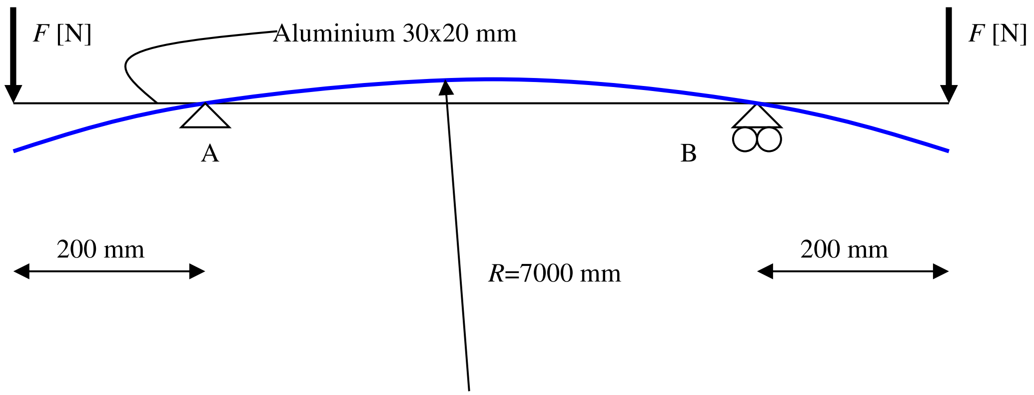
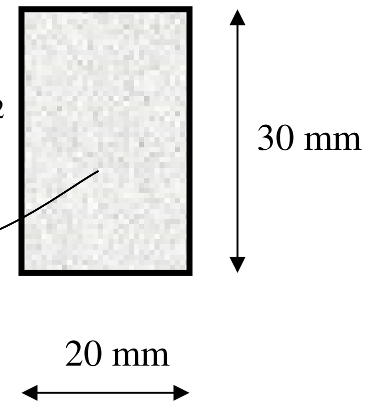
Given:



Aluminium :

$f_y = 60 \text{ N/mm}^2$

$E = 0.8 \times 10^5 \text{ N/mm}^2$



Question:

1) Find the force ***F*** in N on the cantilever for the given *constant* curvature radius between A and B of 7.0 m.
2) Find the bending stiffness *EI* of part AB for this load.

ANALYSIS

Curvature radius	R	
Curvature	κ	
Strain	ε	= *strain diagram* for the cross section
Stress	σ	= *stress diagram* for the cross section

IS THE CROSSSECTION:

- Elastic ?
- Elasto-plastic ?
- Plastic ?

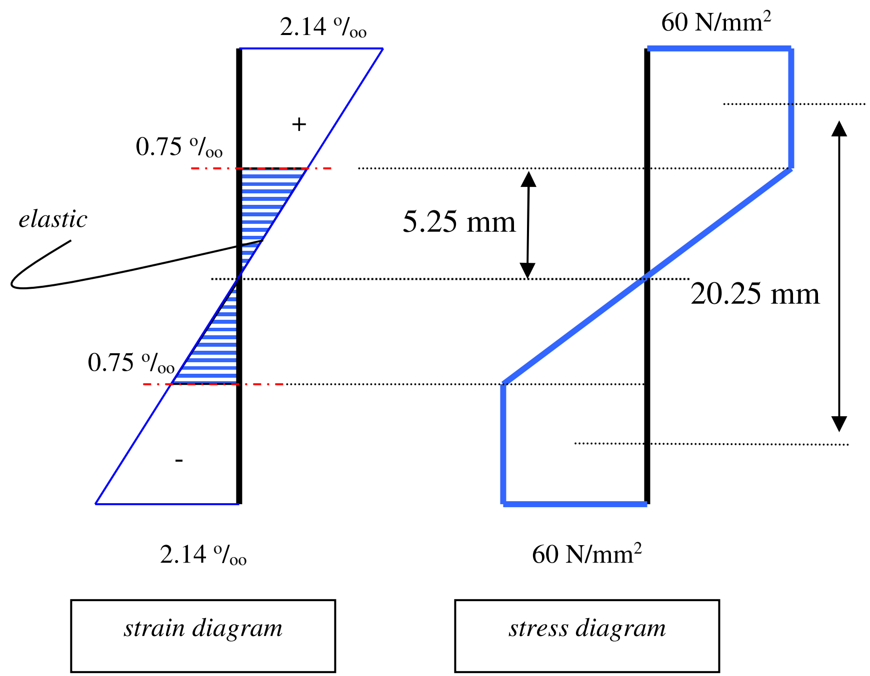
SOLUTION

$$R = 7000\ mm$$

$$\kappa = \frac{1}{R} = \frac{1}{7000}$$

$$\varepsilon_{boven} = \frac{15}{7000} = 2.14\times10^{-3} = 2.14 \quad {}^{o}/_{oo}$$

$$\varepsilon_{y} = \frac{60}{0.8\times10^{5}} = 0.75\times10^{-3} = 0.75 \quad {}^{o}/_{oo}$$



MOMENT IN THE CROSS SECTION BETWEEN A and B:

$$M = -\left(\tfrac{1}{6}\times20\times10.5^{2}\times60+9.75\times20\times20.25\times60\right) = -258.975\times10^{3}\ \text{Nmm}$$

FORCE ON THE CANTILEVER

$$M = -F\times200 = -258.975\times10^{3} \Rightarrow F = 1295\ \text{N}$$

BENDING STIFFNESS OF PART AB

For a rectangular cross section holds:

$$\frac{M}{M_e} = 1.5 - \tfrac{1}{2}\left[\frac{\kappa_e}{\kappa}\right]^2 \quad \Leftrightarrow \quad \text{with: } \kappa_e = \frac{\varepsilon_y}{\tfrac{1}{2}h} = 5\times10^{-5} \text{ en } M_e = \frac{I_{zz} f_y}{\tfrac{1}{2}h} = 180000 \text{ Nmm}$$

$$M = M_p - \tfrac{1}{2}M_e\left[\frac{\kappa_e}{\kappa}\right]^2 \quad \Leftrightarrow \quad \text{with: } M_p = 1.5\times M_e$$

$$EI = \frac{\mathrm{d}M}{\mathrm{d}\kappa} = \frac{M_e\kappa_e^2}{\kappa^3} = \frac{0.00045}{\kappa^3}$$

For the given value of the curvature κ this results in a bending stiffness:

$$EI = 154.35\times10^6 \text{ Nmm}^2$$

The linear elastic bending stiffness is:

$$EI_o = E\times\tfrac{1}{12}bh^3 = 3600\times10^6 \text{ Nmm}^2$$

REDUCTION IN BENDING STIFFNESS DUE TO ELASTO-PLASTICITY

ONLY 4% OF THE ORIGINAL **LE**-BENDING STIFFNESS for R = 7.0 m