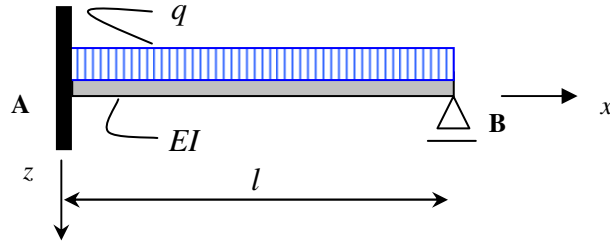


EXERCISE 4 : WORK AND ENERGY

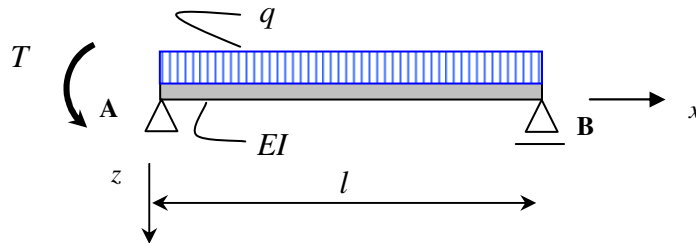
Find by using the energy method, the fixed-end moment at A of the following construction. Answer the two questions in order to solve this problem.



Questions:

- Draw the bending moment diagram as a result of the given distributed load q and the unknown fixed-end moment in A.
- Determine the fixed-end moment using the 2nd law of Castigliano.

- a) The shown structure is *statically indeterminate to the first degree*. Chose the fixed-end moment T as static redundant. The associated degree of freedom of this moment is the rotation at A which is zero in this case zero due to the fixed end. This is the compatibility condition for the statically determinate system.



To solve this condition the moment distribution is required in order to obtain the strain energy. Draw the bending moment diagram by using superposition of the bending moments as a result of the couple T and the distributed load q .

- b) Using Castigliano's 2nd theorem for the the compatibility condition results in:

$$\varphi = \frac{\partial E_v}{\partial T} = 0$$

In the expression for the strain energy you will find the bending stiffness EI , the statically indeterminate T , the distributed load q and the length l . Determine the E_v using:

$$E_v = \int_0^l \frac{M^2}{2EI} dx \quad \varphi = \frac{\partial E_v}{\partial T} = \int_0^l \frac{M}{EI} \times \frac{\partial M}{\partial T} dx$$

The moment distribution as a result of the distributed load q and the statically indeterminate T will be (just like exercise 1 and 2):

$$M(x) = -T + \frac{Tx}{l} + \frac{1}{2}qx(l-x) \quad \text{en} \quad \frac{\partial M}{\partial T} = -1 + \frac{x}{l}$$

With the result:

$$\begin{aligned} \varphi &= \frac{1}{EI} \int_0^l \left(-T + \frac{Tx}{l} + \frac{1}{2}qx(l-x) \right) \times \left(-1 + \frac{x}{l} \right) dx = 0 \\ \frac{1}{3}Tl - \frac{1}{4}ql^3 + \frac{1}{3}ql^3 - \frac{1}{8}ql^3 &= 0 \Leftrightarrow \\ T &= \frac{1}{8}ql^2 \end{aligned}$$

This is just like we had expected. Of course you will be able to find the same result using the force method with *forget-me-nots*, which uses the same compatibility condition.