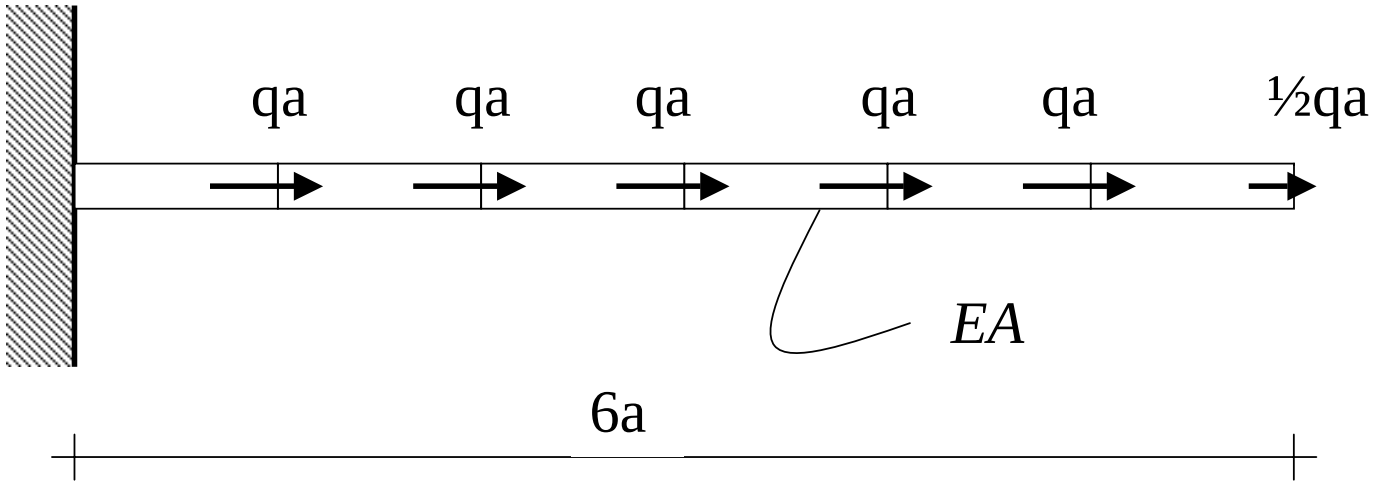
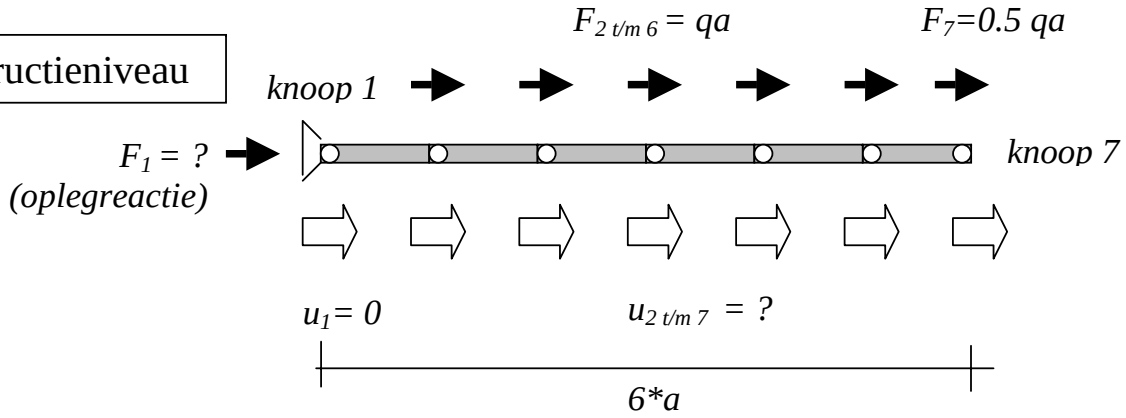
# GEGEVEN CONSTRUCTIE



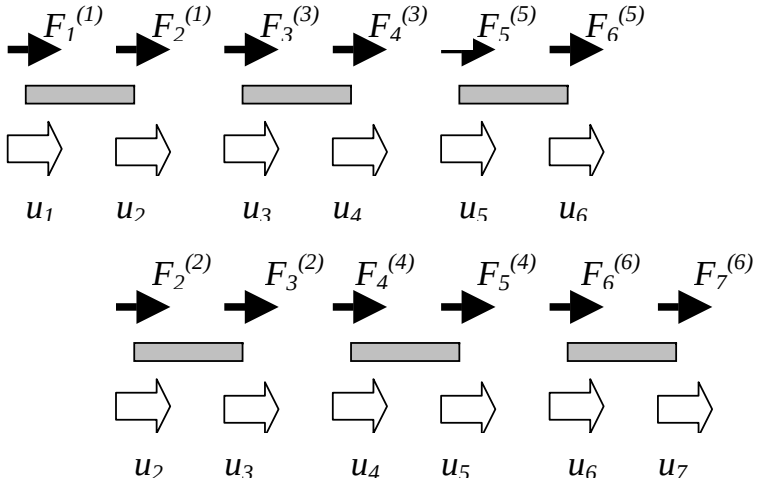

- **Welke vrijheidsgraden ?**
- **Welk element ?**

# MODEL EN ELEMENTASSEMBLAGE

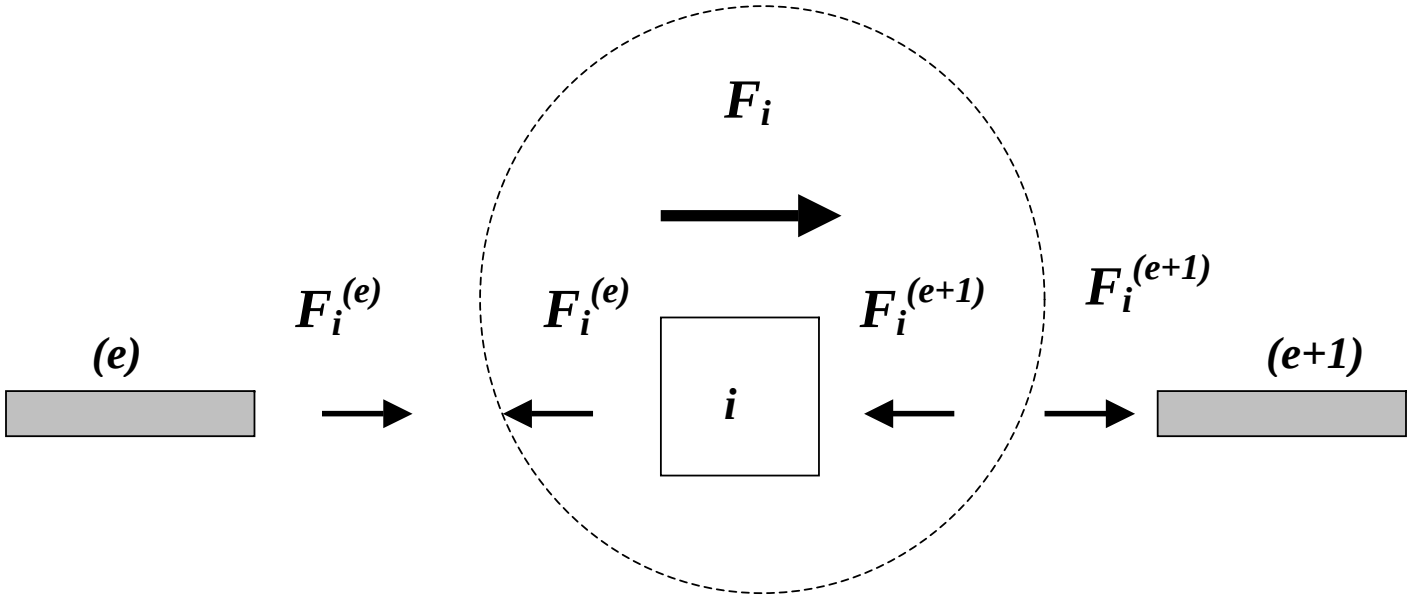
constructieniveau




elementniveau



# KNOOPEVENWICHT (1)




***Knoopevenwicht***

$$F_i = F_i^{(e)} + F_i^{(e+1)}$$

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## KNOOPEVENWICHT (2)

$$F_1 = F_1^{(1)}$$

$$F_2 = F_2^{(1)} + F_2^{(2)} = qa$$

$$F_3 = F_3^{(2)} + F_3^{(3)} = qa$$

$$F_4 = F_4^{(3)} + F_4^{(4)} = qa$$

$$F_5 = F_5^{(4)} + F_5^{(5)} = qa$$

$$F_6 = F_6^{(5)} + F_6^{(6)} = qa$$

$$F_7 = F_7^{(6)} = \tfrac{1}{2} qa$$

$$\begin{Bmatrix} F_i^{(e)} \\ F_j^{(e)} \end{Bmatrix} = \frac{EA}{a} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_i \\ u_j \end{Bmatrix}$$

## KNOOPEVENWICHT (3)

$$F_1 = F_1^{(1)} = \frac{EA}{a} * u_1 - \frac{EA}{a} * u_2$$

$$F_2 = F_2^{(1)} + F_2^{(2)} = -\frac{EA}{a} * u_1 + \frac{EA}{a} * u_2 + \frac{EA}{a} * u_2 - \frac{EA}{a} * u_3 = qa$$

$$F_3 = F_3^{(2)} + F_3^{(3)} = -\frac{EA}{a} * u_2 + \frac{EA}{a} * u_3 + \frac{EA}{a} * u_3 - \frac{EA}{a} * u_4 = qa$$

$$F_4 = F_4^{(3)} + F_4^{(4)} = -\frac{EA}{a} * u_3 + \frac{EA}{a} * u_4 + \frac{EA}{a} * u_4 - \frac{EA}{a} * u_5 = qa$$

$$F_5 = F_5^{(4)} + F_5^{(5)} = -\frac{EA}{a} * u_4 + \frac{EA}{a} * u_5 + \frac{EA}{a} * u_5 - \frac{EA}{a} * u_6 = qa$$

$$F_6 = F_6^{(5)} + F_6^{(6)} = -\frac{EA}{a} * u_5 + \frac{EA}{a} * u_6 + \frac{EA}{a} * u_6 - \frac{EA}{a} * u_7 = qa$$

$$F_7 = F_7^{(6)} = -\frac{EA}{a} * u_6 + \frac{EA}{a} * u_7 = \tfrac{1}{2} qa$$

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**OP TE LOSSEN STELSEL**

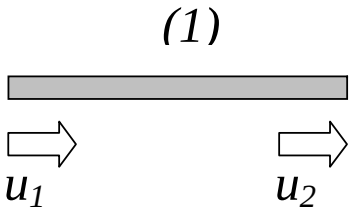
$$\frac{EA}{a}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \begin{bmatrix} F_1 \\ qa \\ qa \\ qa \\ qa \\ qa \\ \frac{1}{2}qa \end{bmatrix}$$

**OPLEGGING MET VOORGESCHREVEN VERPLAATSING 0**

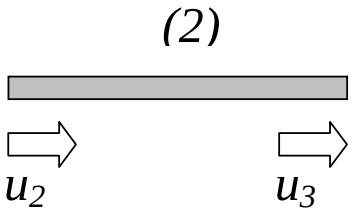
- BEWERKELIJKE METHODE
- GRAAG EEN EENVOUDIG ALTERNATIEF

$$
\begin{matrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \end{matrix}
$$

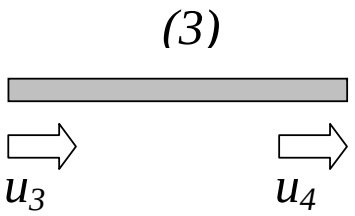
$$
\frac{EA}{a}\begin{bmatrix} 1 & -1 & & & & & \\ -1 & 1 & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}\begin{bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \begin{bmatrix} F_1 \\ qa \\ qa \\ qa \\ qa \\ qa \\ \frac{1}{2}qa \end{bmatrix}
$$



$$\frac{EA}{a}\begin{bmatrix} & & & & & & \\ & & & & & & \\ & & 1 & -1 & & & \\ & & -1 & 1 & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}\begin{bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \begin{bmatrix} F_1 \\ qa \\ qa \\ qa \\ qa \\ qa \\ \frac{1}{2}qa \end{bmatrix}$$



$$\frac{EA}{a}\begin{bmatrix} & & & & & & \\ & & & & & & \\ & & 1 & -1 & & & \\ & & -1 & 1 & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}\begin{bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \begin{bmatrix} F_1 \\ qa \\ qa \\ qa \\ qa \\ qa \\ \frac{1}{2}qa \end{bmatrix}$$



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## OP TE LOSSEN STELSEL

$$\frac{EA}{a}\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \begin{bmatrix} F_1 \\ qa \\ qa \\ qa \\ qa \\ qa \\ \frac{1}{2}qa \end{bmatrix}$$

- BANDSTRUCTUUR
- DET = 0 DUS VERWERKEN VAN DE RANDVOORWAARDEN NOODZAKELIJK ⟶ **STELSEL REDUCEREN**

## GEREDUCEERD STELSEL

$$\frac{EA}{a}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \begin{bmatrix} qa \\ qa \\ qa \\ qa \\ qa \\ \frac{1}{2}qa \end{bmatrix}$$

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# INVERTEREN VAN DE SYSTEEMSTIJFHEIDSMATRIX :

$$\begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{bmatrix} = \frac{a}{EA} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix} . \begin{bmatrix} qa \\ qa \\ qa \\ qa \\ qa \\ \frac{1}{2} qa \end{bmatrix}$$

$$u_2 = 5{,}5 \frac{qa^2}{EA} \qquad u_3 = 10{,}0 \frac{qa^2}{EA} \qquad u_4 = 13{,}5 \frac{qa^2}{EA}$$

$$u_5 = 16{,}0 \frac{qa^2}{EA} \qquad u_6 = 17{,}5 \frac{qa^2}{EA} \qquad u_7 = 18{,}0 \frac{qa^2}{EA} \qquad F_1 = ?$$

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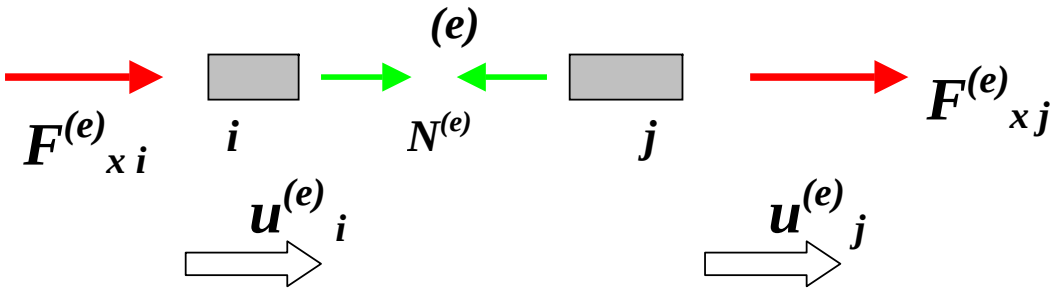
# OPLEGREACTIE

- GEBRUIK PER OPLEGREACTIE DE VERGELIJKING VAN DE VRIJHEIDSGRAAD DIE UIT HET STELSEL IS VERWIJDERD :

$$F_1 = \frac{EA}{a}\left[u_1 - u_2\right] = -\frac{EA}{a} \cdot \frac{5{,}5qa^2}{EA} = -5{,}5qa$$

EINDE VAN DE REKENFASE
( OPLOSSEN VAN DE ONBEKENDEN )

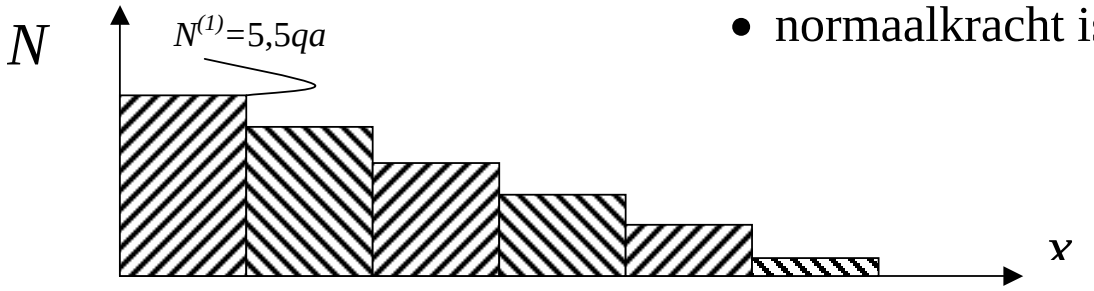
**POSTPROCESSING : KRACHTEN IN DE ELEMENTEN**




twee mogelijkheden

$$N^{(1)} = -F_1^{(1)} = -\frac{EA}{a}\left(u_1^{(1)} - u_2^{(1)}\right) = -\frac{EA}{a}\left(0 - \frac{5{,}5qa^2}{EA}\right) = 5{,}5qa$$

$$N^{(1)} = F_2^{(1)} = \frac{EA}{a}\left(-u_1^{(1)} + u_2^{(1)}\right) = 5{,}5qa$$




- normaalkracht is *constant* over het element !

# AANPAK EEM :

- MODELLEER DE CONSTRUCTIE (BEPAAL DE VRIJHEIDSGRADEN )
- BEPAAL PER ELEMENT DE ELEMENTSTIJFHEIDSMATRIX
- ASSEMBLEER DE SYSTEEMSTIJFHEIDSMATRIX
- STEL DE SYSTEEM-KRACHTVECTOR OP
- VERWERK DE RANDVOORWAARDEN
- LOS HET GEREDUCEERDE STELSEL OP
- BEPAAL DE KRACHTSVERDELING PER ELEMENT