Example 6 : Core of a part of an inhomogeneous cross section

An inhomogeneous cross section contains four different materials 1-4. Each part has a cross section of 40x40 mm^2 . From the entire cross section the cross sectional properties are given as:

$$EA = 624\times10^{6}\ \text{N}$$
$$EI_{yy} = 260{,}43\times10^{9}\ \text{Nmm}^2$$
$$EI_{yz} = EI_{zy} = 59{,}07\times10^{9}\ \text{Nmm}^2$$
$$EI_{zz} = 324{,}76\times10^{9}\ \text{Nmm}^2$$

De location of the normal force centre NC is shown in figure 1.

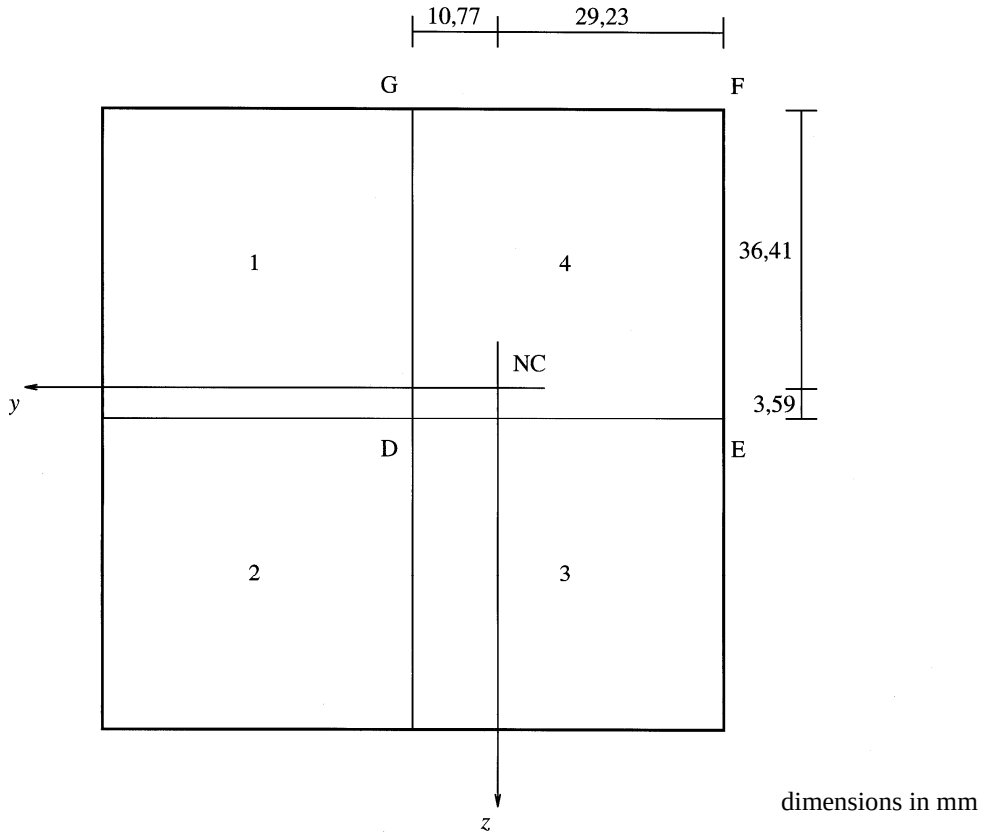

Figure 1 : Inhomogeneous cross section

Question:

The cross section is loaded with a compressive force only. Find the area for the point of application of this compressive force in such a way that material 4 (DEFG) is in compression only.

Solution

Each corner point of the requested area can be found by situating the neutral axis just tangent to the area DEFG. Material 4 has four edges so four situations have to be addressed. The standard relations for the eccentricity of the point of application of the normal force with a neutral axis tangent to the cross section can be used:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{zy} & EI_{zz} \end{bmatrix} \begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

In numbers:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{624\times10^6} \begin{bmatrix} 260,43\times10^9 & 59,07\times10^9 \\ 59,07\times10^9 & 324,76\times10^9 \end{bmatrix} \begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

or:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \begin{bmatrix} 417,36 & 94,66 \\ 94,66 & 520,45 \end{bmatrix} \begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

The computation is summarised in the table below:

Tabel : Results

n.a.	y_1	z_1	e_y	e_z	**"core"**
DE	∞	+3,59	-26,37	-144,97	P
EF	-29,23	∞	+14,28	+3,24	Q
FG	∞	-36,41	+2,60	+14,29	R
GD	+10,77	∞	-38,75	-8,79	S

With this data the core for material 4 can be drawn. The core shown in figure 2 is not the core for the entire cross section, only for material 4. A remarkable result is the fact that the area of the core extends far outside the physical cross section.

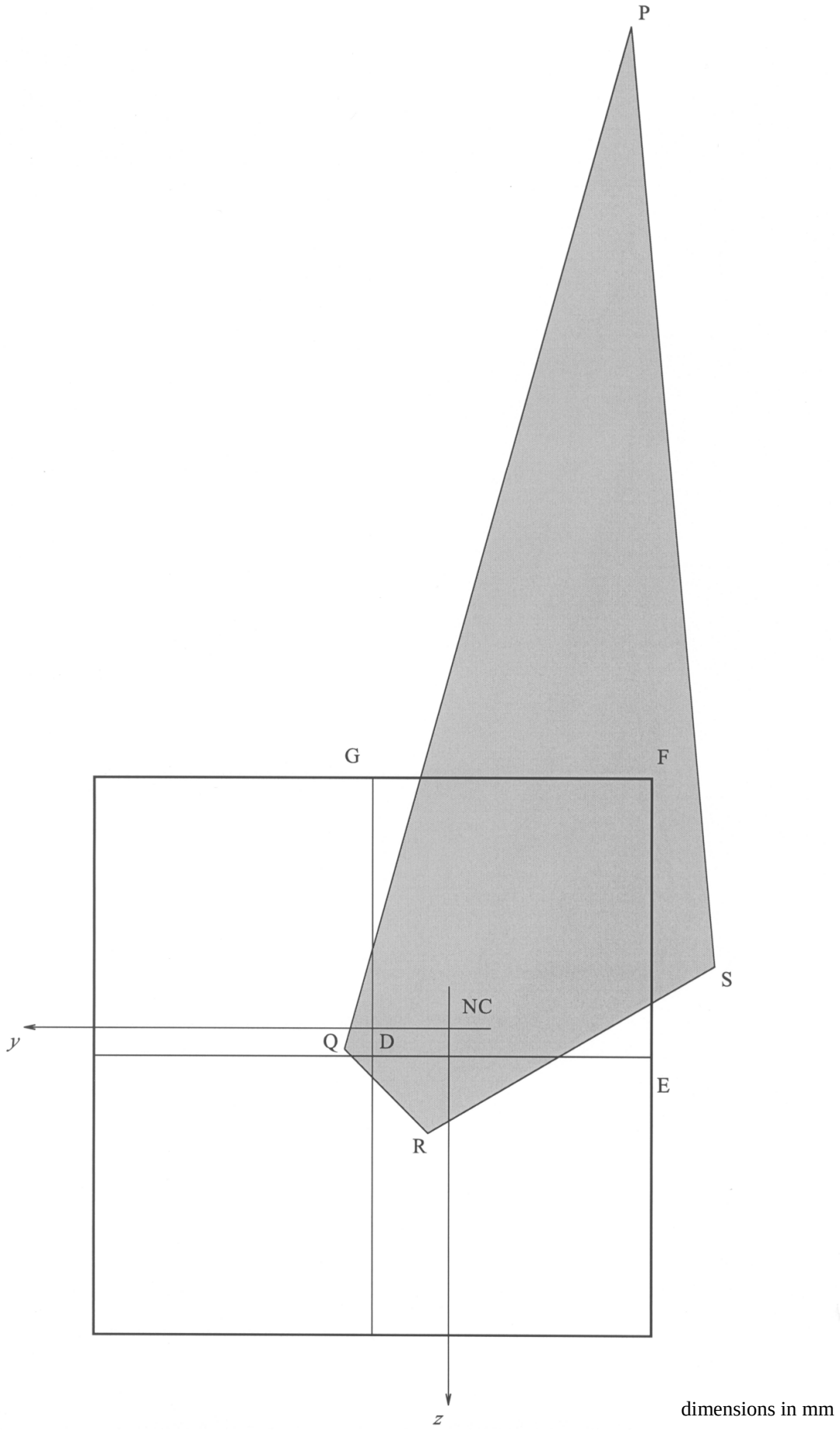

Figure 2: Core of material 4 (DEFG) of the inhomogeneous cross section