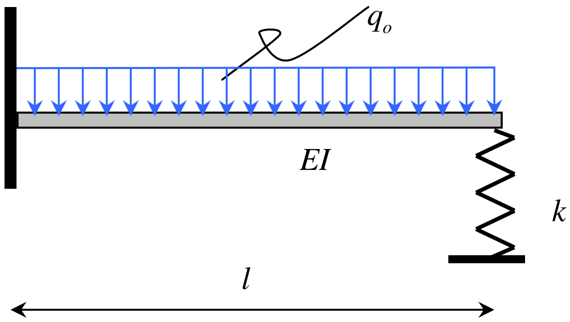
Exercise 6: Work and Energy

In the following figure a clamped beam is shown supported on a spring with stiffness k at the right side. The beam is loaded by a distributed load q_o . For the beam only bending is considered.

The spring stiffness can be expressed in a dimensionless coefficient of the ratio with the bending stiffness and span of the beam:

$$\rho = \frac{kl^3}{EI} \quad \Rightarrow \quad k = \frac{\rho EI}{l^3}$$



spring:

$$E_v = \frac{F_{veer}^2}{2k}$$

Question 1
Please explain how Castigliano's theorems can be used to find the force distribution in the system and make a sketch to underline your explanation.

Question 2
Determine the expression for the force in the spring using your described method and show that the following holds:

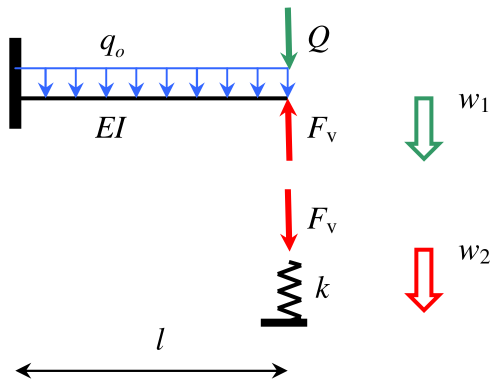
$$F_v = \frac{\rho}{\rho + 3} \times \tfrac{3}{8} ql$$

Question 3
Determine the value of ρ for which holds that the clamping moment in absolute sense is three times the maximal moment at the span.

Question 4
If the following parameters are given: EI = 10000 kNm^2; l = 8 m; q_o = 5 kN/m, please determine the moment distribution for the found value of ρ and draw this, please add the deformation signs and values at characteristic points.

Solution

Question 1



The Strain energy of the structure is composed of two parts, the bending of the beam and the deformation of the spring. Assume an unknown compressive force in the spring F_v and make sure the deflection of the beam end is equal to the deformation of the spring. This can be shown in several ways. Here we choose to apply a dummy load Q at the beam end. For the bending moments the following holds:

$$M(x) = -\tfrac{1}{2}q(l-x)^2 - (Q - F_v)(l-x)$$

The expressions for the components of the strain energy are:

Beam: $$E_{v1} = \int_{x=0}^{x=l} \frac{M(x)^2}{2EI}dx \quad \text{.......Maple}$$

Spring: $$E_{v2} = \frac{F_v^{\ 2}}{2k} = \frac{F_v^{\ 2} l^3}{2\rho EI}$$

Apply Castigliano and differentiate the strain energy of both parts to Q and F_v respectively, then make sure these are equal using the assumption that the dummy force Q is equal to zero:

$$\frac{\mathrm{d}E_{v1}}{\mathrm{d}Q} = \frac{\mathrm{d}E_{v2}}{\mathrm{d}F_v}$$

Merk op:
Door een drukkracht in de veer aan te nemen is de bijbehorende verplaatsing die wordt verkregen m.b.v. Castigliano een verkorting van de veer (zakking). Op deze manier is deze zakking gelijk van teken als de zakking van het liggeruiteinde t.g.v. de dummy last Q. Dit is een handigheidje voor teken-ellende.

Question 2

Solving this equation gives for the force in the spring:

$$F_v = \frac{\rho}{\rho + 3} \times \tfrac{3}{8} ql$$

Note:
For infinite spring stiffness this answer will also result in the correct answer. Check this!

Question 3

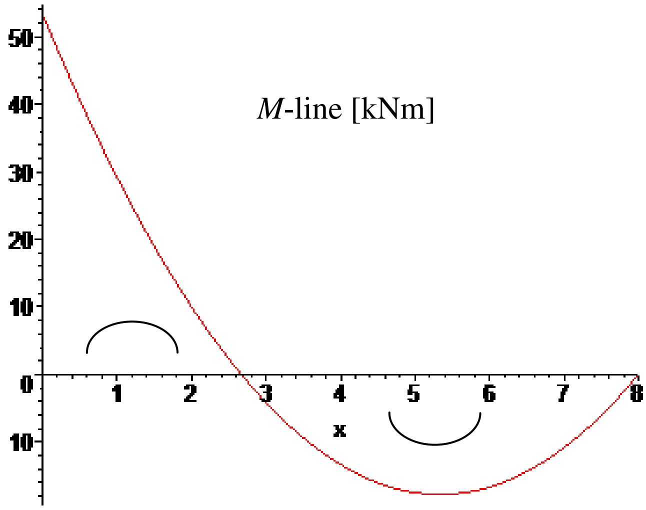
This part is about determining the maximum moment at the span. The location of the extreme moment at the span occurs for zero shear. Using the known force in the spring we can fully determine the bending moment distribution in the beam. The shear force is the first derivative of the bending moment and thus we can find the shear force distribution. The maximum moment is found at the value of x where $V = 0$.

$$x = \frac{(5\rho + 24)}{8(\rho + 3)} \cdot l$$

Using the obtained value for x in the moment distribution results in the maximum moment. By applying the absolute value of the clamping moment and stating that this must equal 3 times the field moment we can solve the equation for ρ. The solution is equal to 24.

Question 4

Using the given parameters the clamping moment is -53,33 kNm.



Complete Maple sheet is shown on following page.

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> restart;
> k:=rho*EI/L^3;
> M:=-(1/2)*q*(L-x)^2-(Q-Fv)*(L-x);
> Ev1:=int(M^2/(2*EI),x=0..L); Ev2:=Fv^2/(2*k);
> w1:=diff(Ev1,Q); w2:=diff(Ev2,Fv);
> Q:=0; #EI:=10000; L:=8; q:=5; rho:=1500;
> eq1:=w1=w2;
> Fv:=solve(eq1,Fv);
> V:=diff(M,x);
> xmax:=solve(V,x);
> x:=xmax; Mmax:=M: evalf(Mmax);
> x:=0; Mi:=M: evalf(Mi);
> eq2:=Mi=(-3*Mmax);
> sol:=solve(eq2,rho); assign(sol);
> evalf(sol[1]); evalf(sol[2]);
> rho:=sol[1];
> evalf(Mi);
> evalf(3*Mmax);
> EI:=10000; L:=8; q:=5;
> evalf(Mi);
> evalf(3*Mmax); evalf(Mmax);
> x:='x';
> plot(-M,x=0..L);
```