BENDING

- ELASTICITY VERSUS PLASTICITY
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- STRUCTURAL BEHAVIOUR (LIMIT ANALYSIS)
  - BEAMS
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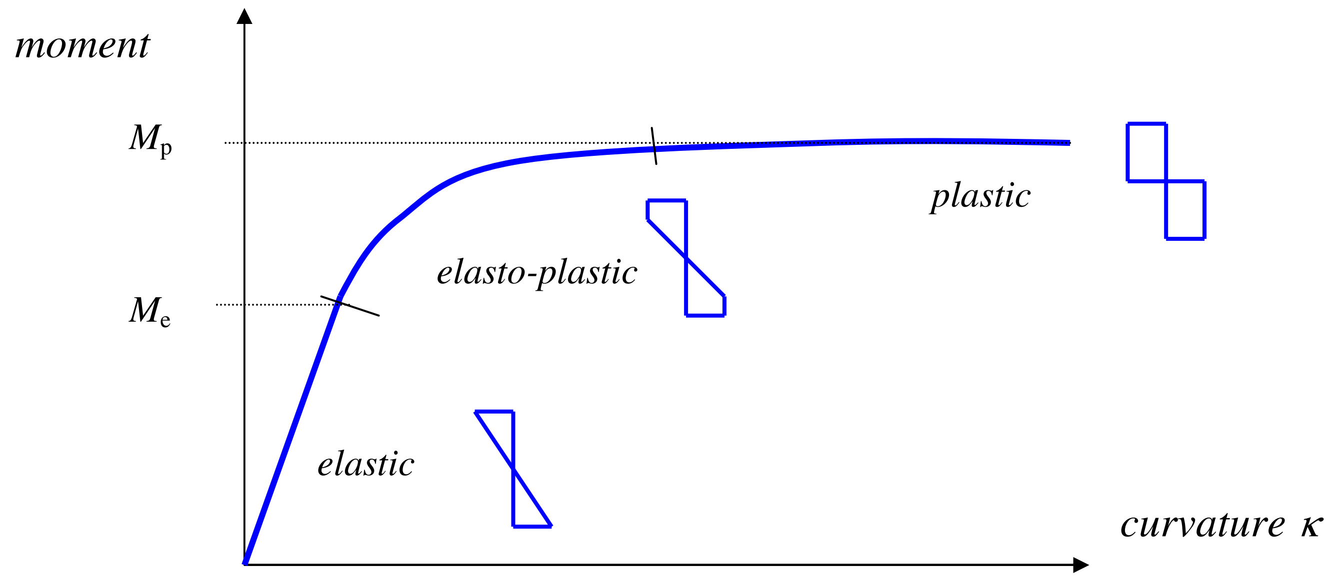
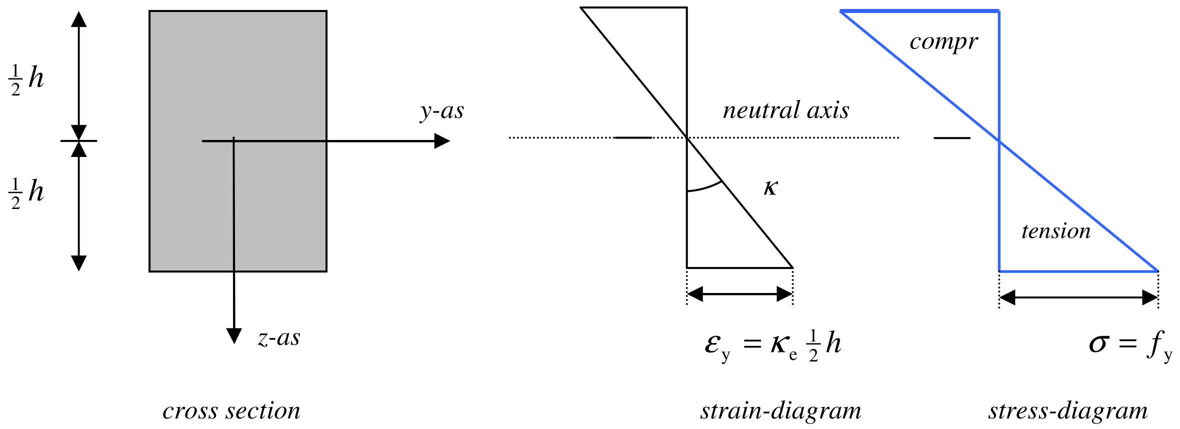
v2021-1

M-κ BEHAVIOUR OF THE CROSS SECTION



MOMENT-CURVATURE

ELASTIC :



start with :

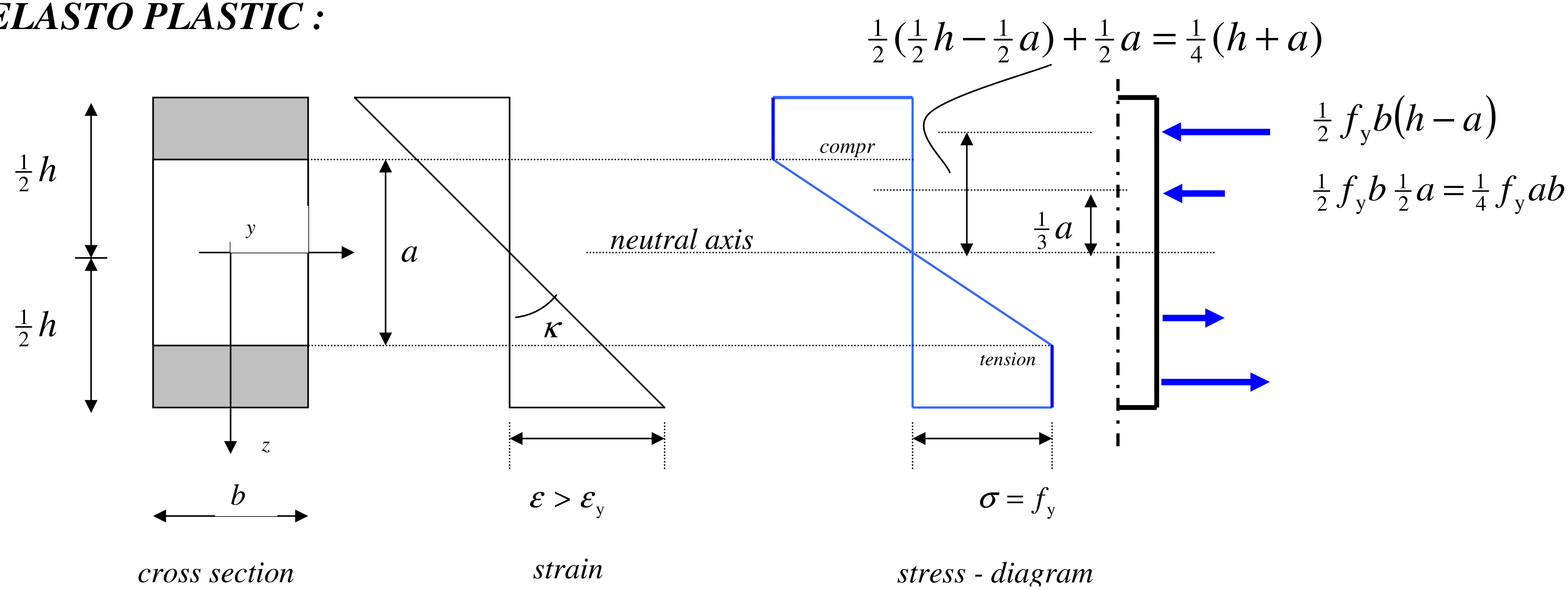
$$\sigma = \frac{M}{W} \Rightarrow M = W\sigma$$

thus :

$$M_e = W f_y$$

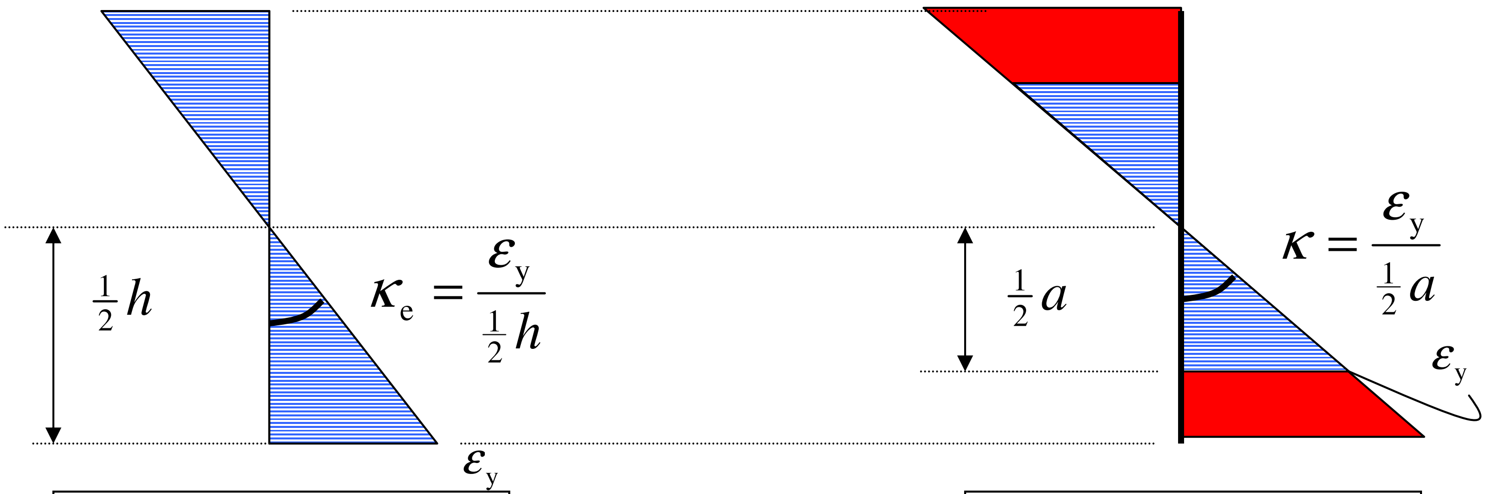
MOMENT-CURVATURE

ELASTO PLASTIC :



$$M = \tfrac{1}{6} f_y b a^2 + \tfrac{1}{2} f_y b(h - a) \times 2 \times \tfrac{1}{4}(h + a) = \tfrac{1}{4} b h^2 \left(1 - \frac{a^2}{3h^2} \right) \times f_y$$

MOMENT – CURVATURE



strain diagram : elastic limit

strain diagram : elasto plastic fase

$$M = \tfrac{1}{4}bh^2\left(1 - \frac{a^2}{3h^2}\right)\times f_y$$

$$M_e = \tfrac{1}{6}bh^2 \times f_y$$

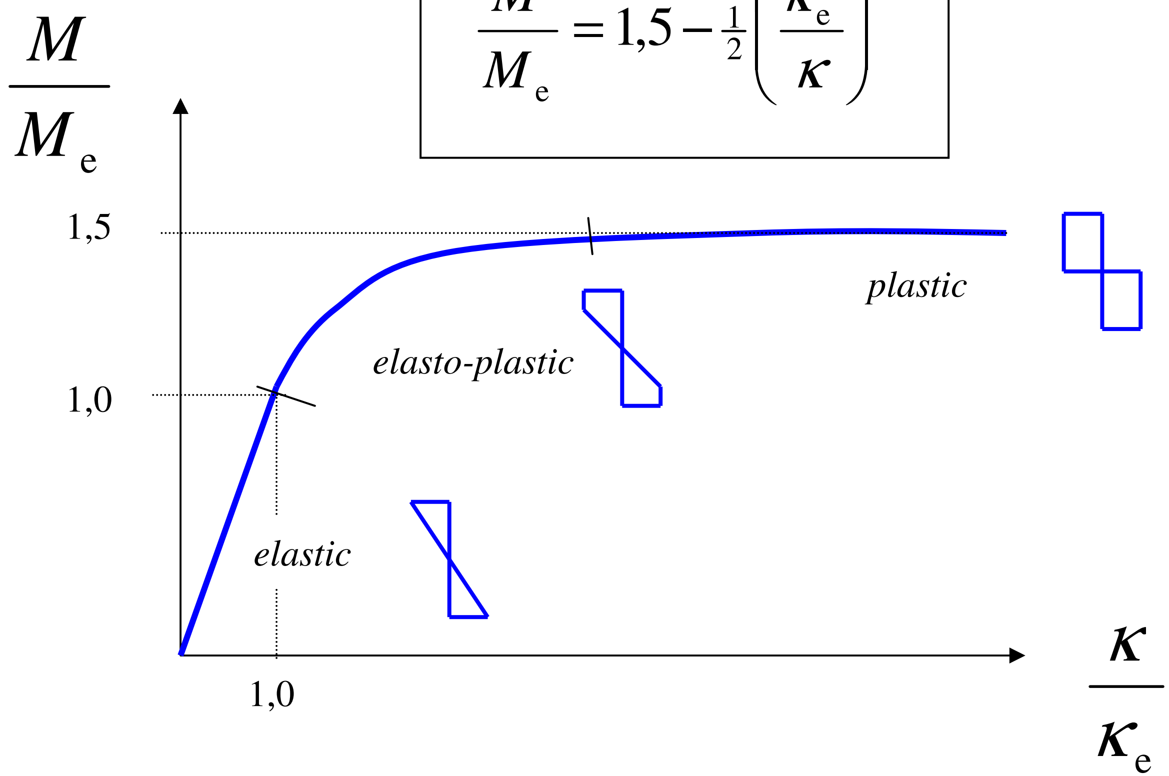
$$\frac{M}{M_e} = \left(1{,}5 - \frac{a^2}{2h^2}\right) = 1{,}5 - \tfrac{1}{2}\left(\frac{\kappa_e}{\kappa}\right)^2$$

MOMENT - CURVATURE

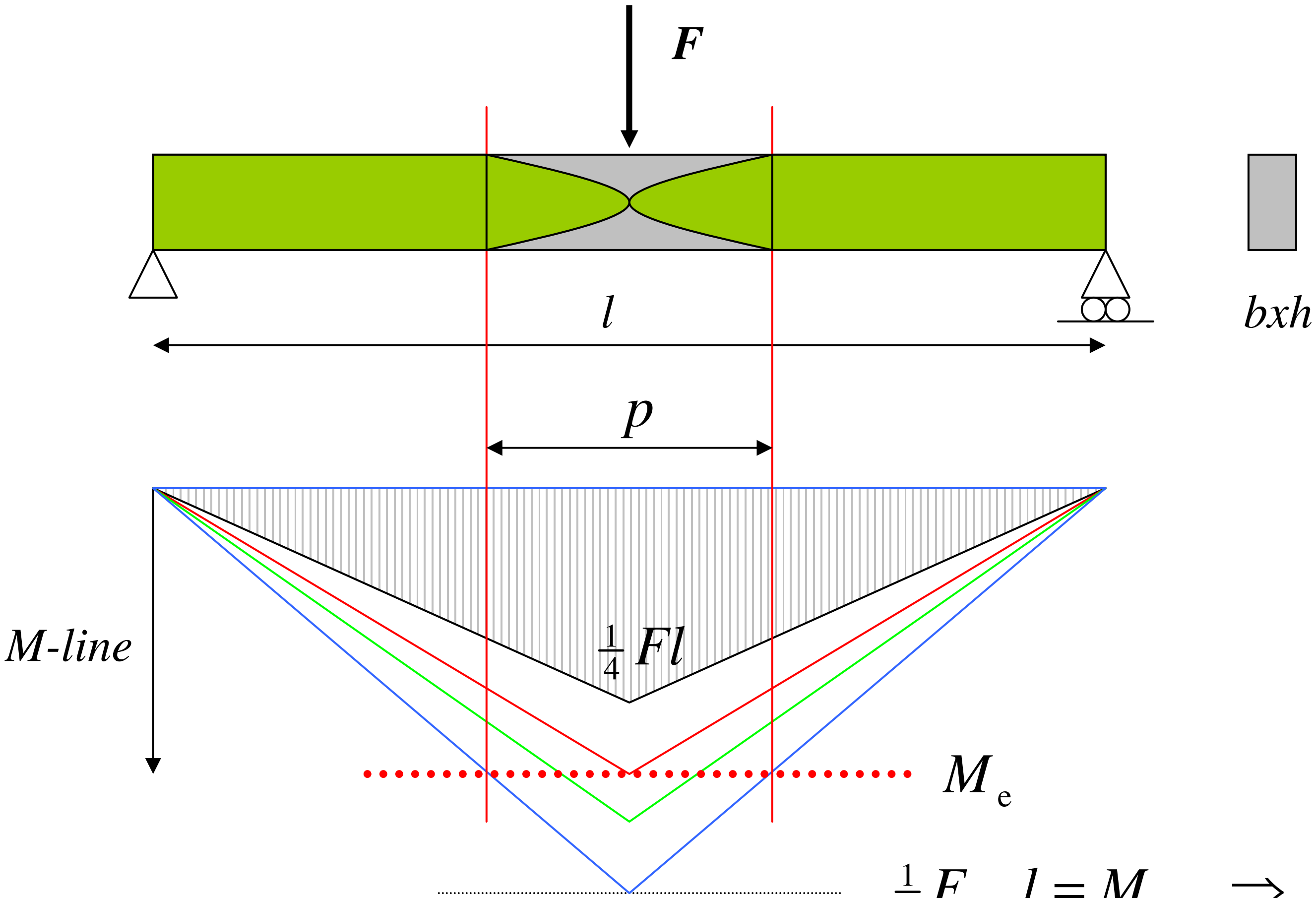
For rectangular cross sections

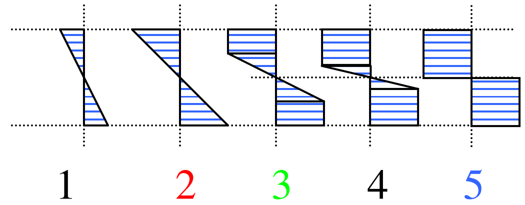
Elasto-plastic :

$$\frac{M}{M_e} = 1{,}5 - \tfrac{1}{2}\left(\frac{\kappa_e}{\kappa}\right)^2$$



PLASTIC ZONE IN THE BEAM



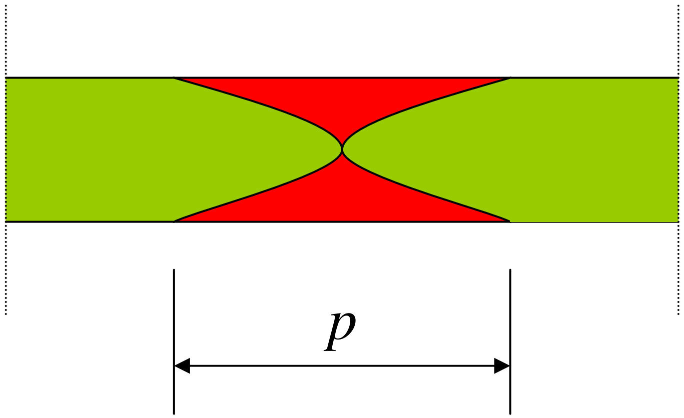


stress in critical cross section due to increasing load

1 = elastic
2 = elastic limit = M_e
3 = elasto plastic
4 = elasto plastic
5 = fully plastic = M_p

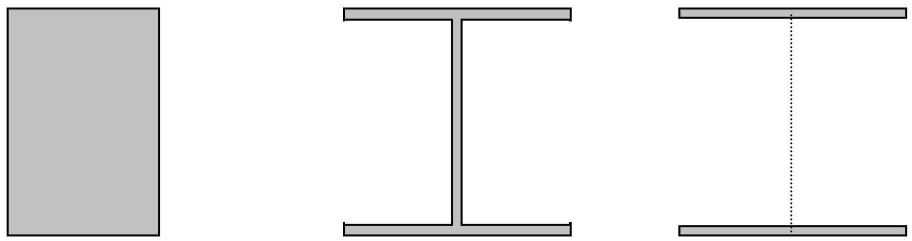
$$\tfrac{1}{4} F_{max} l = M_p \quad \Rightarrow \quad F_{max} = F_p = \frac{4M_p}{l}$$

PLASTIC ZONE



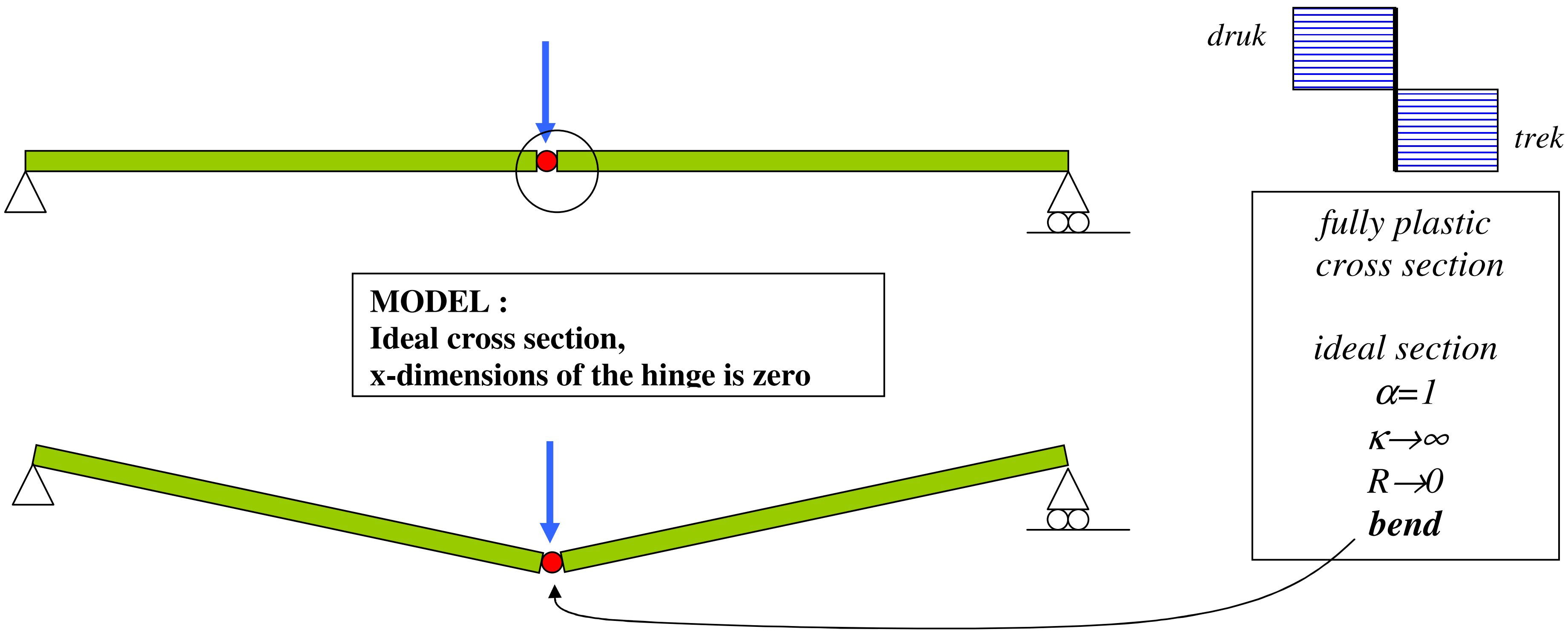
$$p = \frac{\alpha - 1}{\alpha} l$$

$$\alpha = \frac{M_{\mathrm{p}}}{M_{\mathrm{e}}}$$



α=1,5 **p=0,33l**	**α=1,15** **p=0,13l**	**α=1,0** **p=0**

PLASTIC HINGE



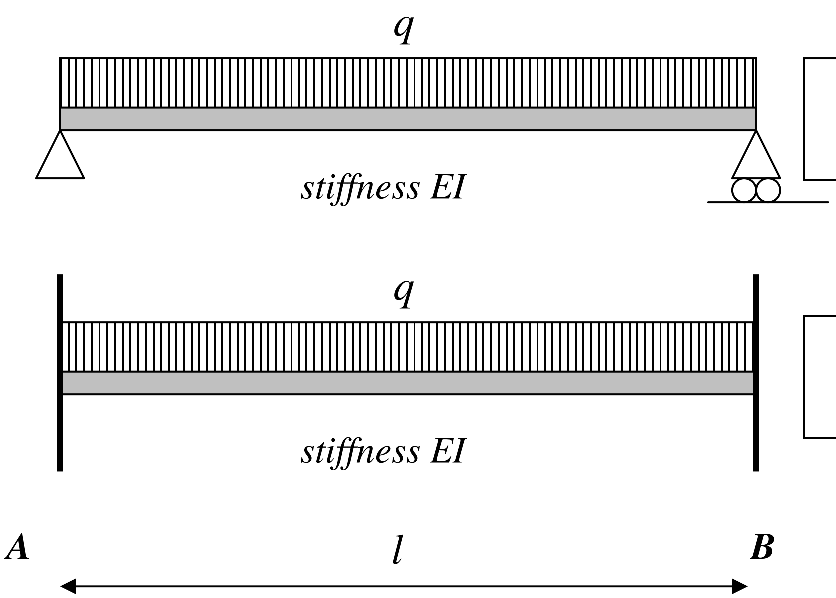
DEVELOPMENT OF CROSS SECTION IN FULL PLASTICITY, STATICALLY DETERMINATE SYSTEM CHANGES INTO A MECHANISM DUE TO THE PRESENCE OF THE PLASTIC HINGE

FAILURE MECHANISM

MODEL FOR THE LIMIT STATE ANALYSIS

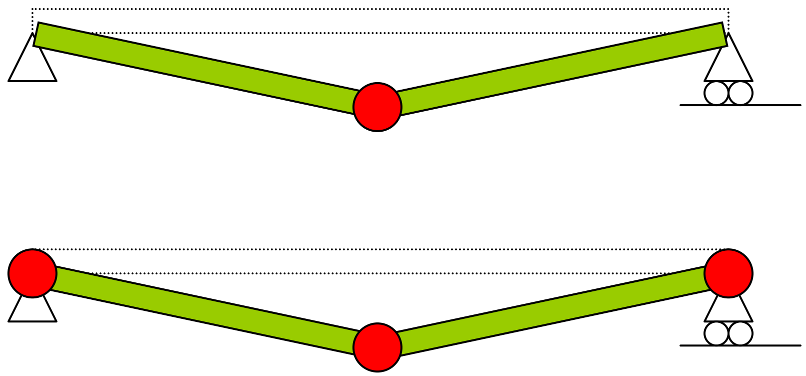
- All deformation concentrated in plastic hinges
- Failure whenever a mechanism occurs : **failure mechanism**

EXAMPLES



Statically determinate
$n = 0$

Statically indeterminate
$n = 2$



Kinematically indeterminate
(mechanism)
$n = -1$

INCREMENTAL ANALYSIS