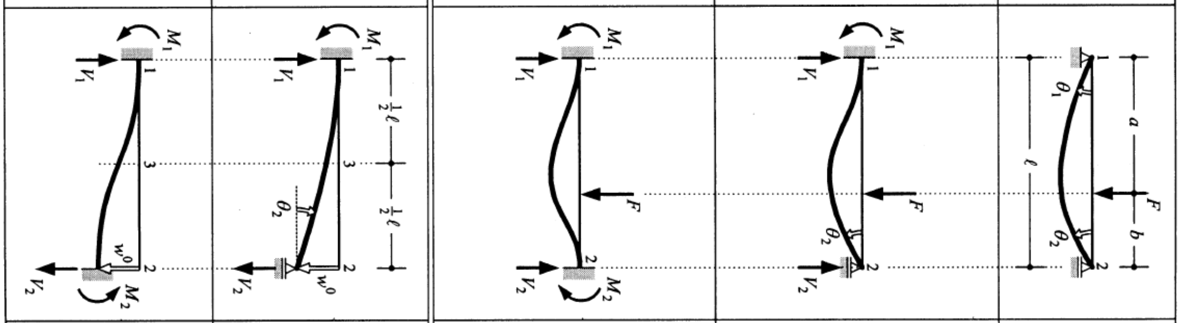
(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

statically indeterminate beam (one fixed end)

(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statically indeterminate beam (two fixed ends)

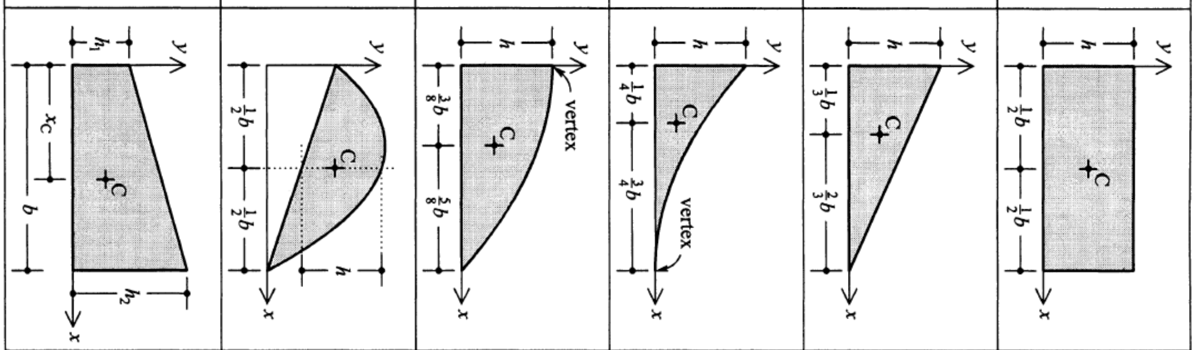
(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$



(c)	$\theta_1 = \dfrac{Fab(\ell+b)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(2\dfrac{a}{\ell} - 3\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fab(\ell+a)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(\dfrac{a}{\ell} - \dfrac{a^3}{\ell^3}\right)$
(d)	$M_1 = \dfrac{Fb(\ell^2-b^2)}{2\ell^2} = F\ell\left(\dfrac{a}{\ell} - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb(3\ell^2-b^2)}{2\ell^3} = F\left(1 - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(3\ell-a)}{2\ell^3} = F\left(\dfrac{3}{2}\dfrac{a^2}{\ell^2} - \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fa^2b}{4EI\ell} = \dfrac{F\ell^2}{4EI}\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$
(e)	$M_1 = \dfrac{Fab^2}{\ell^2} = F\ell\left(\dfrac{a}{\ell} - 2\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb^2(\ell+2a)}{\ell^3} = F\left(1 - 3\dfrac{a^2}{\ell^2} + 2\dfrac{a^3}{\ell^3}\right)$ $M_2 = \dfrac{Fa^2b}{\ell^2} = F\ell\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(\ell+2b)}{\ell^3} = F\ell\left(3\dfrac{a^2}{\ell^2} - 2\dfrac{a^3}{\ell^3}\right)$
(f)	$M_1 = \dfrac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{3EI}{\ell^3}w^0$ $\theta_2 = \dfrac{3}{2}\dfrac{w^0}{\ell}$ $\theta_3 = \dfrac{9}{8}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{5}{16}w^0$
(g)	$M_1 = M_2 = \dfrac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{12EI}{\ell^3}w^0$ $\theta_3 = \dfrac{3}{2}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{1}{2}w^0$

settlements (f, g) — *support reactions and rotations at the beam ends* (c, d, e)

properties of plane figures to be used for the moment-area theorems



(1)	rectangle: $y = h$ $A = bh$ $x_C = \frac{1}{2}b$
(2)	triangle: $y = h\left\{1 - \dfrac{x}{b}\right\}$ $A = \frac{1}{2}bh$ $x_C = \frac{1}{3}b$
(3)	parabola: $y = h\left\{1 - \dfrac{x}{b}\right\}^2$ $A = \frac{1}{3}bh$ $x_C = \frac{1}{4}b$
(4)	parabola: $y = h\left\{1 - \left(\dfrac{x}{b}\right)^2\right\}$ $A = \frac{2}{3}bh$ $x_C = \frac{3}{8}b$
(5)	parabola: $A = \frac{2}{3}bh$ $x_C = \frac{1}{2}b$
(6)	trapezium: $y = h_1 + (h_2 - h_1)\dfrac{x}{b}$ $A = \frac{1}{2}b(h_1 + h_2)$ $x_C = \frac{1}{3}b\dfrac{h_1 + 2h_2}{h_1 + h_2}$

Timoshenko − beam

$$\frac{d^2w(x)}{dx^2} = -\frac{q(x)}{GA_{eff}} - \frac{M(x)}{EI}$$

$$EI\,\frac{d^2\varphi(x)}{dx^2} - GA_{eff}\left(\frac{dw(x)}{dx} + \varphi(x)\right) = 0 \quad (1)$$

$$GA_{eff}\left(\frac{d^2w(x)}{dx^2} + \frac{d\varphi(x)}{dx}\right) = -q(x) \quad (2)$$