Example 7 : Concrete-Steel column

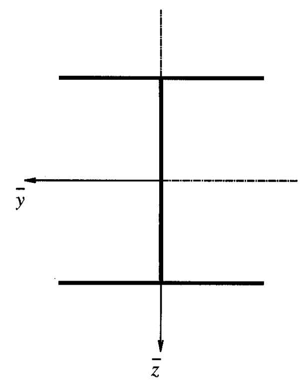
A concrete column AB with length l, is reinforced with a steel section. The steel section is not centred in the column as is shown in figure 1. In this figure all relevant data is given like the dimensions and the cross sectional properties of the steel section. The cantilever column is fully fixed at A and free at B in which the concentrated compressive force acts as indicated. The concrete and steel are perfectly bonded and respond linear elastically. In this example the nett concrete section will be used.

$F = 2500$ kN

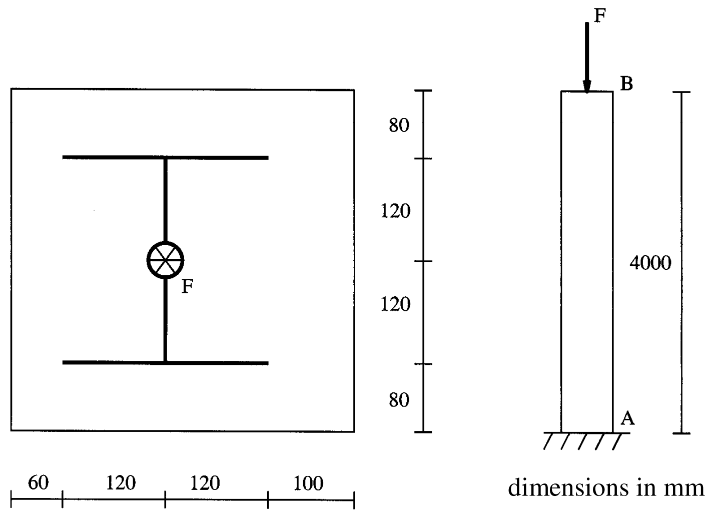
$l = 4000$ mm

Young's modulus concrete $E_c = 20\times10^3$ N/mm^2

Young's modulus steel $E_s = 210\times10^3$ N/mm^2



$A = 10^4$ mm^2
$I_{\bar{y}\bar{y}} = 4\times10^7$ mm^4
$I_{\bar{z}\bar{z}} = 11.25\times10^7$ mm^4



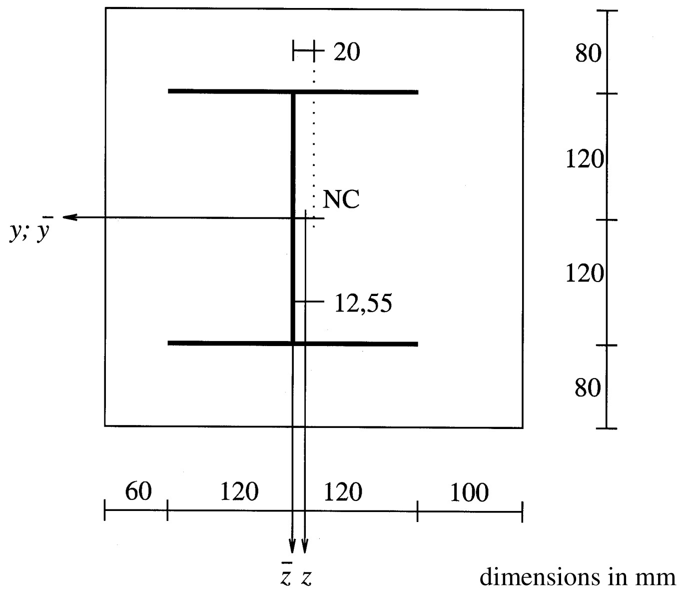
dimensions in mm

Question:

a) Find the location of the normal force centre NC,
b) Compute the axial and bending stiffness properties for the composite cross section,
c) Find the normal force N and the components of the bending moment M both in magnitude and with the correct sign,
d) Draw the strain and stress distribution for the entire cross section,
e) Compute the change of length of this column,
f) Compute the horizontal displacement of point B.

Solution

a)



dimensions in mm

Chose a $\bar{y}-\bar{z}-$coordinate system through the normal force centre of the steel section. The normal force centre NC of the entire cross section must be located at the line of symmetry (which is the $\bar{y}-$axis, thus $\bar{z}_{\text{NC}}=0$).

For the other direction yields: $\bar{y}_{\text{NC}} = ES_{\bar{y}} / EA$.

$$EA = (EA)_c + (EA)_s = 20\times10^3\times(400^2-10^4)+210\times10^3\times10^4 = 5.1\times10^9 \text{ N}$$

$$ES_{\bar{y}} = (ES_{\bar{y}})_c = 20\times10^3\times400^2\times(-20) = -64\times10^9$$

Note: For both the missing concrete and present steel part the $S_{\bar{y}}=0$. The missing concrete part is subtracted from the total (external) concrete dimensions.

$$\bar{y}_{\text{NC}} = -64\times10^9/5.1\times10^9 = -12.55 \text{ mm}$$

b) The axial stiffness *EA* was already computed: $EA = 5.1\times10^9$ N.
The bending stiffness components are EI_{yy} and EI_{zz}, defined in the *y*-*z*-coordinate system through the normal force centre NC.
Since the *y*-axis is a axis of symmetry, both EI_{yy} and EI_{zz} are principal values and the cross product $EI_{yz} = EI_{zy} = 0$.

$$EI_{yy} = E_c \left\{ \frac{1}{12} h^4 + h^2 (20 - 12.55)^2 \right\} + (E_s - E_c)\left(4 \times 10^7 + 10^4 \times 12.55^2\right) =$$

$$= 20 \times 10^3 \left\{ \frac{1}{12} \times 400^4 + 400^2 \times 7.45^2 \right\} + 190 \times 10^3 \times \left(4 \times 10^7 + 10^4 \times 12.55^2\right) =$$

$$= 50.774 \times 10^{12} \text{ Nmm}^2$$

$$EI_{zz} = E_c \times \frac{1}{12} h^4 + (E_s - E_c) \times 11.25 \times 10^7 =$$

$$= 20 \times 10^3 \times \frac{1}{12} \times 400^4 + 190 \times 10^3 \times 11.25 \times 10^7 =$$

$$= 64.042 \times 10^{12} \text{ Nmm}^2$$

c) The normal force N and the bending moment M are constant over the entire length of the column:

$N = -2500 \text{ kN} = -2.5 \times 10^6 \text{ N}$

$e_y = +12.55 \text{ mm}$; $M_y = N \times e_y = -2.5 \times 10^6 \times 12.55 = -31.375 \times 10^6 \text{ Nmm}$

$e_z = 0$; $M_z = N \times e_z = 0$

d) $\varepsilon = N / EA = -2.5 \times 10^6 / 5.1 \times 10^9 = -0.49 \times 10^{-3}$

$\kappa_y = M_y / EI_{yy} = -31.375 \times 10^6 / 50.774 \times 10^{12} = -0.618 \times 10^{-6} \text{ mm}^{-1}$

$\kappa_z = M_z / EI_{zz} = 0$

(The above used expressions for κ_y and κ_z are only valid if EI_{yy} and EI_{zz} are principal values and thus $EI_{yz} = EI_{zy} = 0$!)

$$\varepsilon(y,z) = \varepsilon + \kappa_y y + \kappa_z z$$

$$= -0.49 \times 10^{-3} - 0.618 \times 10^{-6} \times y \quad (y \text{ in mm})$$

The extreme values are found at:

$y = 192.55 \text{ mm} \quad \rightarrow \varepsilon = -0.609 \times 10^{-3}$

$y = -207.45 \text{ mm} \quad \rightarrow \varepsilon = -0.362 \times 10^{-3}$

The neutral axis runs parallel to the z-axis and intersects with the y-axis at

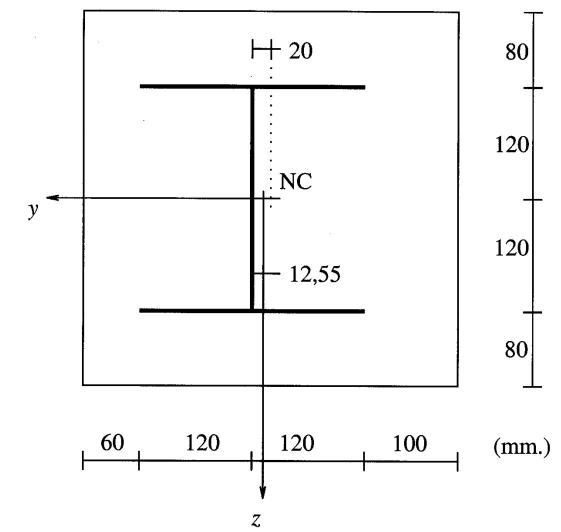
$y_1 = -0.49 \times 10^{-3} / 0.618 \times 10^{-6} = -792.88 \text{ mm}$

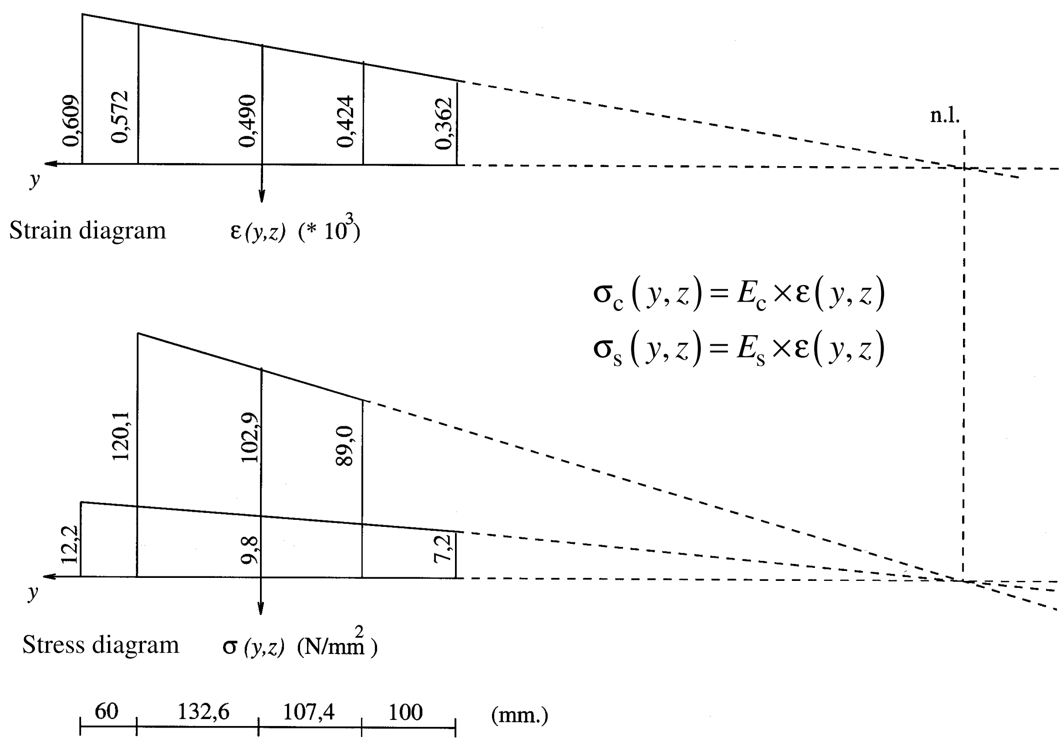
Concrete stress: $\sigma_c(y,z) = E_c \times \varepsilon(y,z)$

Steel stress: $\sigma_s(y,z) = E_s \times \varepsilon(y,z)$

e) $\Delta l = Nl / EA = -2.5 \times 10^6 \times 4000 \ / \ 5.1 \times 10^9 = -1.96 \text{ mm}$

The change of length of the column is negative which, so a shortening of roughly 2 mm.





$$\sigma_c(y,z) = E_c \times \varepsilon(y,z)$$
$$\sigma_s(y,z) = E_s \times \varepsilon(y,z)$$

f) The column bends in the *x*-*y*-plane. Since the *y*-direction is a principal direction the "*forget-me-nots*" can be used to find the displacement of point B:

$$u_y(B) = -\frac{M_y\, l^2}{2EI_{yy}} = \frac{-31.375\times10^6\times4000^2}{2\times50.774\times10^{12}} = +4.94 \text{ mm} \approx 5 \text{ mm}$$

$$u_z(B) = 0$$