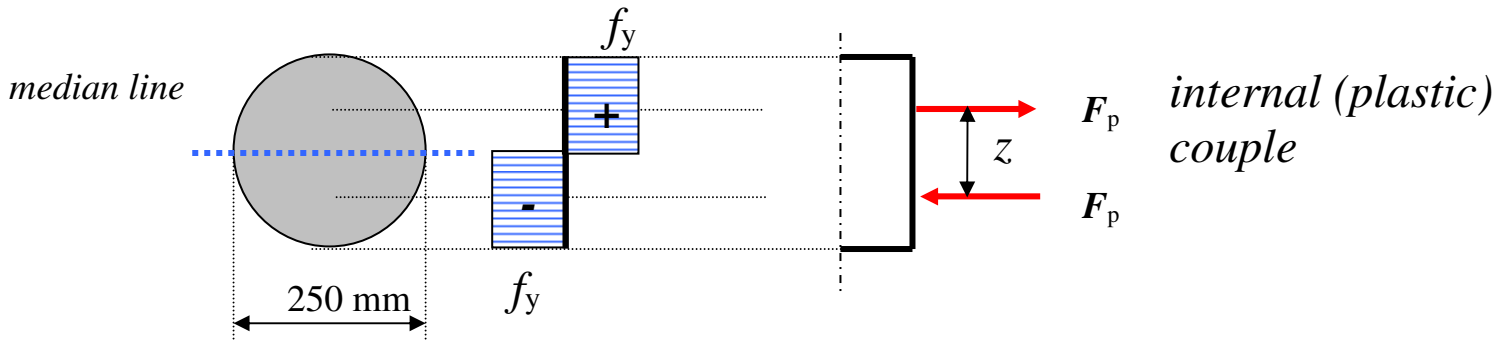
FIND THE SHAPE FACTOR FOR A CIRCULAR CROSS SECTION

- The median line (horizontal equilibrium)
- The full plastic moment M_p
- The elastic moment M_e

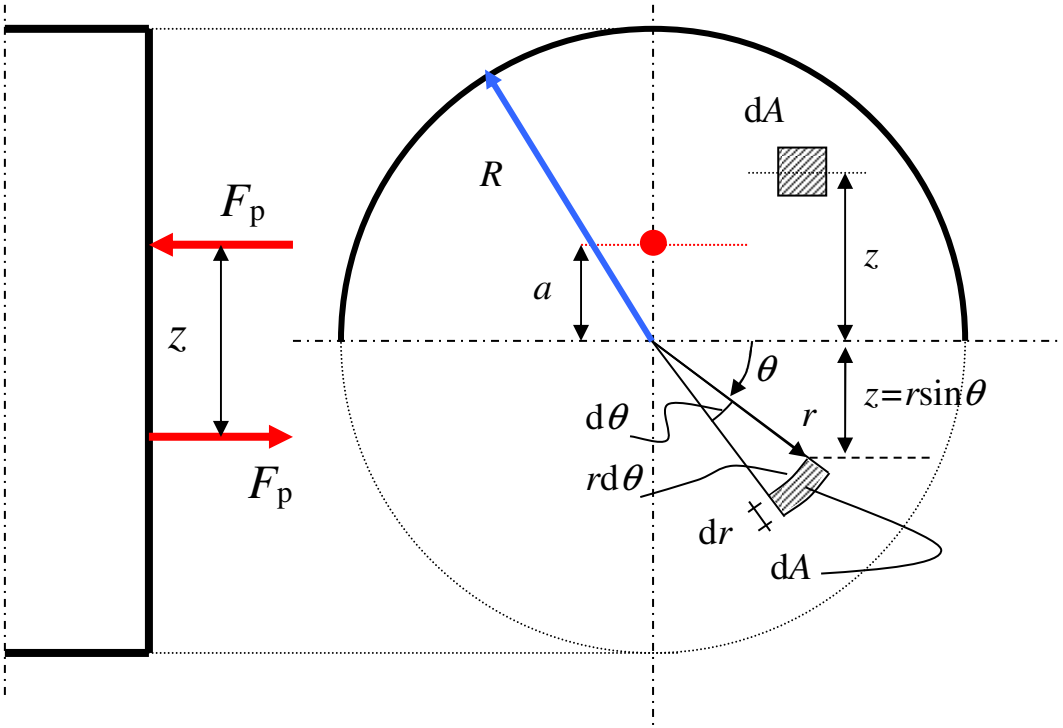
} Shape factor : $\alpha = M_p / M_e$



$$M_p = F_p \, z$$

TU Delft

CENTRE OF GRAVITY & MOMENT OF INERTIA & SHAPE FACTOR



centre of gravity : (half of the circle)

$$a = \frac{\int_A z\,dA}{\int_A dA} = \frac{\int_A z\,dA}{\frac{1}{2}\pi R^2}$$

use polar coordinates : $z = r\sin\theta$ *and* $dA = r\,d\theta\,dr$

$$a = \frac{\iint_{r\,\theta} r^2 \sin\theta\,d\theta\,dr}{\frac{1}{2}\pi R^2} = \frac{\frac{1}{3}R^3\int_0^{\pi}\sin\theta\,d\theta}{\frac{1}{2}\pi R^2} = \frac{\frac{2}{3}R^3}{\frac{1}{2}\pi R^2} = \frac{4R}{3\pi}$$

moment of inertia : (*full circle*)

$$I_{zz} = \int_A z^2 dA$$

use polar coordinates : $z = r\sin\theta$ *and* $dA = r\,d\theta\,dr$

$$I_{zz} = \iint_{r\,\theta} r^2\sin^2\theta\; r\,d\theta\,dr = \iint_{r\,\theta} r^3\sin^2\theta\,d\theta\,dr = \tfrac{1}{4}R^4\int_{\theta=0}^{\theta=2\pi}\sin^2\theta\,d\theta$$

$$I_{zz} = \tfrac{1}{4}R^4\int_{\theta=0}^{\theta=2\pi}\left(\tfrac{1}{2}-\tfrac{1}{2}\cos 2\theta\right)d\theta = \frac{\pi R^4}{4}$$

$$M_e = \frac{I_{zz} f_y}{R} = \frac{\pi R^4 f_y}{4R} = \tfrac{1}{4}\pi R^3 f_y$$

$$\alpha = \frac{M_p}{M_e} = \frac{F_p\,2a}{M_e} = \frac{\frac{1}{2}\pi R^2 f_y\,\frac{8}{3}R/\pi}{\frac{1}{4}\pi R^3 f_y} = \frac{16}{3\pi} = 1.70$$