TU Delft

## General instructions

All pages of your answer sheets must contain your name and your student number. Number each page of your answer sheets and indicate the total number of pages used. Write clearly and in English. Points for each question are in the left margin brackets.

Before proceeding with the derivations, write down all the steps that you are going to take. When necessary, perform, and show on your answer sheets, some "sanity checks" to prove the soundness of your derivations. Explain your answers clearly.

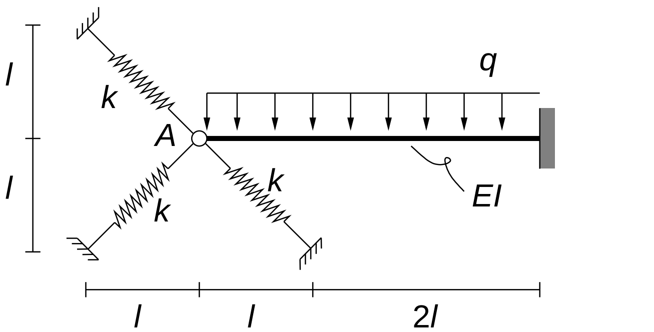
This question sheet **must be returned** before leaving.

## Questions

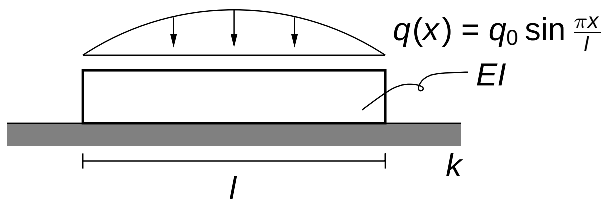
1. Determine rotation and displacement at $A$ using

[2] (a) the differential equation method, and

[2] (b) the matrix method.



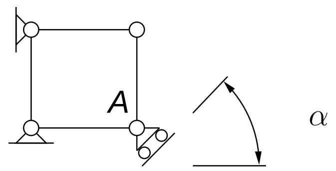

[2] 2. A beam on elastic foundation is loaded by a distributed transverse load $q(x)$. Determine the expression of the transverse deflection $v$. Before proceeding with the solution, derive the governing differential equation starting from $\frac{d^4v}{dx^4} = \frac{q}{EI}$.




[2] 3. A cable supported at both ends is loaded by a uniformly distributed transverse load on the left half of its span. Determine the expression of the cable deflection and the deflection at mid-span. Before proceeding with the solution, derive the governing differential equation.

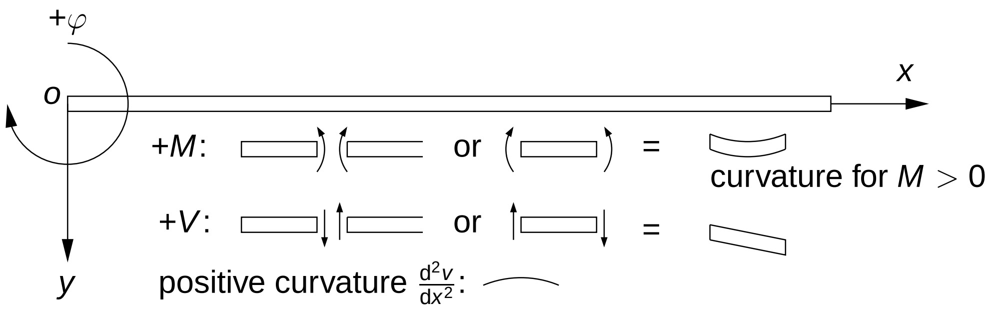
[2] 4. A beam is loaded by a pure moment $M$ and deforms into a circular arc. What does this imply about the deformed shape of a planar cross-section, initially perpendicular to the long axis of the beam?

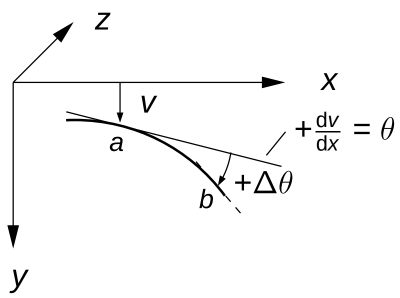
[2] 5. Consider a generic structure that has a translational nodal degree of freedom directed along a line inclined of an angle $\alpha$ with respect to the horizontal axis as shown in the figure below. How would you treat the boundary condition at node $A$? How would you check your solution?

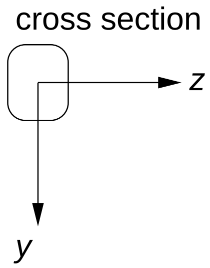


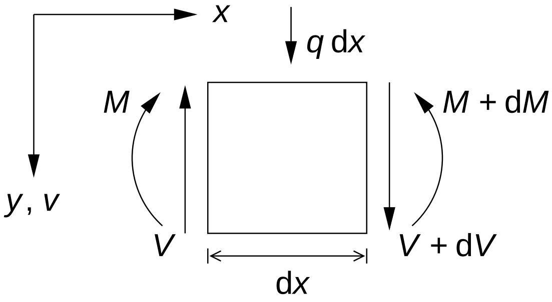

**Useful relations**

- Euler-Bernoulli beam, differential equation



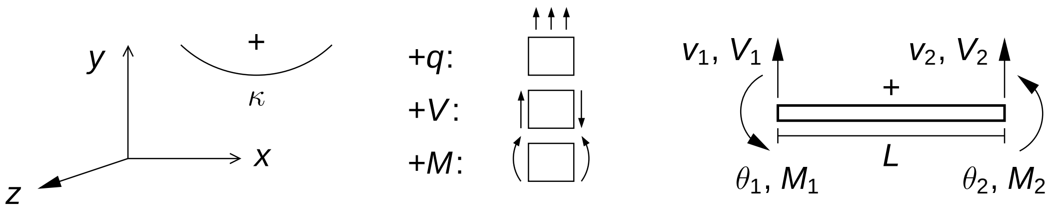






$$\frac{d^2v}{dx^2} = -\frac{M}{EI}, \qquad \frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) = q, \qquad \frac{d^4v}{dx^4} = \frac{q}{EI} \tag{1}$$

- Euler-Bernoulli beam, matrix analysis




$$\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix}. \tag{2}$$