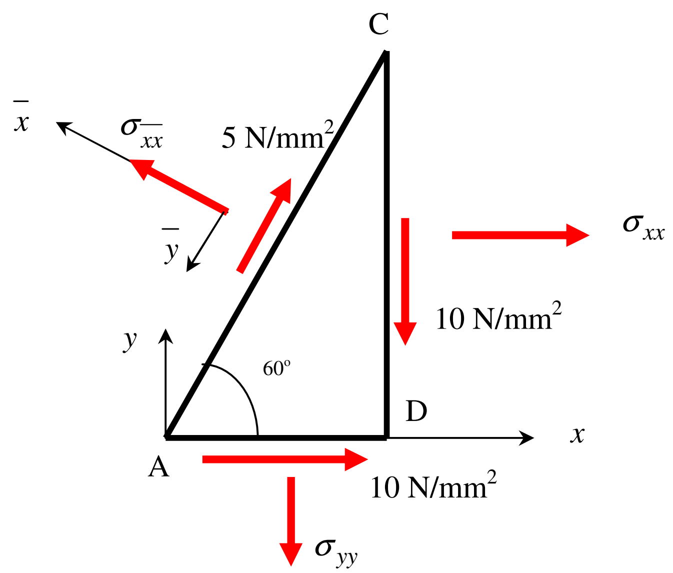
Answer to Problem 2 (page UK49 and NL56)

a) Reduce the given problem to a much simpler problem as shown in the figure below. The shear stress at plane CD is equal to the stress at AD (moment of equilibrium). This problem now has three unknown normal forces at the planes AC, AD and CD. We already have used one equilibrium equation so based upon equilibrium only two unknowns can be found (rigid block in 2D has three equations of equilibrium).



In order to solve this problem a third equation is required and for that we use the given fact that the strain in *y*-direction is given. This results in:

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu\sigma_{xx}}{E} = -1{,}5\times10^{-3} \qquad (1)$$

In order to solve the two unknown normal stresses in *x*- and *y*- direction we need an additional equation with the same unknowns. Using the tensor transformation expression of page 26 we observe:

$$\sigma_{\overline{xy}} = -\tfrac{1}{2}\left(\sigma_{xx} - \sigma_{yy}\right)\sin 2\alpha + \sigma_{xy}\cos 2\alpha$$

The angle α is 150 degrees, the shear stress on the rotated face is -5 and the shear stress on the original *x*- and *y*-plane is -10. This results in:

$$\begin{aligned} -5 &= -\tfrac{1}{2}\left(\sigma_{xx} - \sigma_{yy}\right)\sin 300^{\circ} - 10\cos 300^{\circ} \quad \Leftrightarrow \\ -5 &= -\tfrac{1}{2}\left(\sigma_{xx} - \sigma_{yy}\right)\times\tfrac{1}{2}\sqrt{3} - 10\times\tfrac{1}{2} \qquad \Rightarrow \sigma_{xx} = \sigma_{yy} \end{aligned} \qquad (2)$$

With (2) the result from (1) becomes:

$$\sigma_{xx} = \sigma_{yy} = -10$$

The stress tensor in the x-y- coordinate system is $\sigma = \begin{bmatrix} -10 & -10 \\ -10 & -10 \end{bmatrix}$.

Mohr's circle for the stresses can be constructed. With the stress strain relations the strain tensor can be obtained from the stress tensor and Mohr's circle for the strains can be constructed.