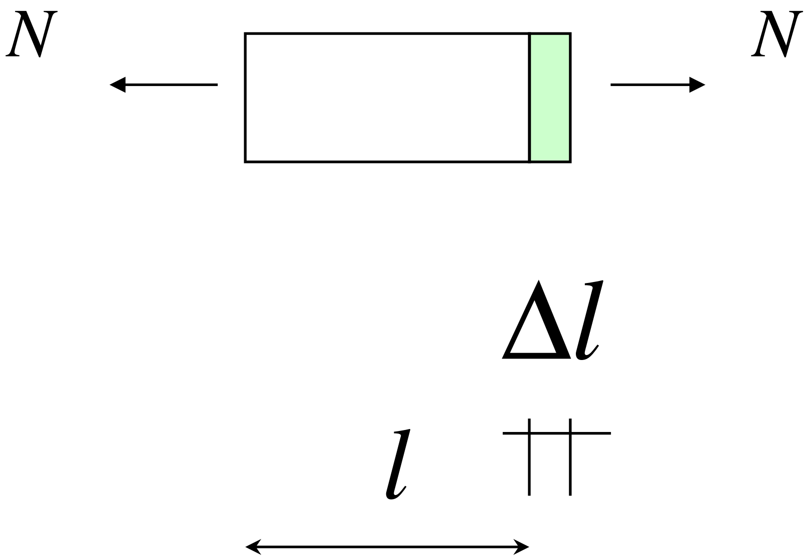
COLLEGE ONDERWERPEN

- 1 Spanningstensor
  - Spanningsdefinitie
  - Spanningstoestanden en voorbeelden
- 2 Rektensor
  - Relatieve verplaatsingen
  - Rekdefinities
  - Rektensor
- 3 Tensoreigenschappen
  - Introductie van tensoren
  - Transformatieregels
  - Cirkel van Mohr
  - Voorbeeld
- 4 Spannings-rek relatie
  - Cirkel van Mohr voor spanning en rek
  - Voorbeelden
- 5 Bezwijktoestanden
  - Spanningen in 3D
  - Von Mises en Tresca

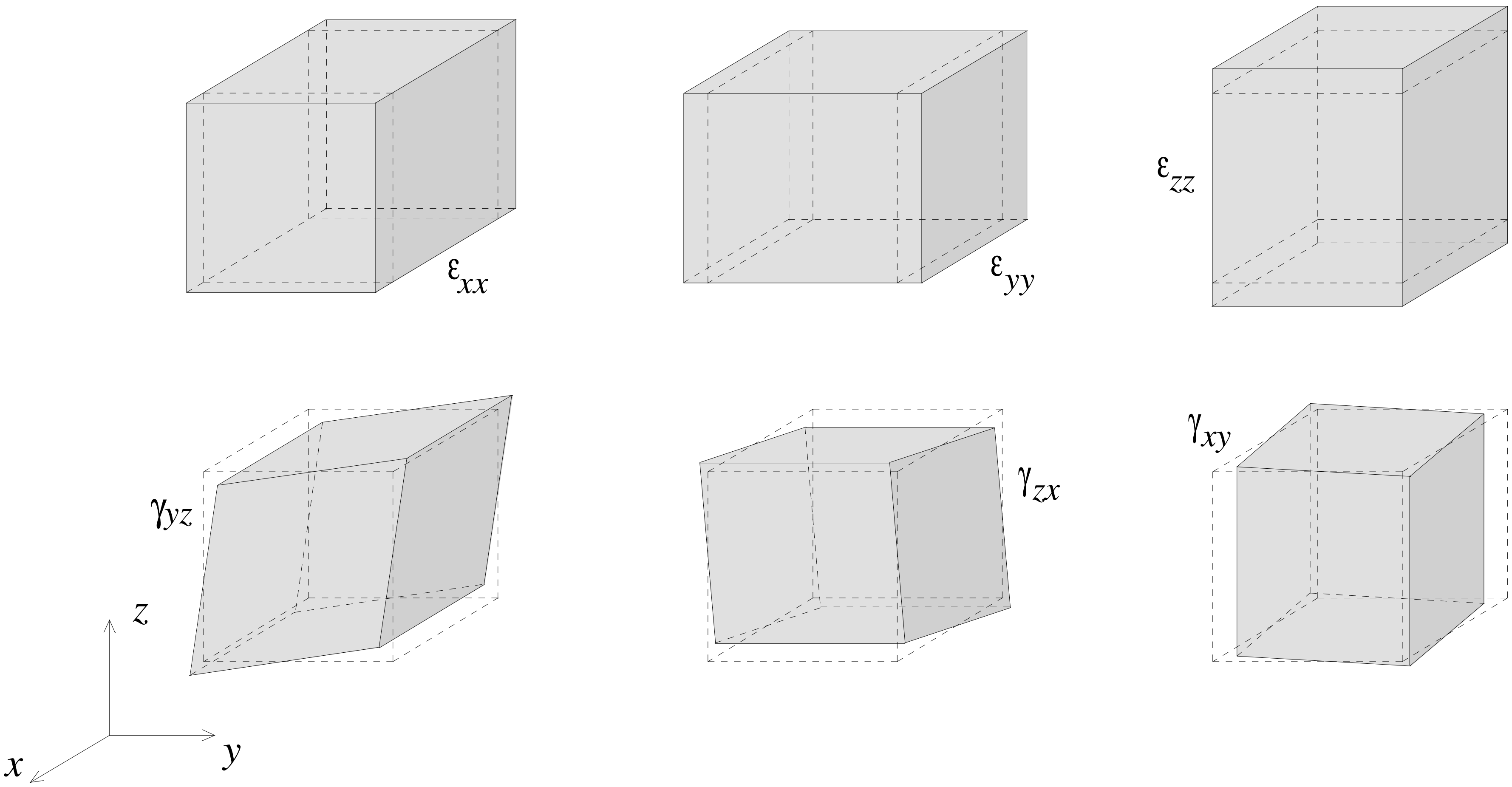
REKKEN



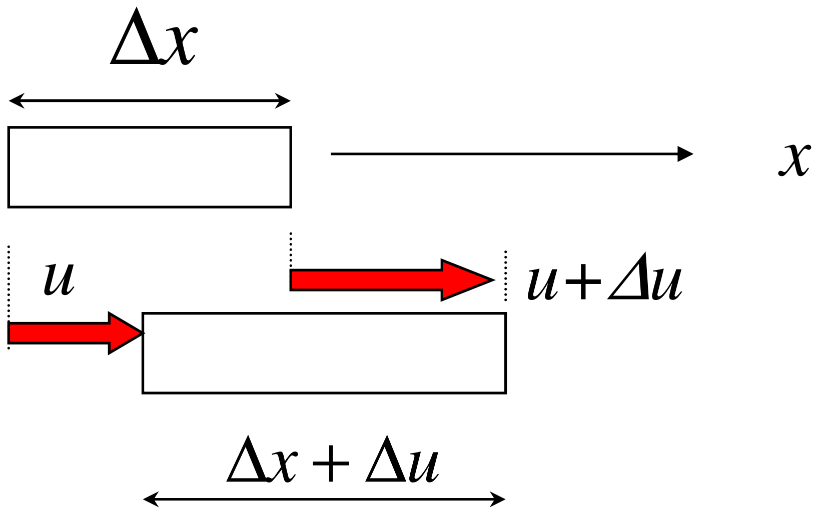
Bekende uitdrukking:

$$\varepsilon = \frac{\Delta l}{l}$$

3D SITUATIE



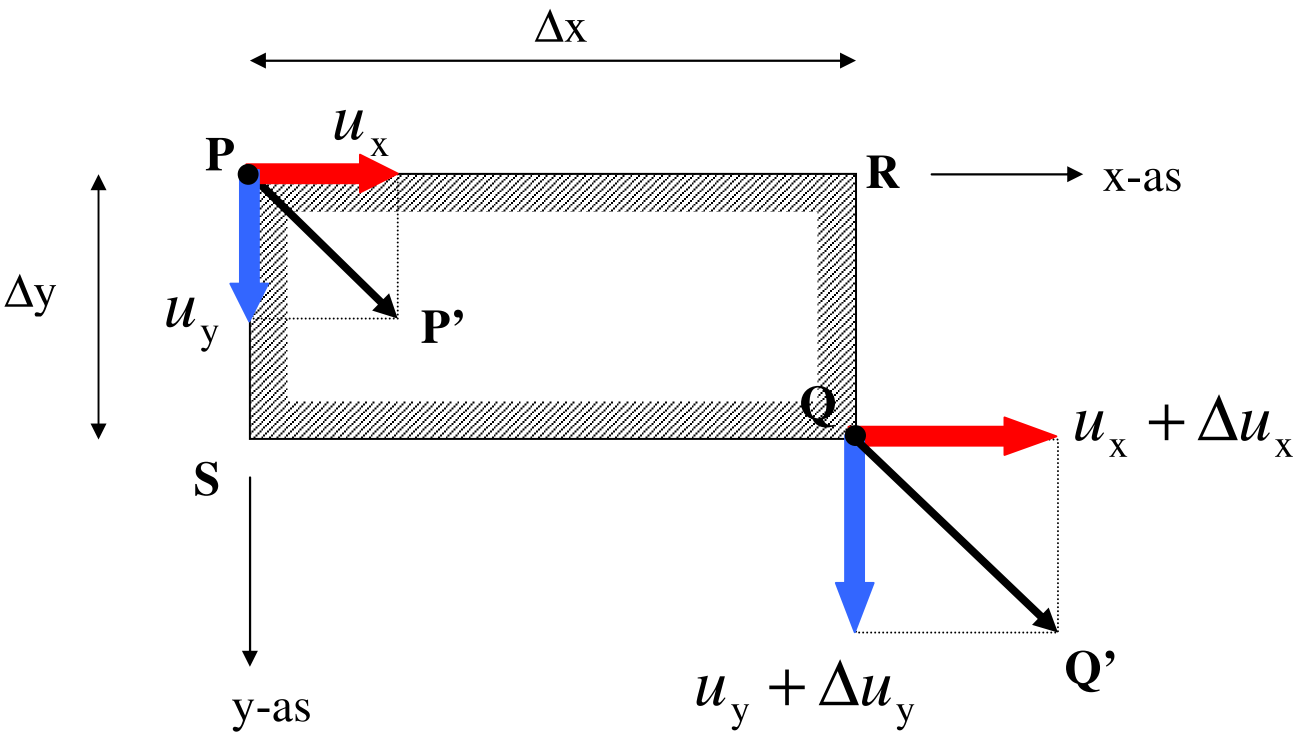
"NETTE" BESCHRIJVING



$$\varepsilon = \frac{\Delta l}{l} = \frac{\Delta u}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{\mathrm{d}u}{\mathrm{d}x}$$

RELATIEVE VERPLAATSINGEN IN 2D



Van het weergegeven stuk materiaal **PRSQ** is een verplaatsingveld gegeven :

$$\begin{cases} u_x = u_x(x, y) \\ u_y = u_y(x, y) \end{cases}$$

Dit verplaastingsveld is continu en continu differentieerbaar.

$$\begin{cases} \Delta u_x = \dfrac{\partial u_x}{\partial x}\Delta x + \dfrac{\partial u_x}{\partial y}\Delta y \\ \Delta u_y = \dfrac{\partial u_y}{\partial x}\Delta x + \dfrac{\partial u_y}{\partial y}\Delta y \end{cases}$$

RELATIEVE VERPLAATSINGEN (VERVOLG)

$$\begin{bmatrix} \Delta u_x \\ \Delta u_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u_x}{\partial x} & \dfrac{\partial u_x}{\partial y} \\ \dfrac{\partial u_y}{\partial x} & \dfrac{\partial u_y}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

LET OP :

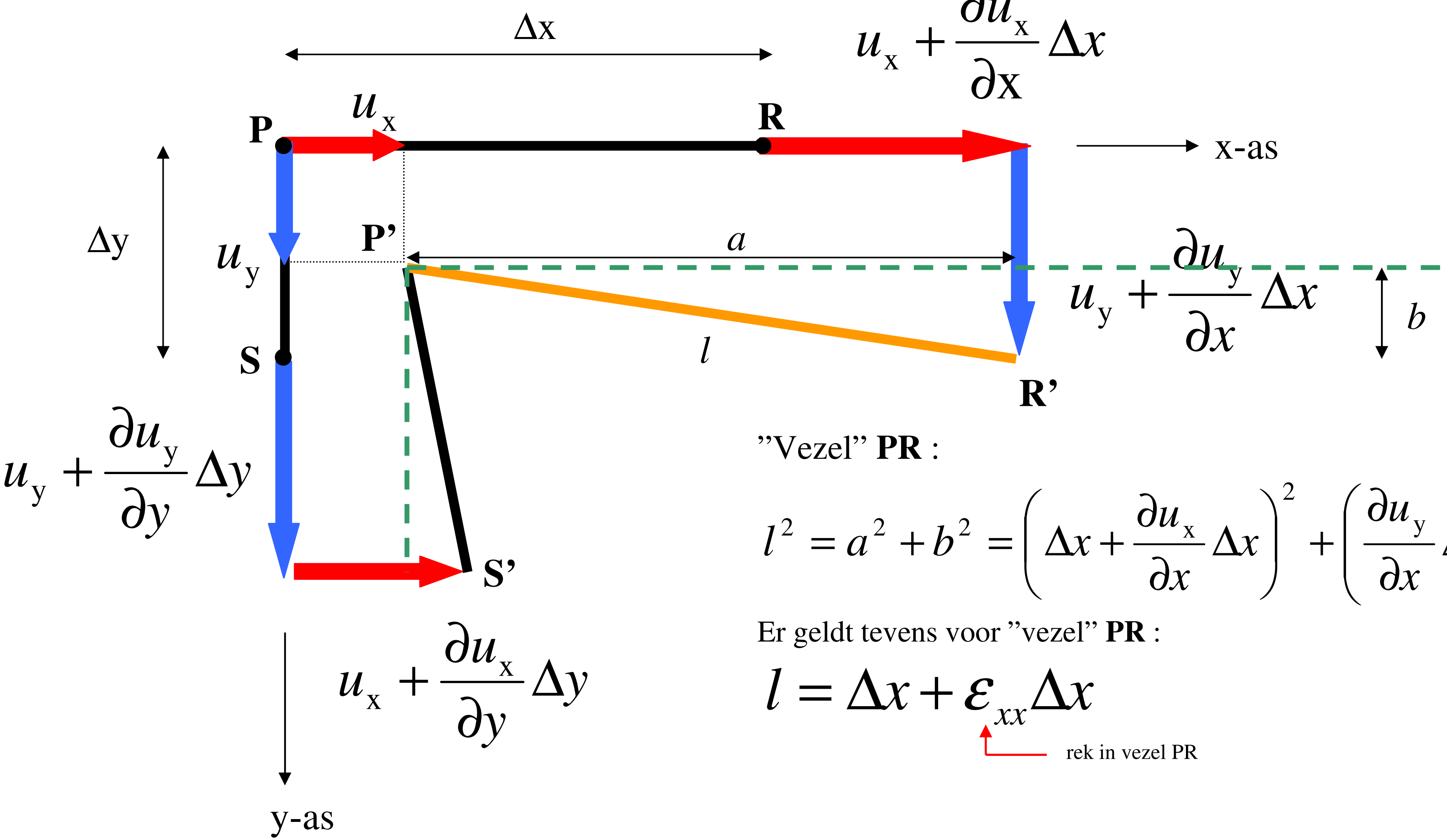
$$\frac{\partial u_x}{\partial y} \neq \frac{\partial u_y}{\partial x}$$

MATRIX MET VERPLAATSINGSGRADIENTEN

vector

vector

RELATIEVE VERPLAATSING VERSUS REK



"Vezel" **PR** :

$$l^2 = a^2 + b^2 = \left(\Delta x + \frac{\partial u_x}{\partial x}\Delta x\right)^2 + \left(\frac{\partial u_y}{\partial x}\Delta x\right)^2$$

Er geldt tevens voor "vezel" **PR** :

$$l = \Delta x + \varepsilon_{xx}\Delta x$$

rek in vezel PR

REKKEN IN DE VEZELS :

UITWERKEN LEVERT :

$$(1+\varepsilon_{\mathrm{xx}})^2.\Delta x^2 = \left(1+\frac{\partial u_{\mathrm{x}}}{\partial x}\right)^2.\Delta x^2 + \left(\frac{\partial u_{\mathrm{y}}}{\partial x}\right)^2.\Delta x^2$$

$$\varepsilon_{\mathrm{xx}} = \sqrt{\left[1+\left(\frac{\partial u_{\mathrm{x}}}{\partial x}\right)\right]^2 + \left(\frac{\partial u_{\mathrm{y}}}{\partial x}\right)^2} - 1 \quad \text{zie bijlage dictaat}$$

Taylor benadering : $\boxed{\varepsilon_{\mathrm{xx}} = \frac{\partial u_{\mathrm{x}}}{\partial x} + \frac{1}{2}\left(\frac{\partial u_{\mathrm{y}}}{\partial x}\right)^2 + \ldots}$

gelineariseerde rek (kleine verplaatsingsgradiënten):

$$\boxed{\varepsilon_{\mathrm{xx}} = \frac{\partial u_{\mathrm{x}}}{\partial x} \quad \text{en dus ook:} \; \varepsilon_{\mathrm{yy}} = \frac{\partial u_{\mathrm{y}}}{\partial y}}$$

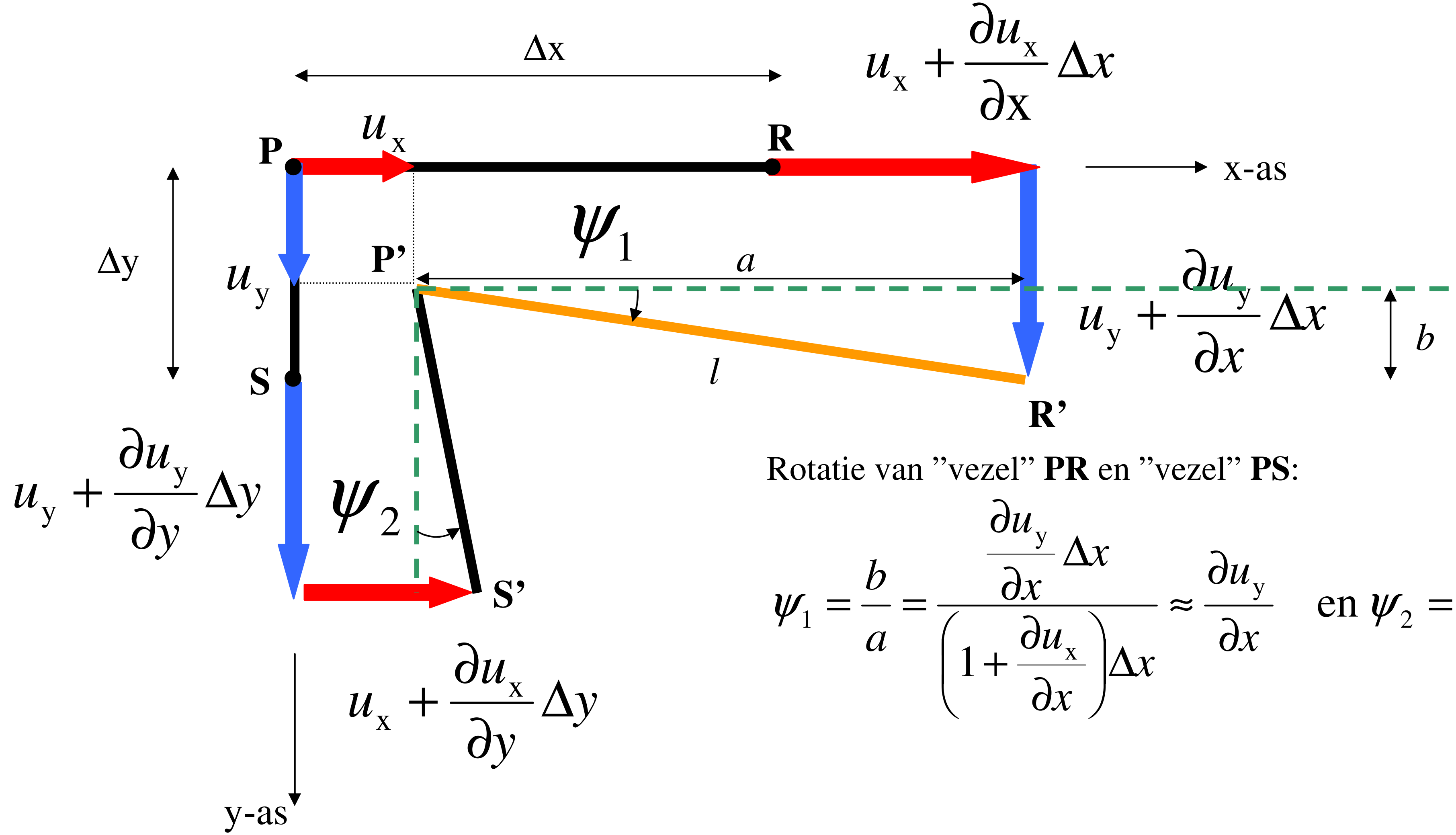
RELATIEVE VERPLAATSING

$$\begin{bmatrix} \Delta u_x \\ \Delta u_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u_x}{\partial x} & \dfrac{\partial u_x}{\partial y} \\ \dfrac{\partial u_y}{\partial x} & \dfrac{\partial u_y}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \qquad \frac{\partial u_x}{\partial y} \neq \frac{\partial u_y}{\partial x}$$

$$\begin{bmatrix} \Delta u_x \\ \Delta u_y \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \boxed{\dfrac{\partial u_x}{\partial y}} \\ \boxed{\dfrac{\partial u_y}{\partial x}} & \varepsilon_{yy} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

WAT STELEN DE NIET DIAGONAAL-TERMEN VOOR VAN DEZE NIET-SYMMETRISCHE MATRIX ?

NIET-DIAGONAALTERMEN IN DE TENSOR VAN VERPLAATSINGSGRADIENTENTEN



Rotatie van "vezel" **PR** en "vezel" **PS**:

$$\psi_1 = \frac{b}{a} = \frac{\dfrac{\partial u_y}{\partial x}\Delta x}{\left(1+\dfrac{\partial u_x}{\partial x}\right)\Delta x} \approx \frac{\partial u_y}{\partial x} \quad \text{en } \psi_2 = \frac{\partial u_x}{\partial y}$$

TU Delft

RESULTAAT

$$\begin{bmatrix} \Delta u_x \\ \Delta u_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u_x}{\partial x} & \dfrac{\partial u_x}{\partial y} \\ \dfrac{\partial u_y}{\partial x} & \dfrac{\partial u_y}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \qquad \frac{\partial u_x}{\partial y} \neq \frac{\partial u_y}{\partial x}$$

invullen :

$$\begin{bmatrix} \Delta u_x \\ \Delta u_y \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \psi_2 \\ \psi_1 & \varepsilon_{yy} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \qquad \psi_1 \neq \psi_2$$

HELP:

Deze matrix is niet symmetrisch !

TRUC : symmetrisch maken

SYMMETRISCH MAKEN

$$\begin{bmatrix} \varepsilon_{xx} & \psi_2 \\ \psi_1 & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \tfrac{1}{2}\psi_1 + \tfrac{1}{2}\psi_2 \\ \tfrac{1}{2}\psi_1 + \tfrac{1}{2}\psi_2 & \varepsilon_{yy} \end{bmatrix} + \begin{bmatrix} 0 & -\tfrac{1}{2}\psi_1 + \tfrac{1}{2}\psi_2 \\ \tfrac{1}{2}\psi_1 - \tfrac{1}{2}\psi_2 & 0 \end{bmatrix}$$

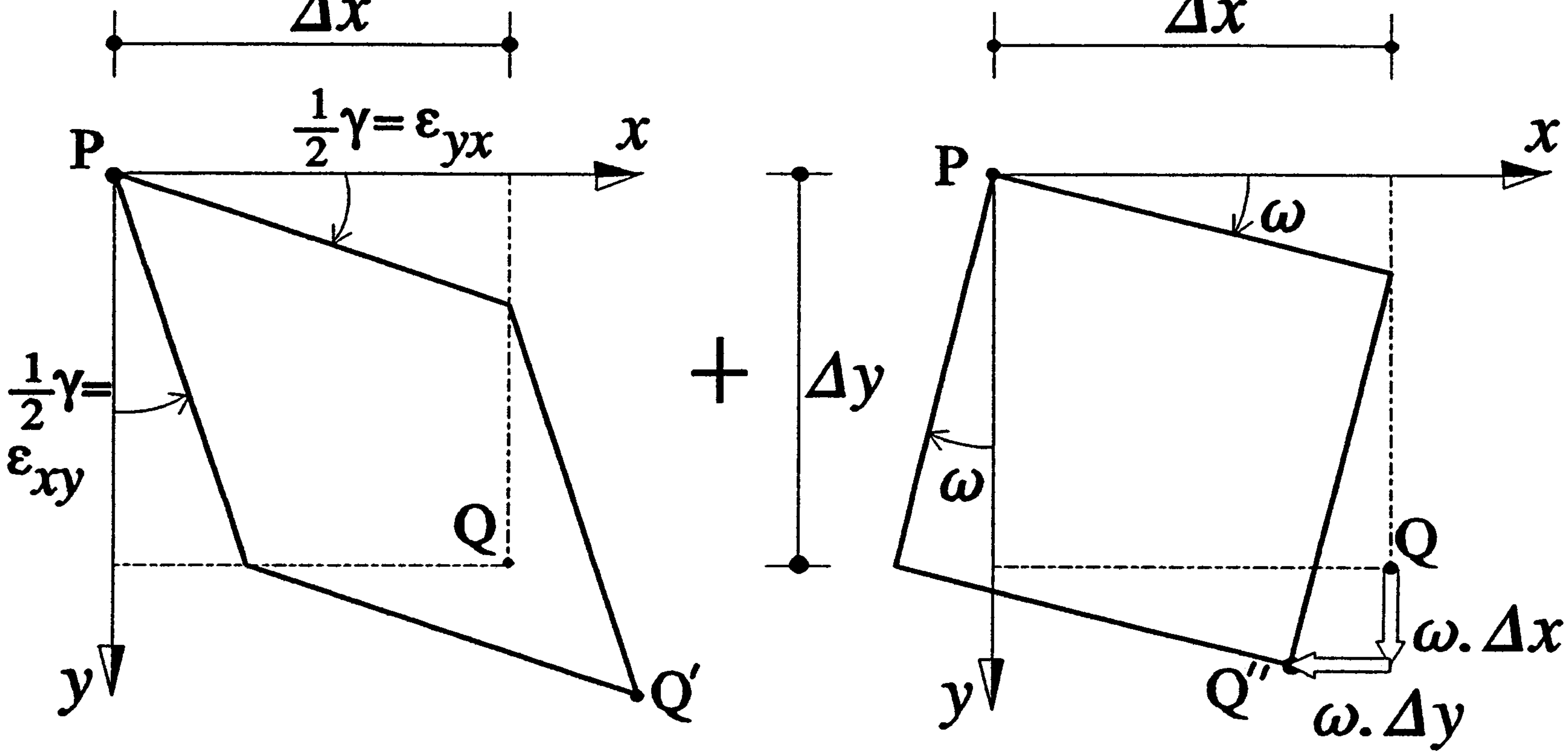
$$\begin{bmatrix} \varepsilon_{xx} & \psi_2 \\ \psi_1 & \varepsilon_{yy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix} + \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \quad \text{met}: \varepsilon_{xy} = \varepsilon_{yx}$$

REKTENSOR
(symmetrisch)

STARRE ROTATIE
levert geen **vervorming** op, wel een verplaatsing !

TU Delft

VERVORMING EN STARRE ROTATIE



(zie ook dictaat)

TU Delft

REKKEN

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) = \tfrac{1}{2}\gamma_{xy}$$ ← AFSCHUIFVERVORMING

TENSORNOTATIE:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial j} + \frac{\partial u_j}{\partial i}\right) \qquad (i, j) = (x, y)$$

STARRE ROTATIE

$$\omega_{xy} = -\omega = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x}\right)$$

TENSORNOTATIE:

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial j} - \frac{\partial u_j}{\partial i}\right) \qquad (i, j) = (x, y)$$

TU Delft

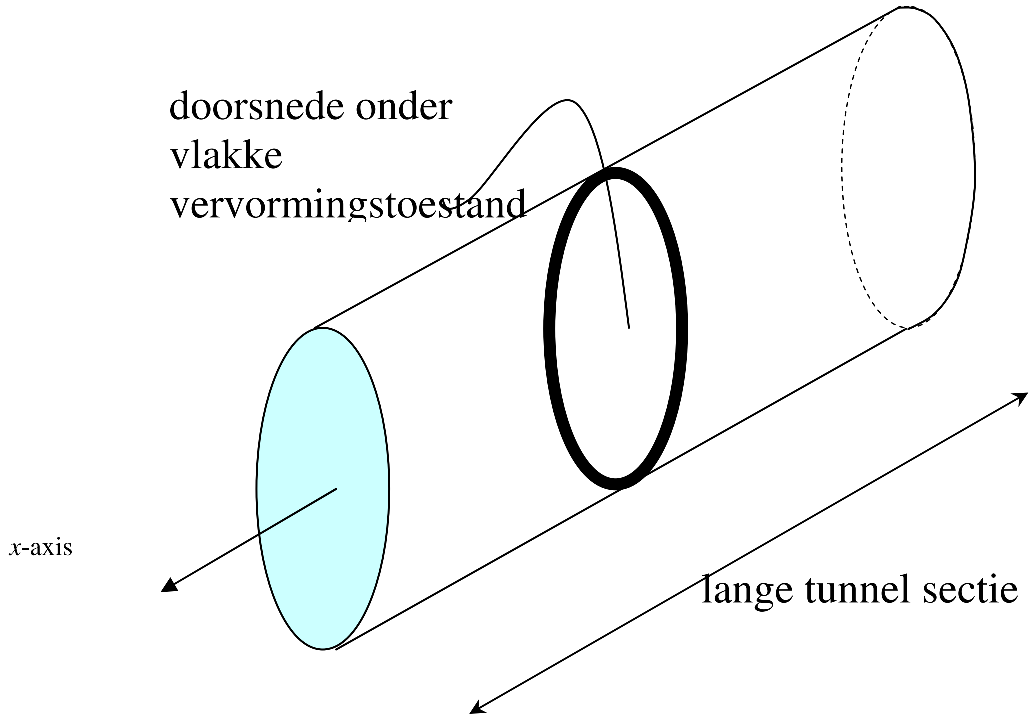
GENERALISATIE NAAR 3D

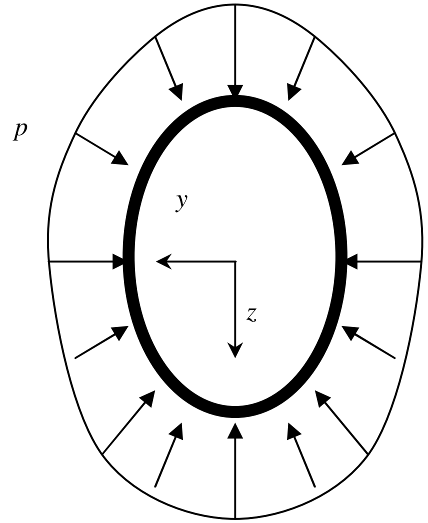
$$\begin{bmatrix} \Delta u_x \\ \Delta u_y \\ \Delta u_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \begin{bmatrix} 0 & \omega_{xy} & \omega_{xz} \\ \omega_{yx} & 0 & \omega_{yz} \\ \omega_{zx} & \omega_{zy} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

rek matrix *starre lichaam rotaties*

(deformaties) *(geen deformatie)*

BIJZONDER GEVAL : Vlakke vervormingstoestand

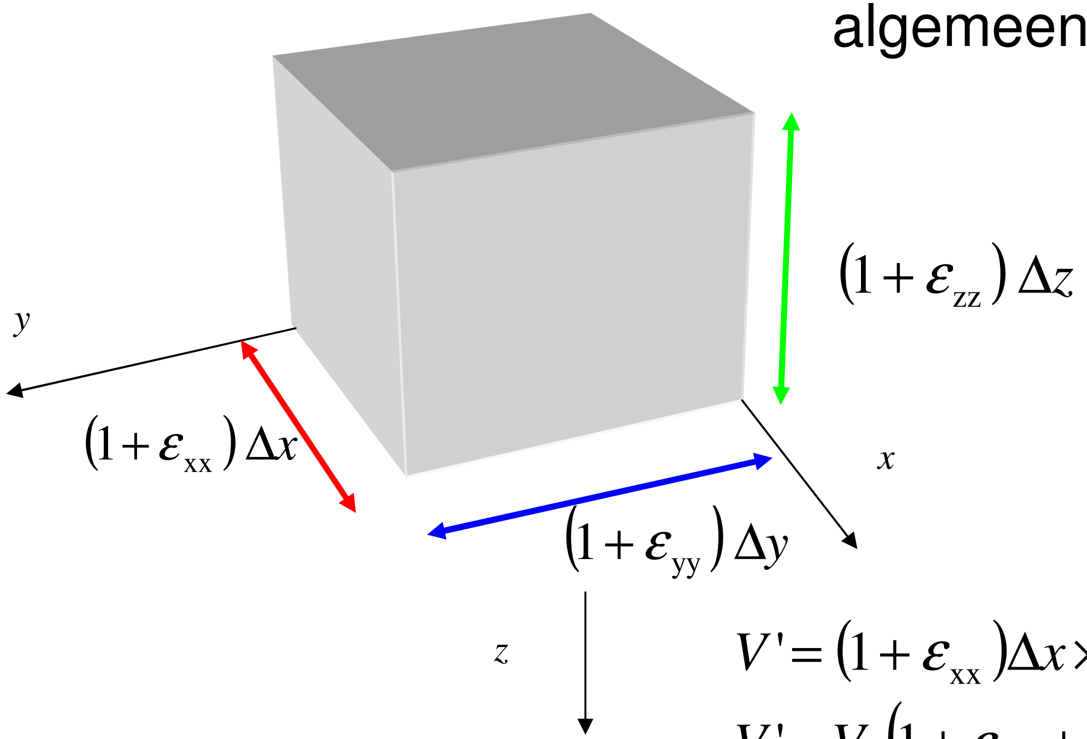




tunnel doorsnede

In een vlak geen rek, b.v. $\varepsilon_{xx} = \varepsilon_{xy} = \varepsilon_{xz} = 0$

algemeen in 3-D



Oorspronkelijk volume :

$$V = \Delta x.\Delta y.\Delta z$$

$$V' = (1+\varepsilon_{xx})\Delta x \times (1+\varepsilon_{yy})\Delta y \times (1+\varepsilon_{zz})\Delta z$$

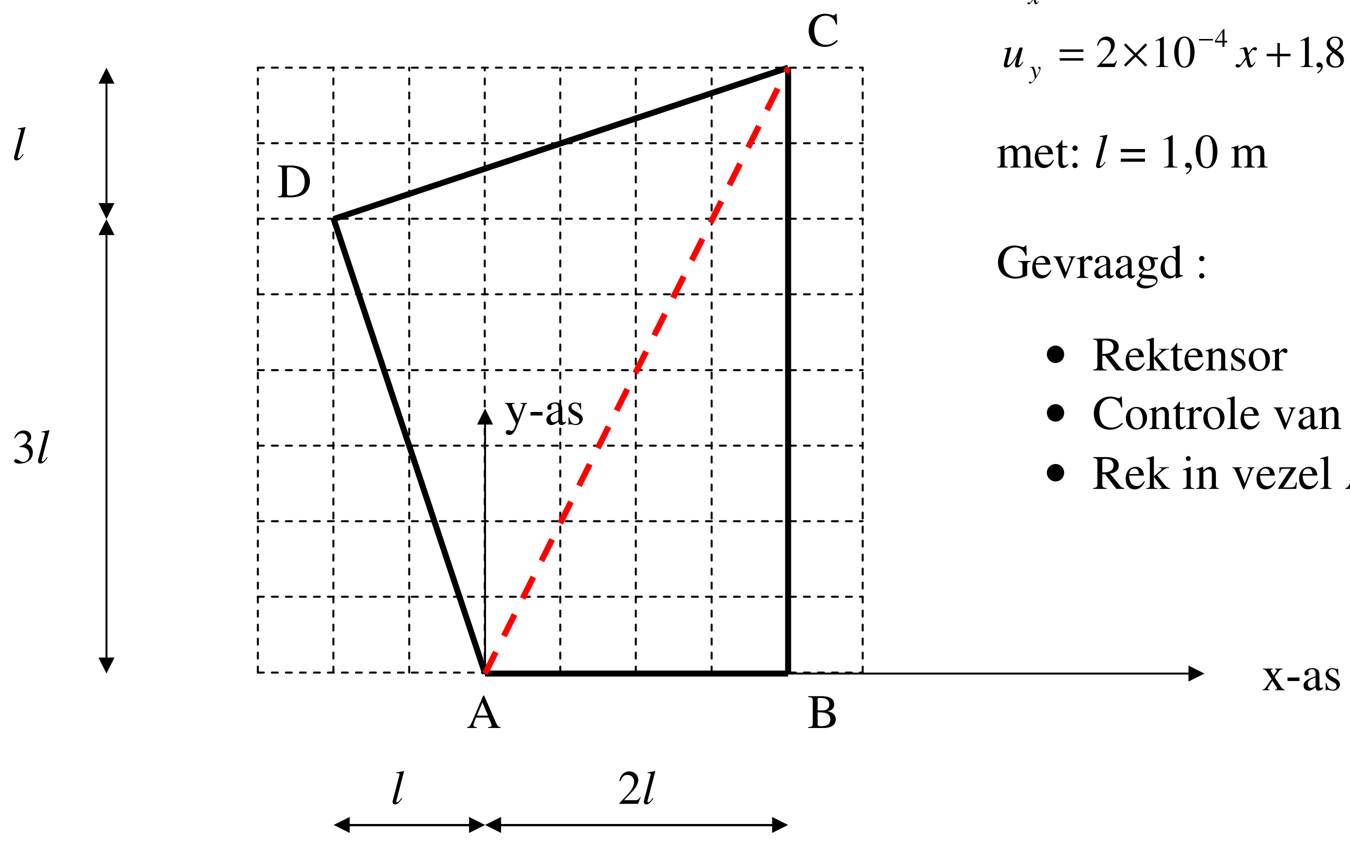
$$V' = V\left(1+\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}+\varepsilon_{xx}\varepsilon_{yy}+\varepsilon_{yy}\varepsilon_{zz}+\varepsilon_{zz}\varepsilon_{xx}+\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz}\right)$$

$$V' \cong V\left(1+\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}\right)$$

dus

$$e_o = \frac{\Delta V}{V} = \frac{V'-V}{V} = \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}$$

VOORBEELD



Gegeven:

$$u_x = 0{,}2 \times 10^{-4} + 0{,}3 \times 10^{-4}\, x$$

$$u_y = 2 \times 10^{-4}\, x + 1{,}8 \times 10^{-4}\, y$$

met: l = 1,0 m

Gevraagd :

- Rektensor
- Controle van verplaatsingen
- Rek in vezel AC