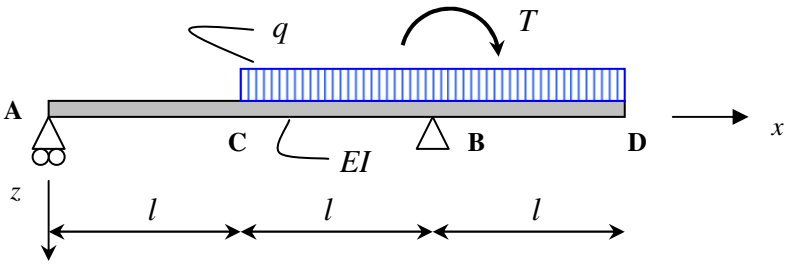
EXERCISE 2 : WORK AND ENERGY

Determine of the following beam the rotation at B with the energy method. Answer the following questions.

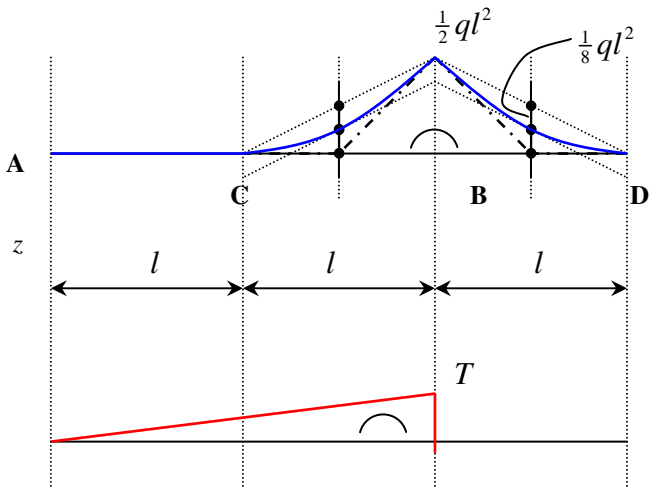


Questions:

a) Draw the bending moment diagram of the given distributed load q and the couple T applied at point B.
b) Determine the rotation at B using Castigliano's 2nd theoreme.

HINT: Use the previous obtained functions for the moment distribution of exercise 1.

The bending moment diagram for the distributed load q en the couple T are displayed below.



M-diagram for q

M-diagram for T

The total bending moment diagram is a superposition of both bending moment diagrams.

a) According to Castigliano: $\varphi = \dfrac{\partial Ev}{\partial T}$ *with*: $E_v = \displaystyle\int_{x=0}^{3l} \frac{M^2(x)\mathrm{d}x}{2EI} = \int_{x=0}^{3l} \frac{(M_T + M_q)^2 \mathrm{d}x}{2EI}$

HINT: Split the contribution in three parts and determine for every part the contribution to this integral.

Part 1: $(0 < x < l)$

$$M_q(x) = 0$$

$$M_T(x) = -\frac{Tx}{2l}$$

Part 2: $(0 < x < l)$

$$M_q(x) = -\frac{1}{2}qx^2$$

$$M_T(x) = -\frac{T}{2} - \frac{Tx}{2l}$$

Part 3: $(0 < x < l)$

$$M_q(x) = -\frac{1}{2}q(l-x)^2$$

$$M_T(x) = 0$$

$\varphi = \dfrac{7ql^3 + 32Tl}{48EI}$, check is possible using MatrixFrame