Answer to Problem 3 (page UK50 NL57)

a) Find the stress tensor for this stress situation in the given coordinate system and determine the principal stresses.

From strain gauge G0 and G90 we can compute the normal stresses in the *x*-*y* coordinate system with:

$$\sigma_{xx} = \frac{E}{1-\nu^2}\left(\varepsilon_0 + \nu\varepsilon_{90}\right) = -10 \text{ N/mm}^2$$
$$\sigma_{yy} = \frac{E}{1-\nu^2}\left(\varepsilon_{90} + \nu\varepsilon_0\right) = 20 \text{ N/mm}^2$$

From the two other strain gauges (one reading missing) we can compute the stresses in a 45 degrees rotated coordinate system:

$$\sigma_{\overline{xx}} = \frac{E}{1-\nu^2}\left(\varepsilon_{45} + \nu\varepsilon_{135}\right) = 40000\varepsilon_{45} + 5 \text{ N/mm}^2$$
$$\sigma_{\overline{yy}} = \frac{E}{1-\nu^2}\left(\varepsilon_{135} + \nu\varepsilon_{45}\right) = 10000\varepsilon_{45} + 20 \text{ N/mm}^2$$

The stress invariant I_1 demands:

$$-10 + 20 = 40000\varepsilon_{45} + 5 + 10000\varepsilon_{45} + 20 \quad \Leftrightarrow \quad \varepsilon_{45} = -0{,}0003$$

From this follows:

$$\sigma_{\overline{xx}} = -7 \text{ N/mm}^2$$
$$\sigma_{\overline{yy}} = 17 \text{ N/mm}^2$$

This shows that only three strain readings are required. The fourth in a rozet can be computed. However four readings will certainly improve the quality of the experiment.

To find the shear stress in the rotated system we can use the tensor transformation rule for the shear stress:

$$\sigma_{\overline{xy}} = -\tfrac{1}{2}(\sigma_{xx} - \sigma_{yy})\sin 2\alpha + \sigma_{xy}\cos 2\alpha$$

This results in:

$$\sigma_{\overline{xy}} = -\tfrac{1}{2}(-10 - 20)\sin 90^o + \sigma_{xy}\cos 90^o$$

From this follows:

$$\sigma_{\overline{xy}} = 15 \text{ N/mm}^2$$

To find the required shear stress in the original coordinate system we can use this transformation rule again:

$$\sigma_{xy} = -\tfrac{1}{2}(\sigma_{\bar{x}\bar{x}} - \sigma_{\bar{y}\bar{y}})\sin(-90^o) + \sigma_{\bar{x}\bar{y}}\cos(-90^o) = -\tfrac{1}{2}\left(-7-17\right)(-1) = -12 \text{ N/mm}^2$$

In this way the complete tensors for the original and rotated coordinate system have been obtained:

x-y coordinate system : $\sigma_{ij} = \begin{bmatrix} -10 & -12 \\ -12 & 20 \end{bmatrix}$ N/mm^2

$\bar{x} - \bar{y}$ coordinate system: $\sigma_{ij} = \begin{bmatrix} -7 & 15 \\ 15 & 17 \end{bmatrix}$ N/mm^2

Mohrs circle will result in (or use the analytical expressions):

$\sigma_1 = 24,21$ N/mm^2;
$\sigma_2 = -14,21$ N/mm^2

plane stress!

b) Show with Mohr circle for stresses and strains that the results are consistent with the theory.

With the stress–strain relation the strains can be obtained:

x-y coordinate system : $\varepsilon_{ij} = \begin{bmatrix} -4,0 & -4,0 \\ -4,0 & 6,0 \end{bmatrix} \cdot 10^{-4}$

Consistent requires to show that the principal directions for strains are the same as for the stresses.

c) Find the shear deformation of this specimen.

The shear deformation is twice the ε_{xy} (definition) thus $\gamma_{xy} = -8,0 \times 10^{-4}$

d) Find the safety factor for this stress situation using the von Mises criterion.

The Von Mises criterion is a **quadratic stress** criterion. Since the problem is a plane stress situation, the third principal stress must be zero:

$$\gamma^2 \times \frac{1}{6}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\} - \tfrac{1}{3} f_y^2 \le 0$$

$$\gamma = 1,04$$