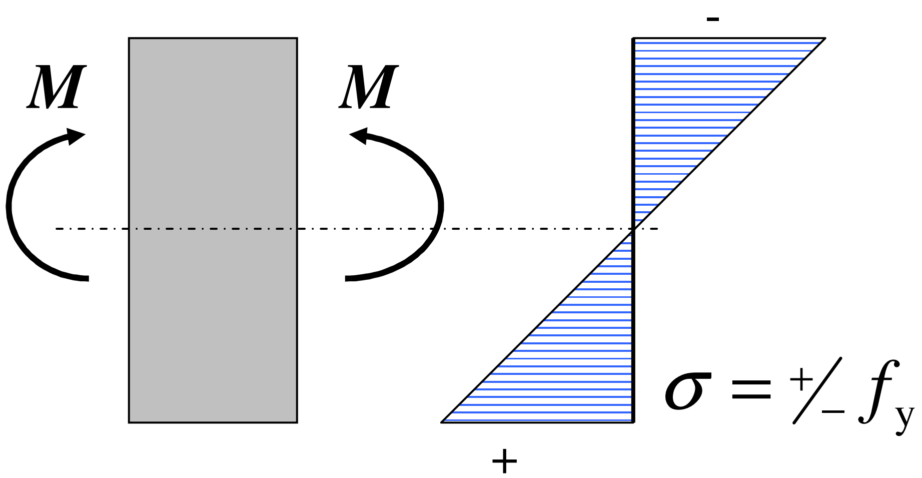
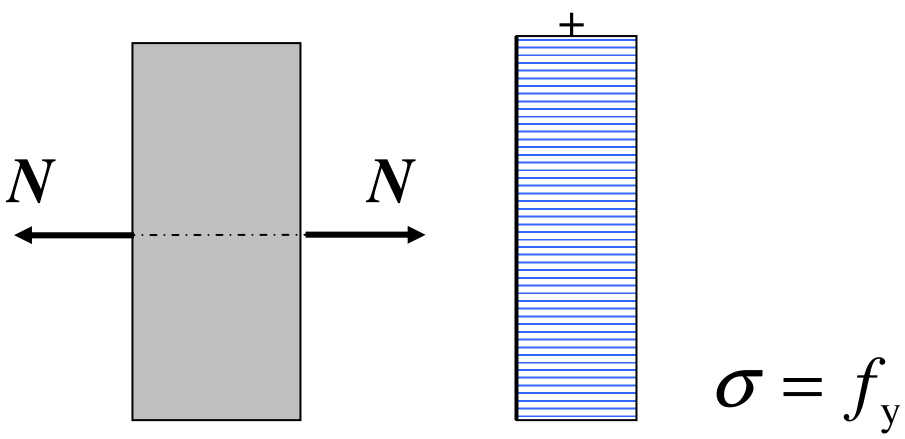
THEME

IS FIRST OCCURRENCE OF YIELDING THE LIMIT?

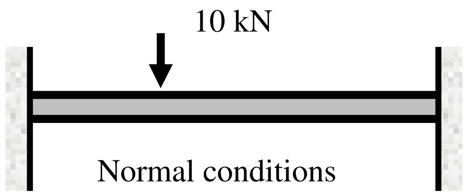


BENDING

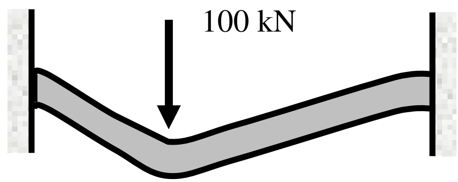


EXTENSION

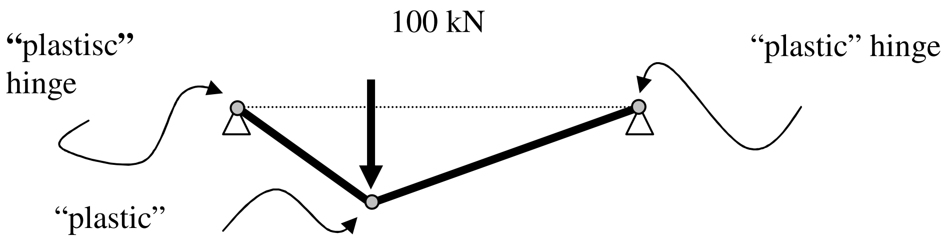
v2021-1



Normal conditions
during the life time



Failure due to
maximum load



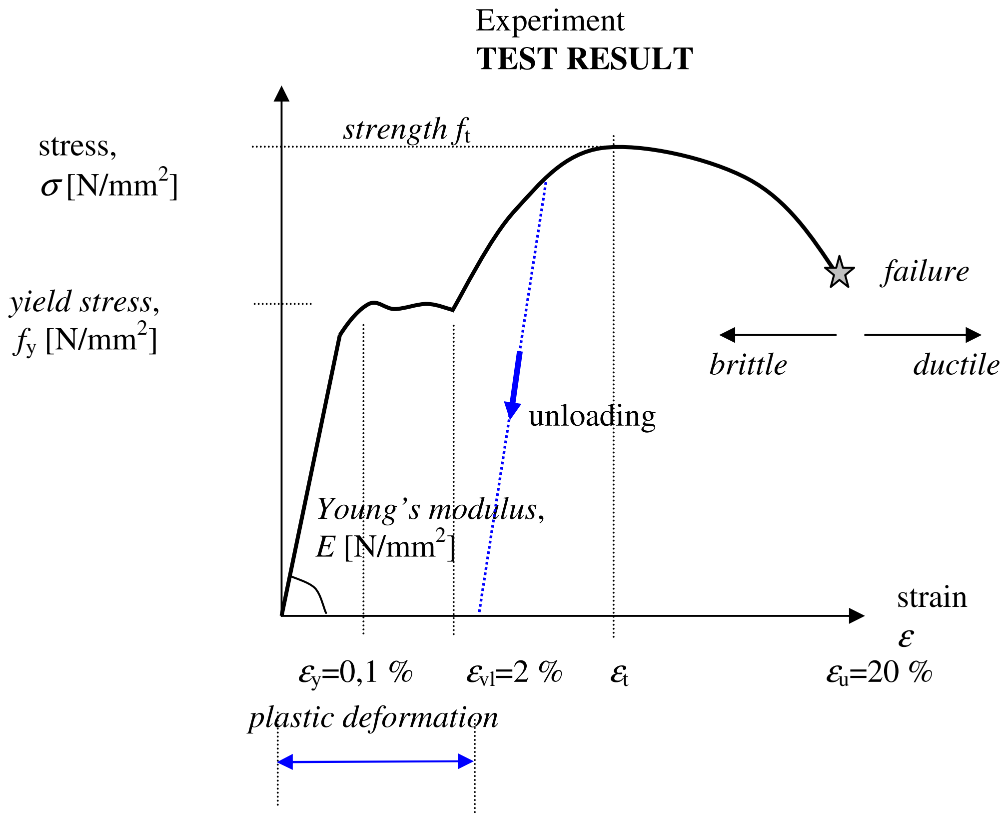
Analysis model with “plastic”
hinges

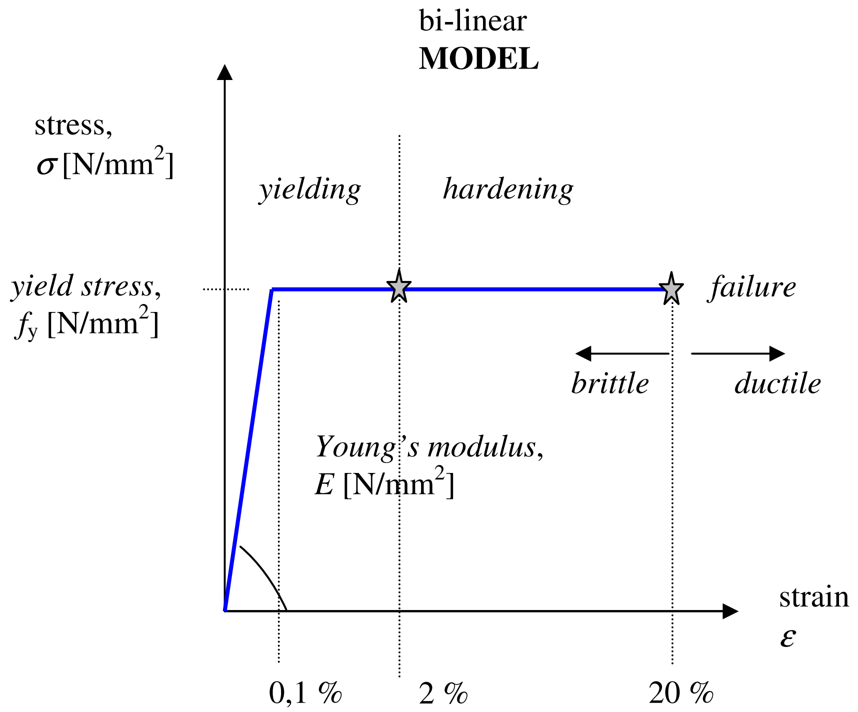
WHAT HAPPENS DUE TO LOADING BEYOND THE ELASTIC LIMIT?

- MATERIAL ?
- CROSS SECTION ?
- STRUCTURE ?

(ULTIMATE LOAD)
(LIMIT STATE ANALYSIS)

RELEVANT MATERIAL PROPERTIES (STEEL)





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(STRUCTURAL) STEEL

name	Yield stress N/mm^2	Strength N/mm^2	yield-strain (ε_y) %	limit-strain (ε_u) %
S235	235	360	≈0,1	19
S275	275	430	≈0,1	16
S355	355	510	≈0,2	16

Young's modulus : E=2,1×10^5 N/mm^2 = 210 GPa

ELASTIC OR PLASTIC ANALYSIS ?

"Piece of Art"

situation 1

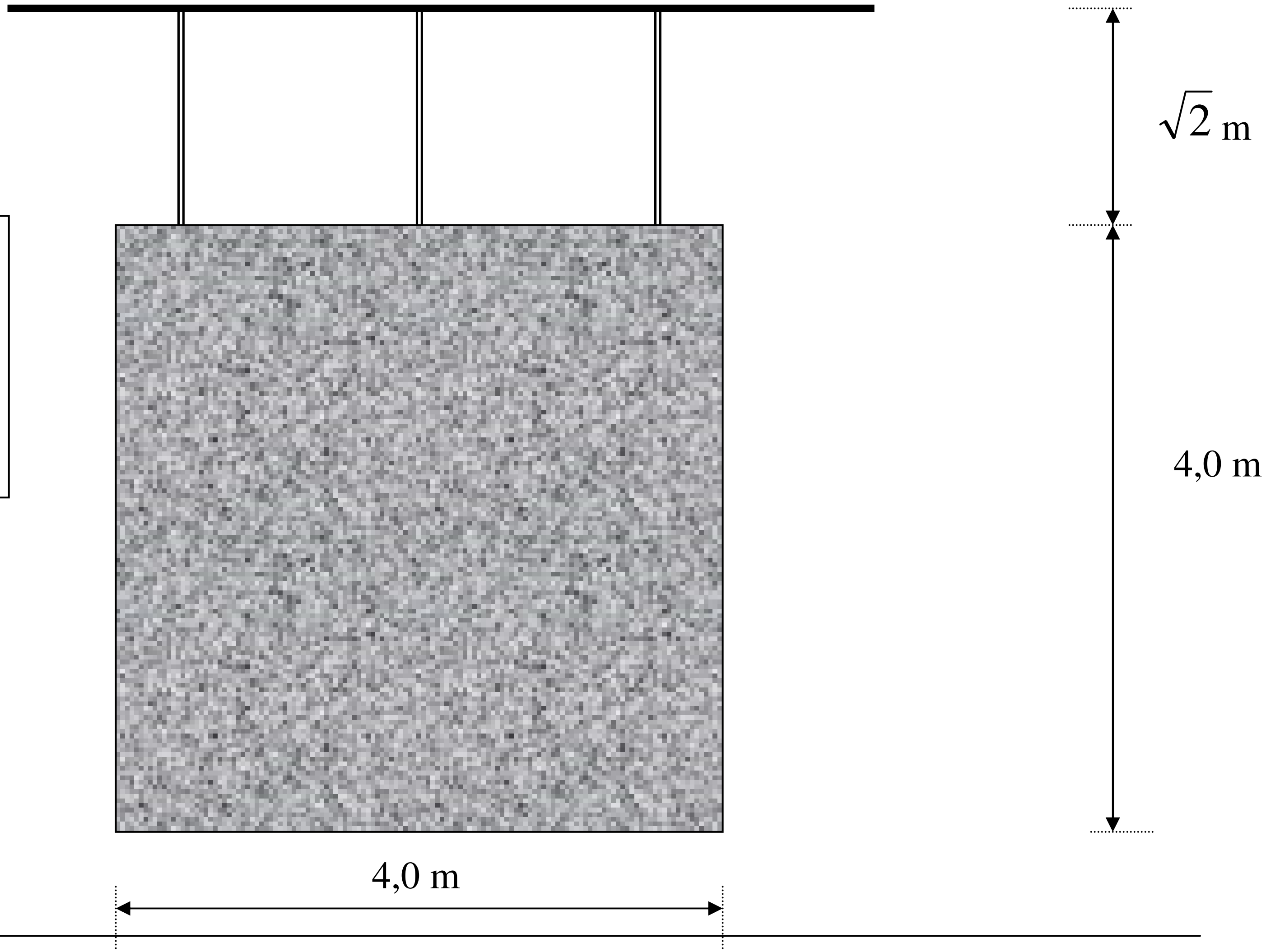
3 cables S355

Cross section $60\sqrt{2}$ mm^2

mass : 5500 kg (55 kN)

Question :

Is this safe ?



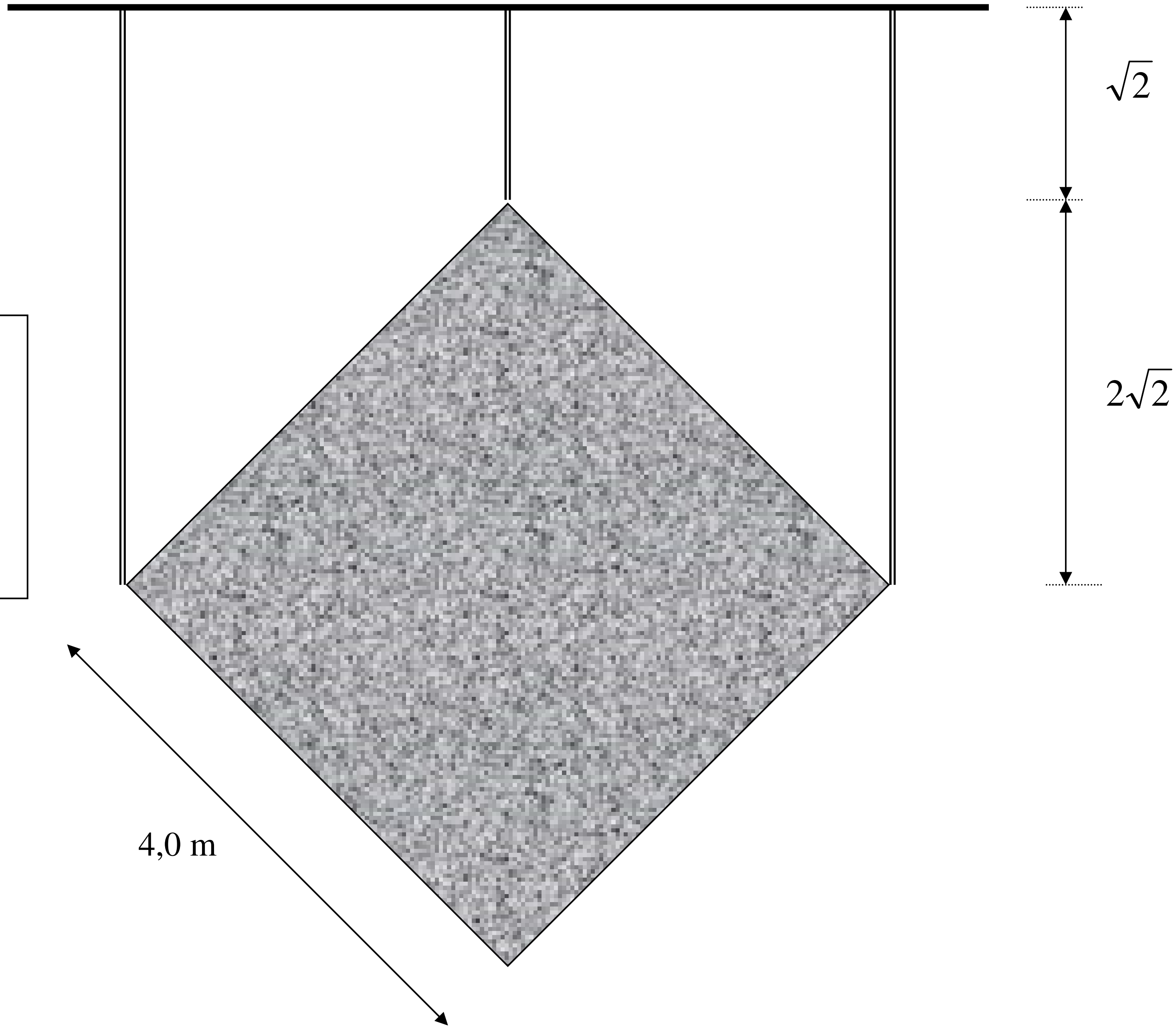
"Piece of Art"

situation 2

3 cables S355

Cross section $60\sqrt{2}$ mm^2

mass : 5500 kg (55 kN)



Question :

Is this allowed ?

ELASTIC LIMIT (CAPACITY)

$$k = \frac{EA}{l} = \frac{E \times 60\sqrt{2}}{\sqrt{2}} = 60E$$

situation 1 :

$$F_p = A \times f_y = 60\sqrt{2} \times 355 = 30{,}1 \times 10^3 \text{ N}$$

$$u_e = u_p = \frac{F_p}{k} = \frac{30{,}1 \times 10^3}{60 \times 2{,}1 \times 10^5} = 2{,}39 \text{ mm}$$

$$G_e = \sum k \times u_e = 3 \times k \times u_e \qquad \Leftrightarrow$$

$$G_e = 3 \times 30{,}1 \times 10^3 = 90{,}3 \times 10^3 \text{ N} \quad (SAFE)$$

(all cables simultaneous loaded up to capacity)

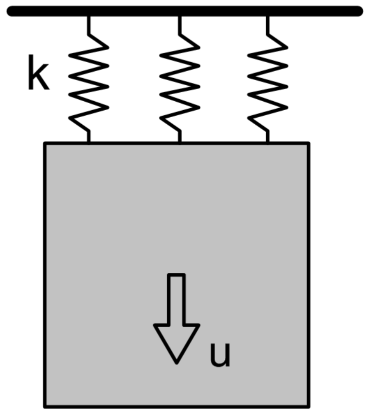
situation 2 :

$$G_e = \tfrac{1}{3} k \times u_e + k \times u_e + \tfrac{1}{3} k \times u_e = \tfrac{5}{3} k \times u_e = 50{,}2 \times 10^3 \text{ N}$$

(centre wire at 100% of capacity but

outer two loaded at 33% of capacity)

Statically indeterminate structure !!

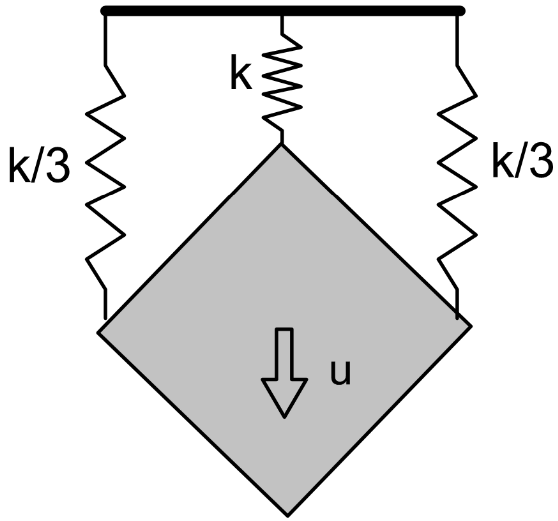


Situation 1

load distribution per cable (LE)

33% 33% 33%

elastic limit is failure load



Situation 2

load distribution per cable (LE)

20% 60% 20%

after elastic limit all additional load is taken by the outer two cables. The centre cable remains at 100% load and is not allowed to break before the outer two cables are loaded up to 100% of the capacity. (required ductility) Failure occurs when all three cables are loaded up to their capacity. The outer cables will have an elongation of 3 × 2,39 = 7,16 mm

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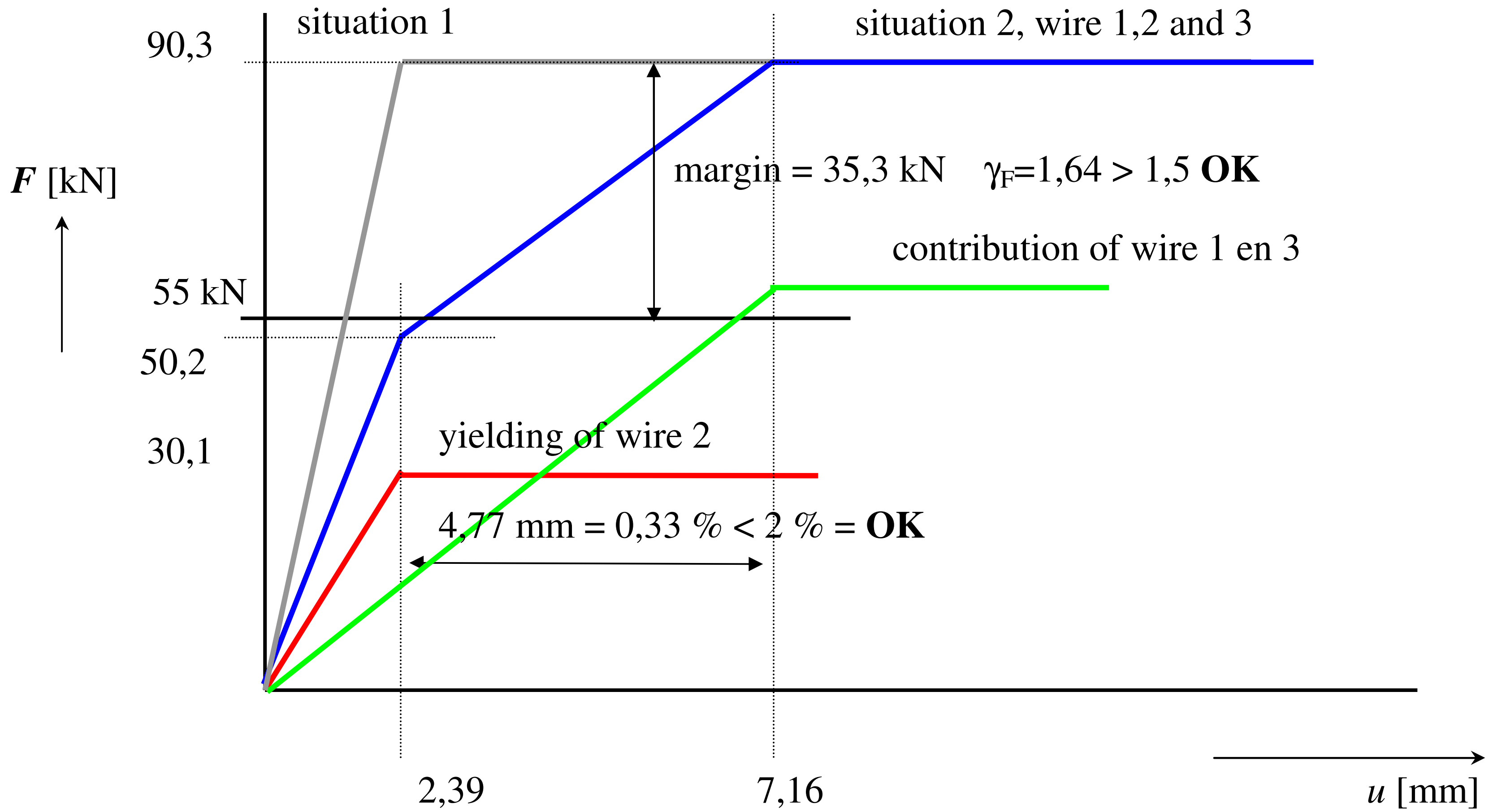
RESULT



PRELIMINARY CONCLUSIONS

PLASTIC ANALYSIS SHOWS ADDITIONAL LOAD BEARING CAPABILITIES FOR STATIC INDETERMINATE STRUCTURES AFTER THE ELASTIC LIMIT HAS BEEN REACHED

DUE TO PLASTICITY THE STATIC SYSTEM CHANGES AND THUS A REDISTRIBUTION OF THE FORCES OCCURS IN CASE OF A STATICALLY INDETERMINATE STRUCTURE

PLASTIC DEFORMATION CAPACITY IS ESSENTIAL IF THE ADDITONAL LOAD BEARING CAPACITY IS USED AND SHOULD BE CHECKED

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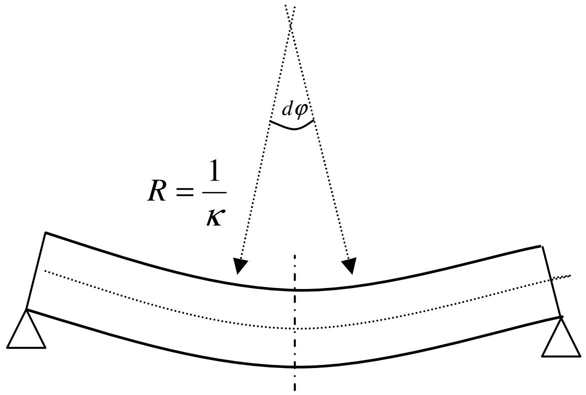
BENDING

- **ELASTICITY VERSUS PLASTICITY**
  - **FULLY PLASTIC MOMENTCAPACITY**
  - **SHAPE FACTOR**
  - **EXAMPLES**
- BEHAVIOUR OF THE CROSS SECTION
  - MOMENT-CURVATURE
  - PLASTIC ZONES
  - IDEAL PLASTIC HINGE
- STRUCTURAL BEHAVIOUR (LIMIT STATE ANALYSIS)
  - BEAMS
  - FRAMES

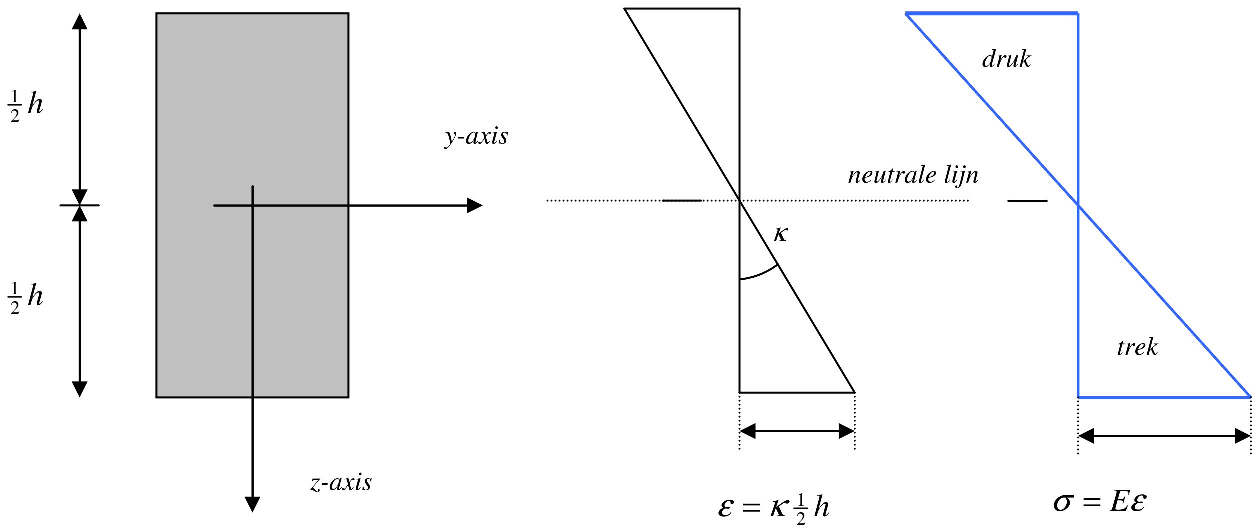
BENDING



Cross section I

Linear strain distribution over the depth of the beam :

$$\varepsilon(z) = \kappa \times z$$



Cross section

strain

stress

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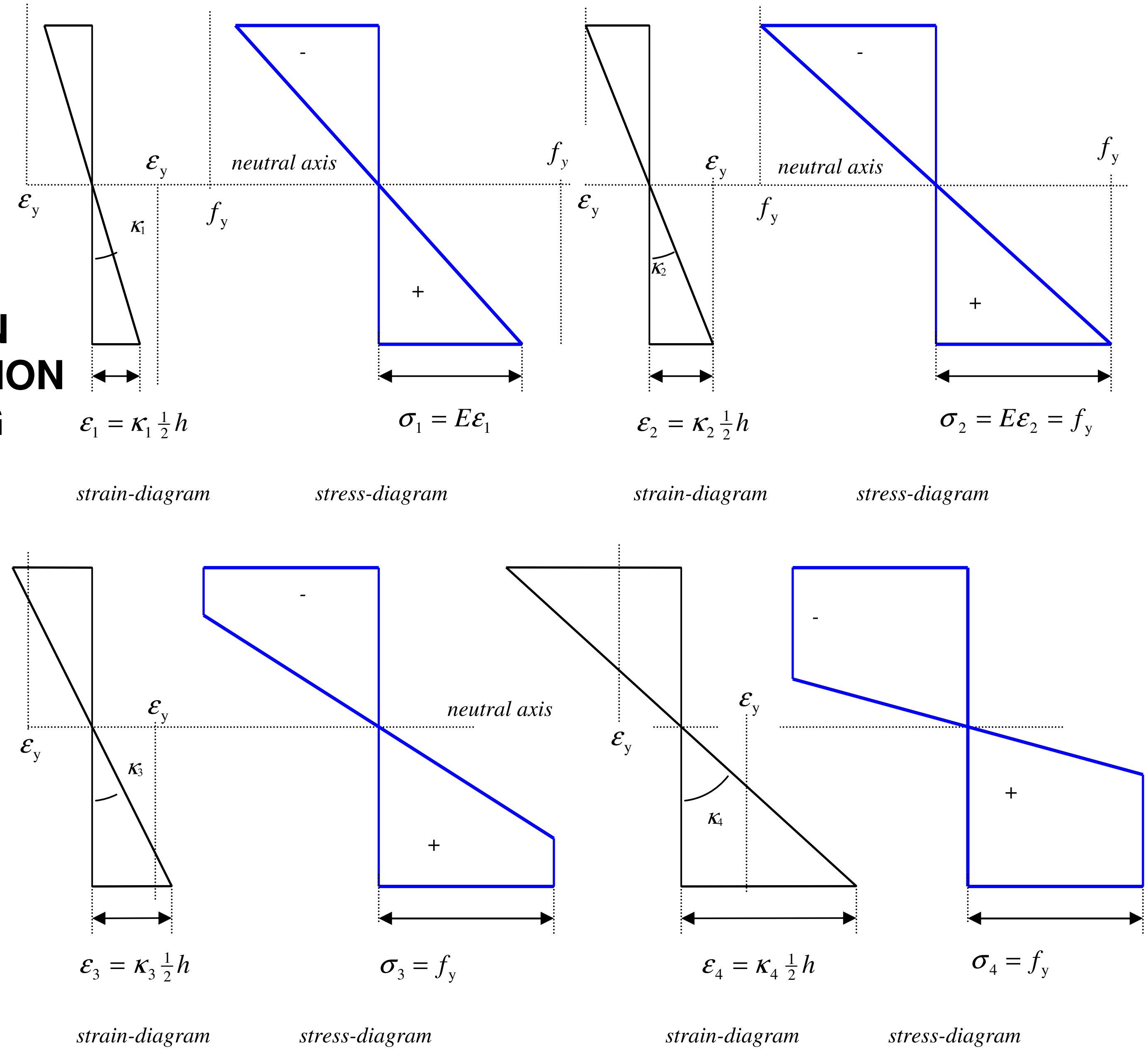
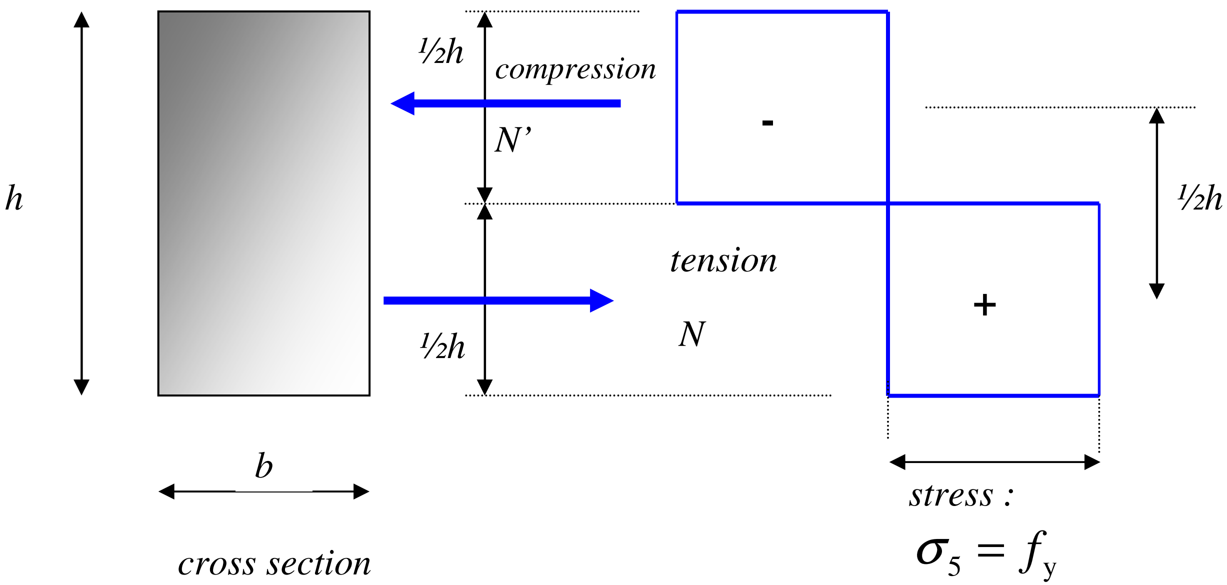
STRESS AND STRAIN IN THE CROSS SECTION DUE TO INCREASING CURVATURE

see figure 3.3
par 3.1.1 (Dutch Book)



FULLY PLASTIC MOMENT



$$\sum H = 0 \Rightarrow N' = N$$

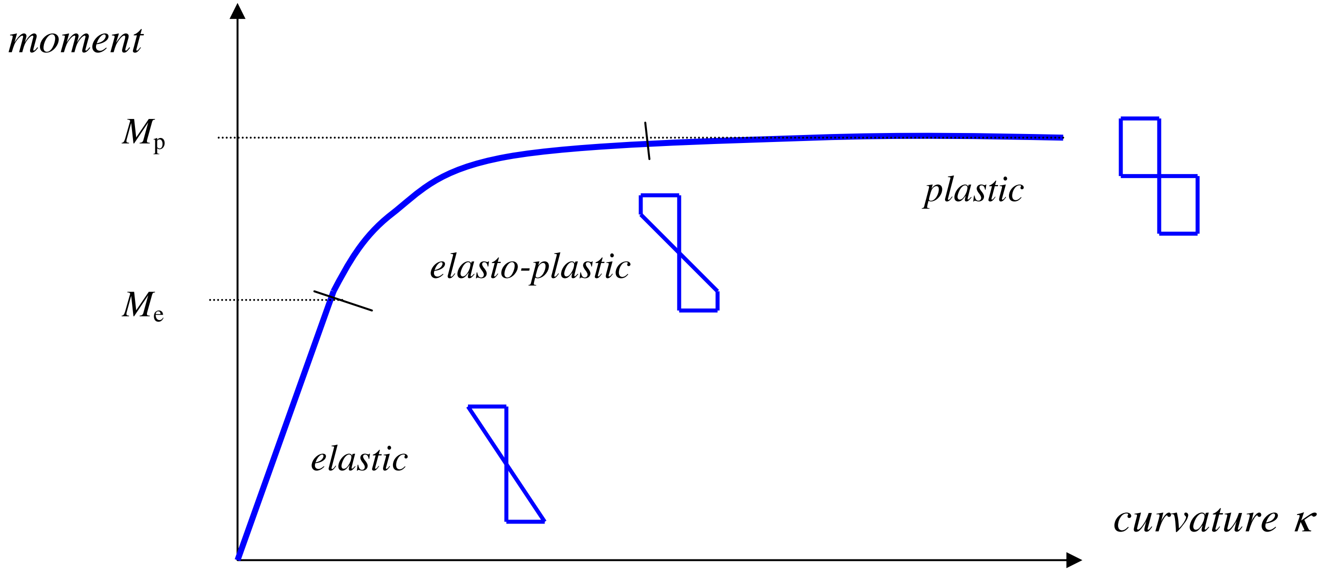
$$N = f_y A = \tfrac{1}{2} bh\, f_y$$

$$M_p = N.\tfrac{1}{2} h = \tfrac{1}{4} bh^2 f_y$$

DEMAND : HORIZONTAL EQUILIBRIUM

THUS : RESULTANT **COMPRESSION** = RESULTANT **TENSION**

MOMENT – CURVATURE DIAGRAM
(M-κ diagram)



$$\text{SHAPE FACTOR}: \alpha = \frac{M_p}{M_e} \quad \text{RECTANGLE}: \alpha = \frac{\frac{1}{4} bh^2 f_y}{\frac{1}{6} bh^2 f_y} = 1{,}5$$

FULLY PLASTIC MOMENT

Symmetrical section, load along the axis of symmetry

NO
Not part of these lectures

homogeneous material

NO

Determine the "medium line" such that :

$F_{\text{compression}} = F_{\text{tension}}$

Determine the cross sectional area :

A_{total}

"medium line splits the cross section in two equal parts :

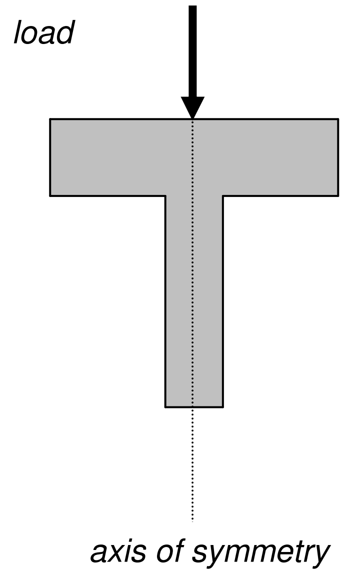
$A_{\text{compression}} = A_{\text{tension}} = ½A_{\text{total}}$

$F_{\text{comp}} = F_{\text{tension}} = \frac{1}{2} A_{\text{total}} f_{\text{y}}$

Determine the resultant compressive and tensile forces acting on the cross section and the internal lever (arm) z.

Fully plastic moment capacity of the cross section becomes :

$M_{\text{p}} = F_{\text{tension}} \times z$



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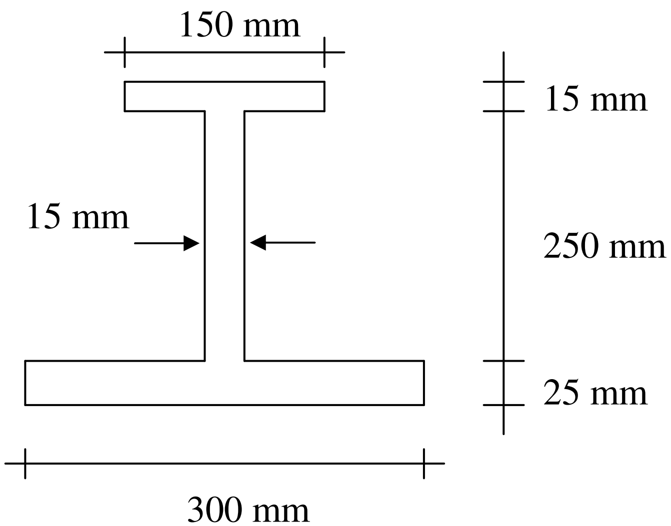
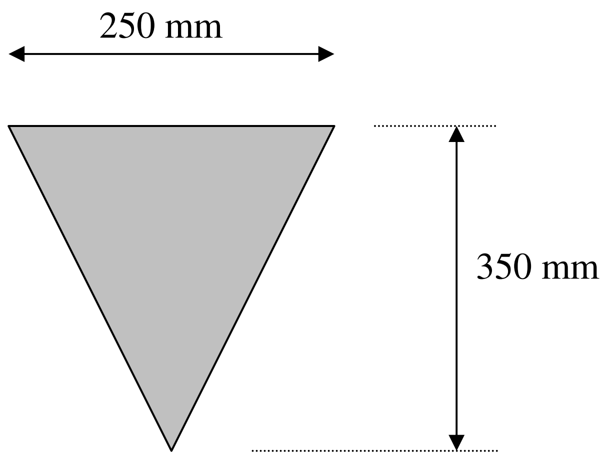
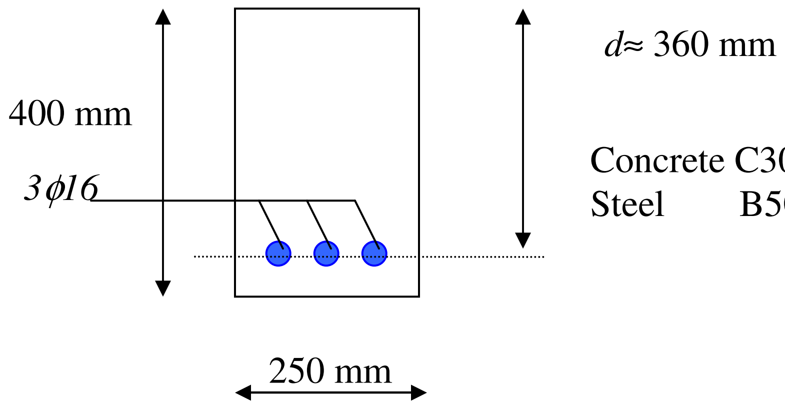
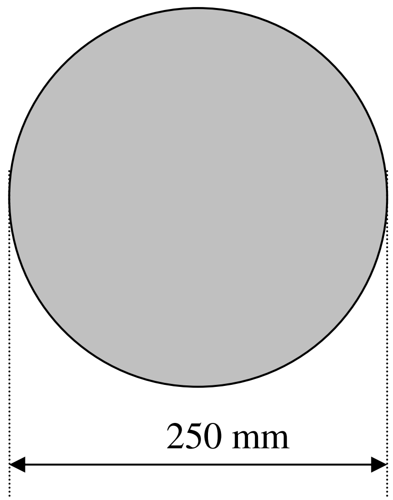
EXAMPLES OF PLASTIC MOMENT





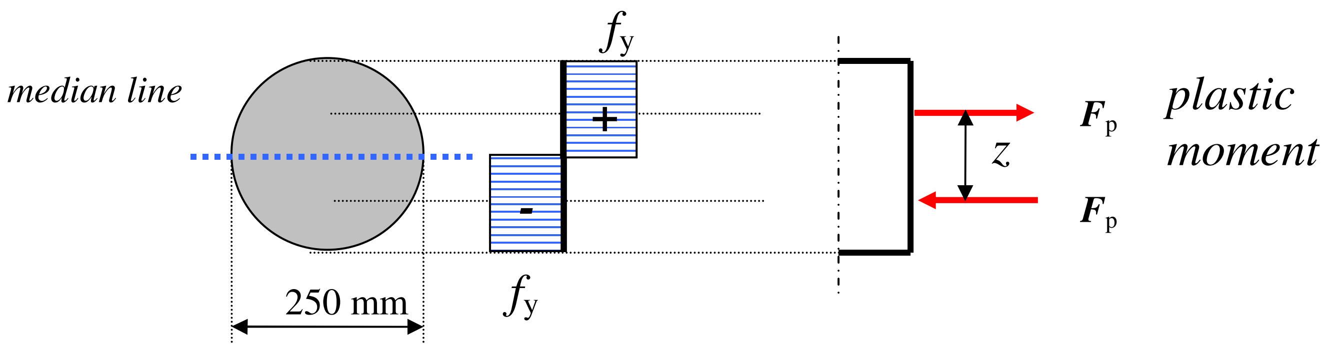


Concrete C30/37 $f_{cd} = 20$ N/mm^2
Steel B500 $f_{yd} = 435$ N/mm^2



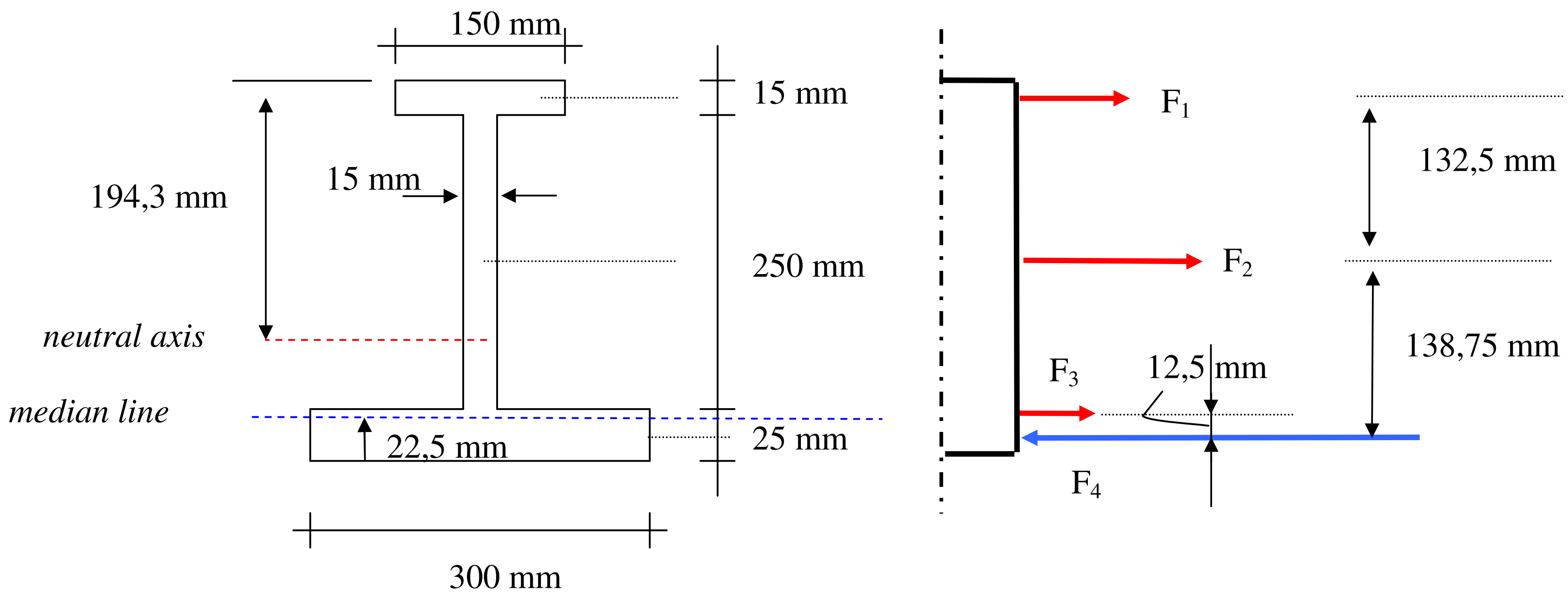
SOLID CIRCULAR CROSS SECTIONS

- *elastic moment*
- *median line*
- *plastic moment*
- *shape factor*



$$M_p = F_p \times z$$

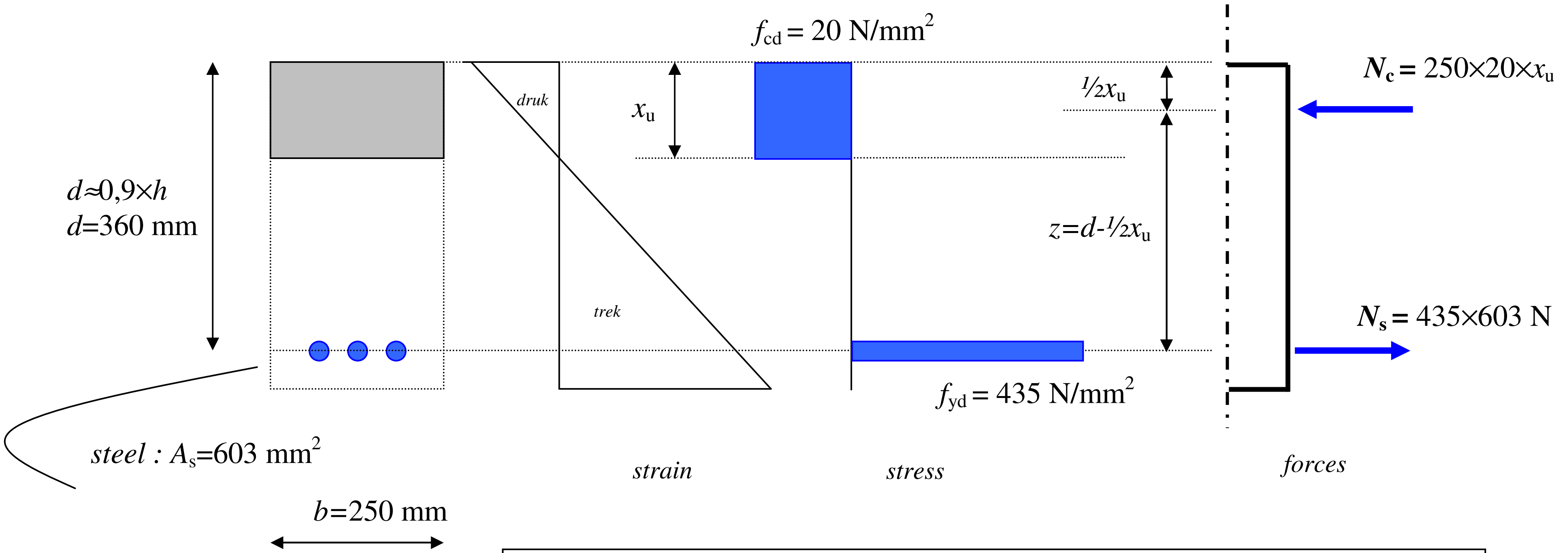
I-SECTION



$N = 0$!

M = sum of the moments

CONCRETE BEAMS (hxb=400x250 mm^2)



$$\sum H = 0 \quad \Rightarrow \quad N_c = N_s \quad \Rightarrow \quad x_u = \frac{A_s f_{yd}}{b f_{cd}}$$

$$M_p = N_s \times z = A_s f_{yd} \times \left(d - \tfrac{1}{2} x_u \right)$$

SOME SHAPE FACTORS