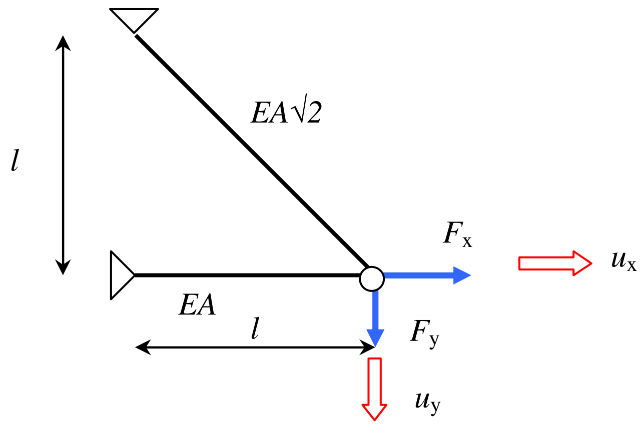
Exercise 3 : WORK AND ENERGY

Answer the following questions about the truss shown below:



a) If the forces are only applied at the bar ends of the prismatic truss elements, prove the stored strain energy as a result of the Δl equals to:

$$E_V = \tfrac{1}{2}\frac{EA}{l}\times(\Delta l)^2$$

b) Express the forces F_x and F_y in terms of the deflections u_x and u_y using the 1st law of Castigliano.

Hint: Start with the strain energy.

c) Determine this equation in the classic method as well. (homework)

a) You can find the answer in the lecture notes.

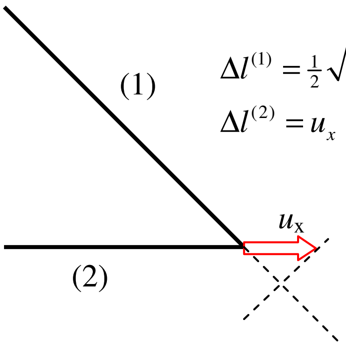
b) We will use the 1st law of Castigliano:

$$F_x = \frac{\partial E_v}{\partial u_x}$$

$$F_y = \frac{\partial E_v}{\partial u_y}$$

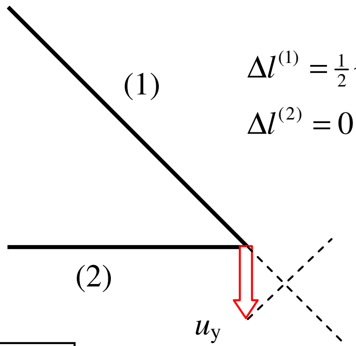
You will need the strain energy E_{v} expressed in terms of the deflections u_{x} and u_{y}. You can determine these like this:

First apply a displacement in the *x*-direction and determine the extension of both bars. Now apply a displacement in the *y*-direction and determine again the extension in both bars. The total extension for every bar is determined as a result of an unknown displacement in the *x*-direction and *y*-direction. *(We use the concept of the displacement method.)*



$$\Delta l^{(1)} = \tfrac{1}{2}\sqrt{2}\, u_x$$

$$\Delta l^{(2)} = u_x$$



$$\Delta l^{(1)} = \tfrac{1}{2}\sqrt{2}\, u_y$$

$$\Delta l^{(2)} = 0$$

$$\Delta l^{(1)} = \tfrac{1}{2}\sqrt{2}(u_x + u_y)$$

$$\Delta l^{(2)} = u_x$$

With this expression of the extension of the bars, the strain energy can be obtained:

$$E_v = \frac{EA^{(1)}}{2l^{(1)}} \times (\Delta l^{(1)})^2 + \frac{EA^{(2)}}{2l^{(2)}} \times (\Delta l^{(2)})^2 = \frac{EA}{2l} \times \left(\tfrac{3}{2} u_x^2 + u_x u_y + \tfrac{1}{2} u_y^2 \right)$$

Applying the first law of Castigliano:

$$F_x = \frac{\partial E_v}{\partial u_x} = \frac{EA}{2l} \times \left(3u_x + u_y \right)$$

$$F_y = \frac{\partial E_v}{\partial u_y} = \frac{EA}{2l} \times \left(u_x + u_y \right)$$

or in a matrix: $\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{EA}{2l} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} u_x \\ u_y \end{bmatrix}$

In fact this is the stiffness matrix of this construction: $K = \frac{EA}{2l} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$.

Isn't this beautiful?