BENDING

- ELASTICITY VERSUS PLASTICITY
  - FULLY PLASTIC MOMENT
  - SHAPE FACTOR
  - EXAMPLES
- BEHAVIOUR OF THE CROSS SECTION
  - MOMENT-CURVATURE
  - PLASTIC ZONES
  - IDEAL PLASTIC HINGE
- **STRUCTURAL BEHAVIOUR (LIMIT ANALYSIS)**
  - **BEAMS**
  - **FRAMES**

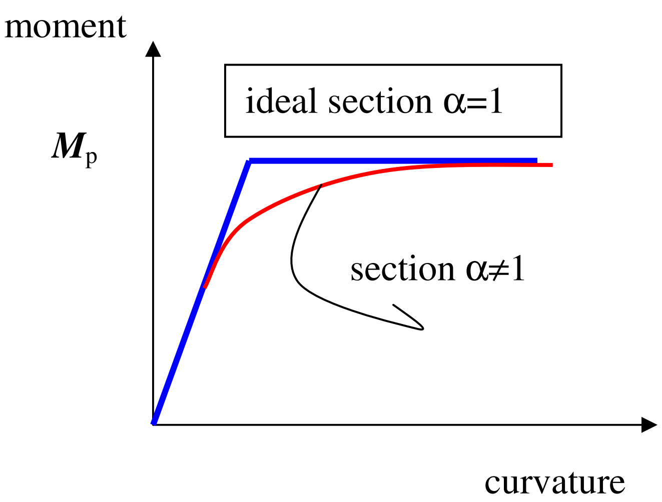
v2021-1

MODEL FOR THE LIMIT STATE ANALYSIS

- Concentrate all plastic deformation in one cross section, the **plastic hinge**
- Failure whenever a mechanism occurs : **failure mechanism**

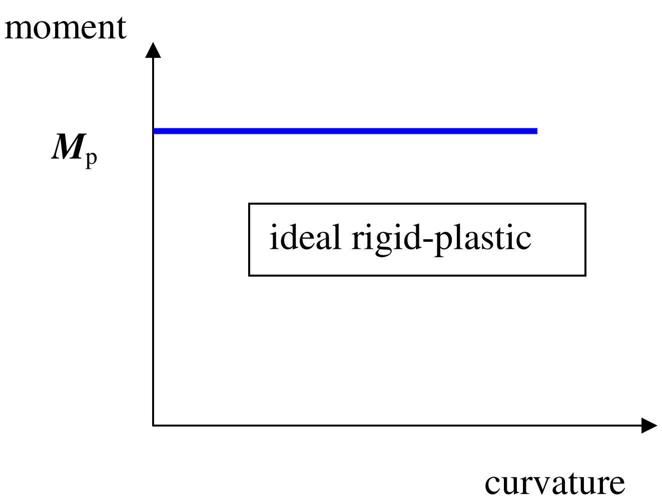
APPROACH SO FAR

- INCREMENTAL METHOD
  - result : Load – Deformation Diagram
- DIRECT (EQUILIBRIUM) METHOD
  - result : Collapse or Limit Load



NEW APPROACH

- STATIC CHECK FOR ALL ASSUMED MECHANISMS USING VIRTUAL WORK
  - result : Collapse or Limit Load



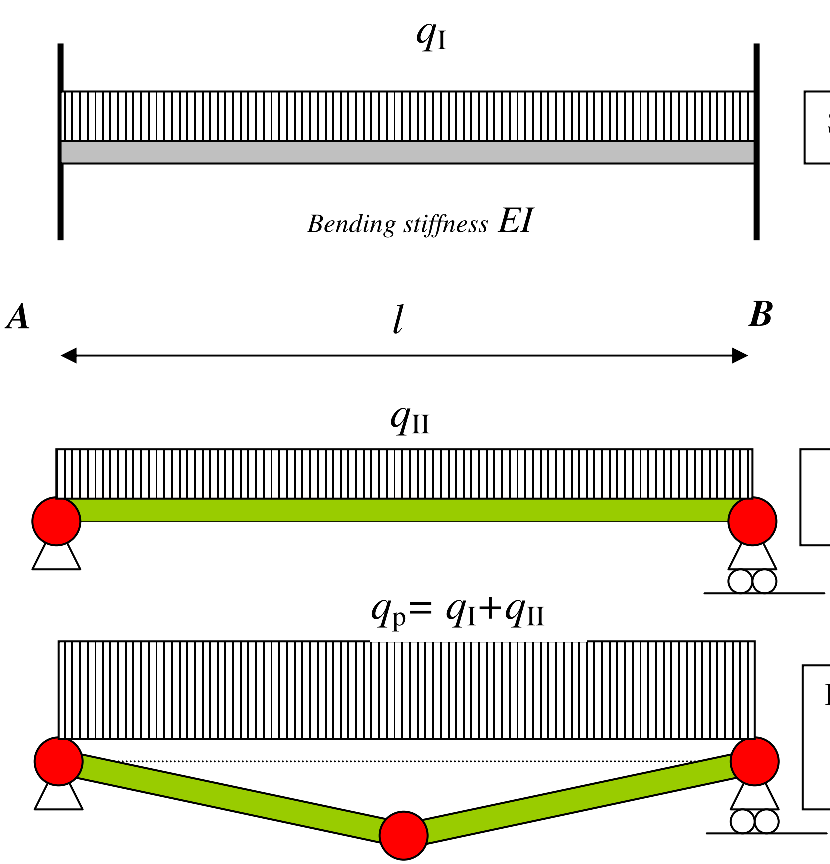
INCREMENTAL METHOD WITH PROPORTIONAL LOADING

- Increase the load until the bending moment at a cross section reaches the full plastic moment. This is the location of the first plastic hinge.

- Change the structural model by adding a hinge and increase the load. Compute the additional moment distribution with the new structural model and determine the location of the next plastic hinge.

- Repeat step 2 until the occurrence of a mechanism. The load is the collapse load or limit load and the mechanism is the collapse mechanism.

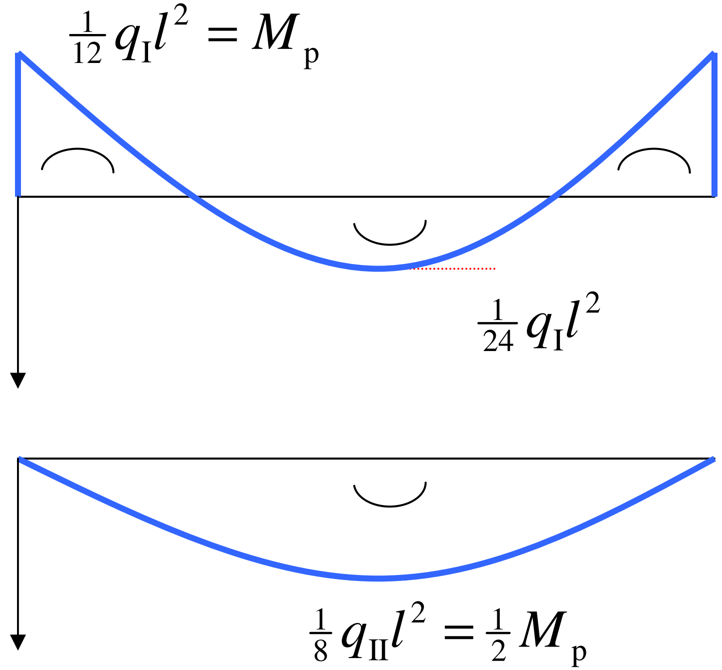
EXAMPLE of the INCREMENTAL METHOD



Statically indeterminate $n=2$

Statically determinate
$n = 0$

Kinematically indeterminate
(mechanism)
$n = -1$

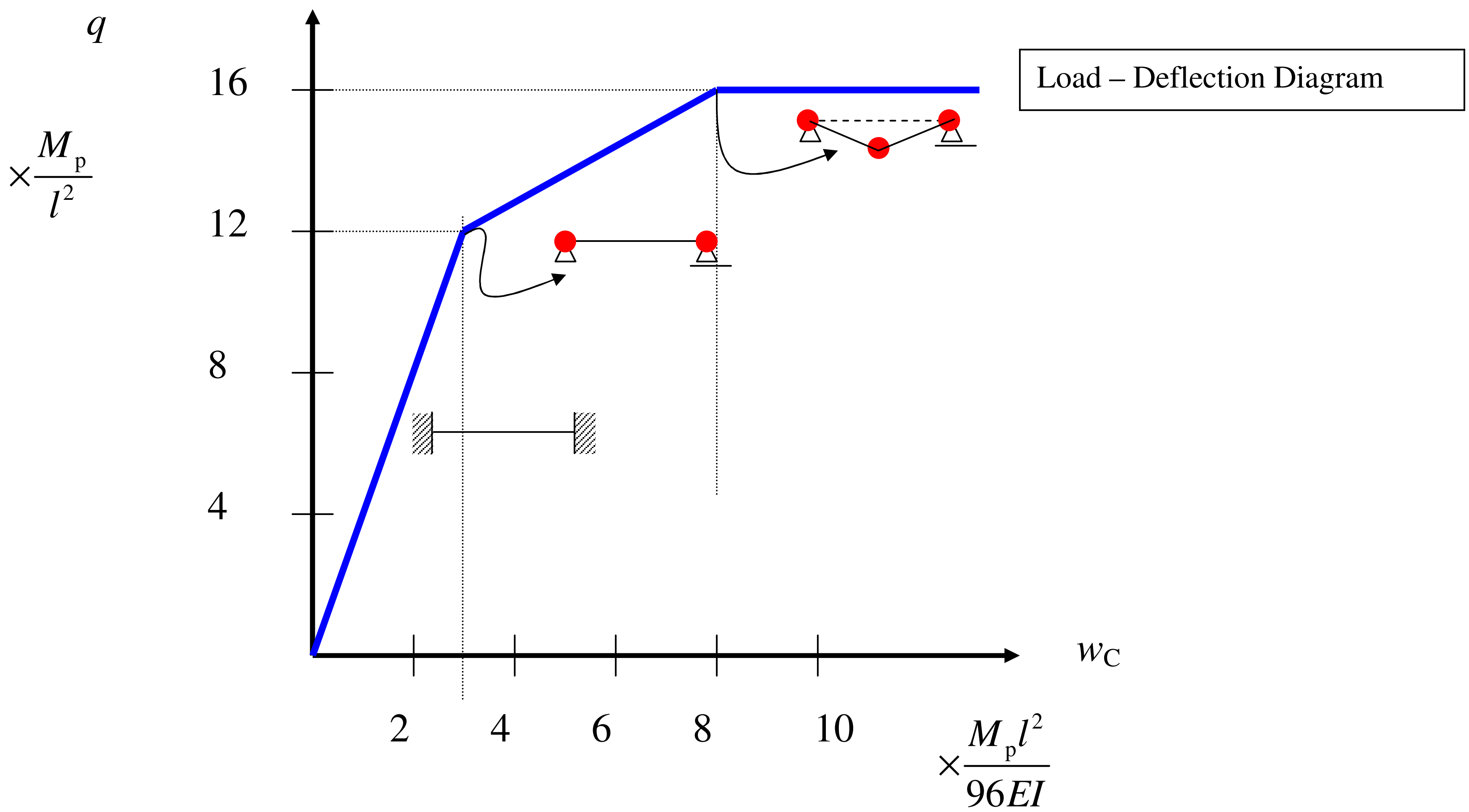


M-line

M-line

$$q_{\mathrm{p}} = q_{\mathrm{I}} + q_{\mathrm{II}} = \frac{16 M_{\mathrm{p}}}{l^2}$$

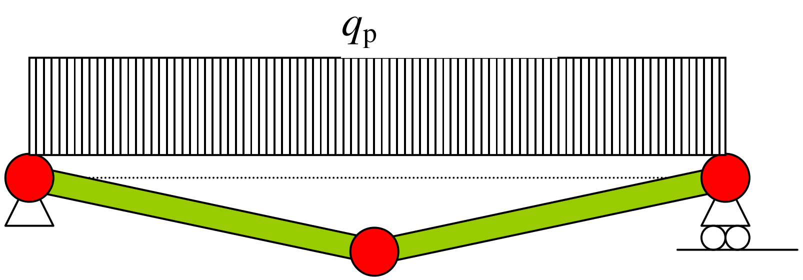
RESULTS OF THE INCREMENTAL METHOD



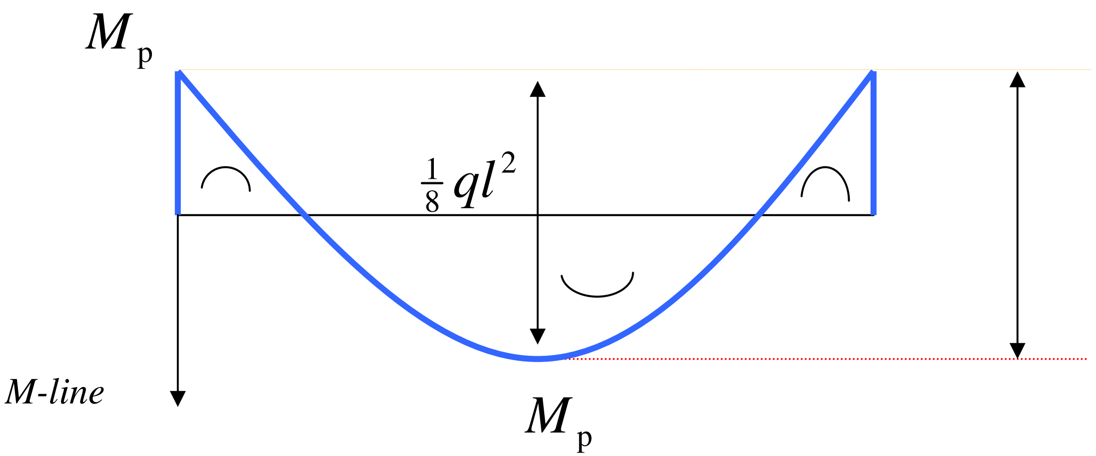
EQUILIBRIUM METHOD (lower bound, chpt 7)

M-distribution at collapse

SAFE ($M < M_p$) AND STATICALLY ADMISSIBLE (equilibrium)
COLLAPSE LOAD IS EQUAL OR LARGER THAN THE OBTAINED LOAD



kinematically indeterminate
(mechanism)
$n = -1$



$$2M_p = \tfrac{1}{8} q l^2$$

$$q_p = \frac{16 M_p}{l^2}$$

NEW APPROACH BASED ON COLLAPSE MECHANISMS (upper bound, chpt 6)

KINEMATICALLY ADMISSIBLE
(mechanism)
COLLAPSE LOAD IS SMALLER OR
EQUAL THAN THE OBTAINED LOAD

1. Determine the max *number of required* plastic hinges to obtain a mechanism
2. Determine the *number of possible positions* for a plastic hinge to occur
3. Find the *number* of possible *mechanisms*
4. Compute for **each mechanism** the collapse load using the principle of *Virtual Work*

COLLAPSE LOAD (upper bound method)

- Of all possible mechanisms the mechanism with the smallest load will be the collapse mechanism.

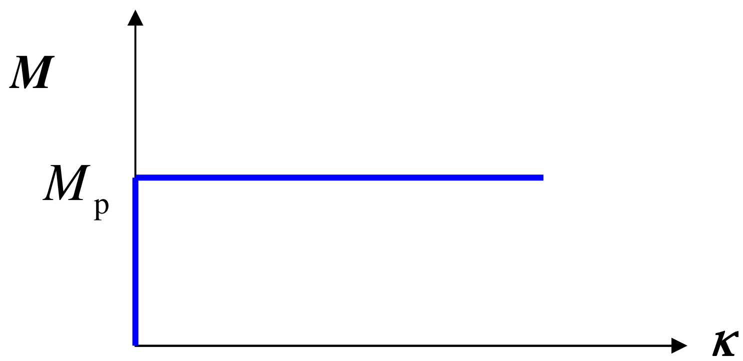
- The actual collapse load will be smaller or equal to the found collapse load, never larger!
  (**Prager's upper-bound-theorema**)

- For all cross sections the moment distribution at collapse must be bounded by the strength of the section. In other words, at no cross section the moments may exceed the plastic limit. If so, the mechanism cannot be the collapse mechanism! (**uniqueness theorem**)

TU Delft

ASSUMPTIONS

- LIMIT STATE AT COLLAPSE
- NO INFO ON DEFORMATION
- IDEAL RIGID-PLASTIC MATERIAL BEHAVIOUR

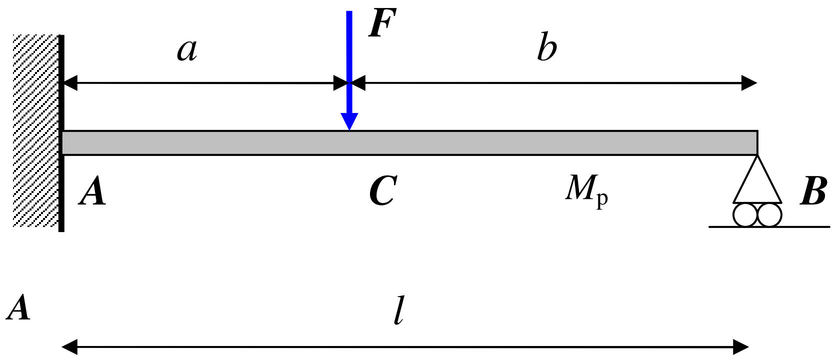


NO ADDITIONAL LOAD CAPACITY DUE TO THE SHAPE FACTOR !!

$\alpha = 1$

- ALL PLASTIC DEFORMATION CONCENTRATED AT A CROSS SECTION, **PLASTIC HINGE**

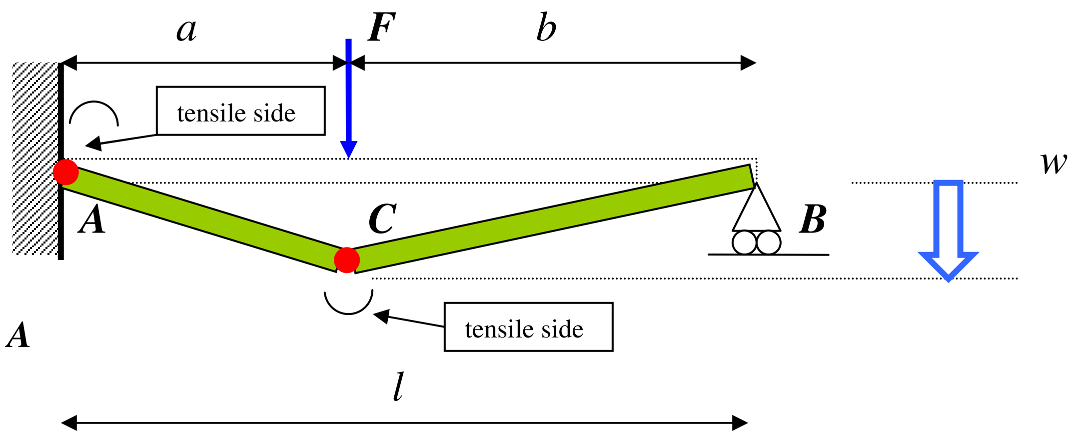
EXAMPLE



Statically indeterminate $n = 1$

- Nr of required hinges for a mechanism : 2
  (1-**2** = -1 = mechanisme)
- Nr of possible position for a hinge : ***A*** and ***C***
- Nr of mechanisms : 1

MECHANISM



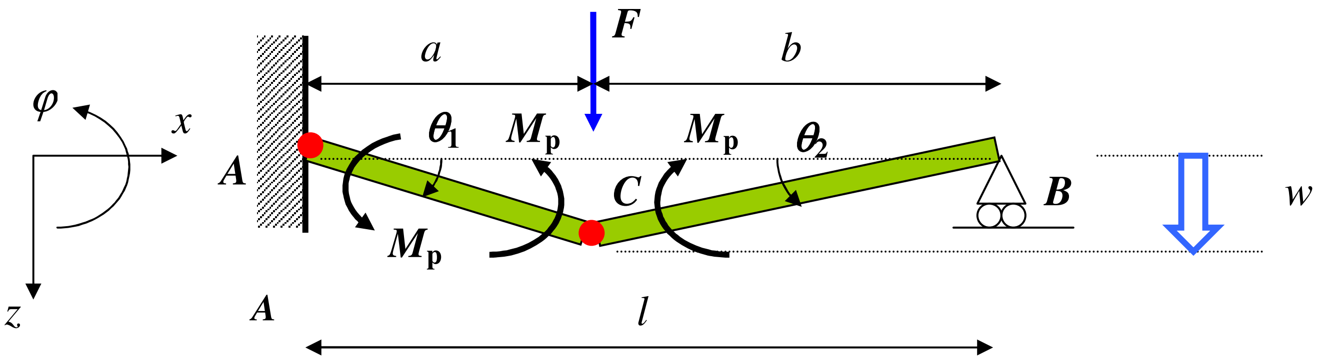
mechanisme n = -1

MOMENTS MUST BE ASSUMED IN THE CORRECT DIRECTION, CORRESPONDING TO THE ASSUMED DISPLACEMENT

Principle of Virtual Work:

Equilibrium of a rigid body for any kinematically admissible displacement only for zero virtual work. (CTB1110 and CTB2210)

VIRTUAL WORK



mechanism n = -1

$$\delta A = -M_p \delta\theta_1 - M_p \delta\theta_1 - M_p \delta\theta_2 + F \times \delta w = 0$$

$$\delta A = -2M_p \frac{\delta w}{a} - M_p \frac{\delta w}{b} + F \delta w = 0$$

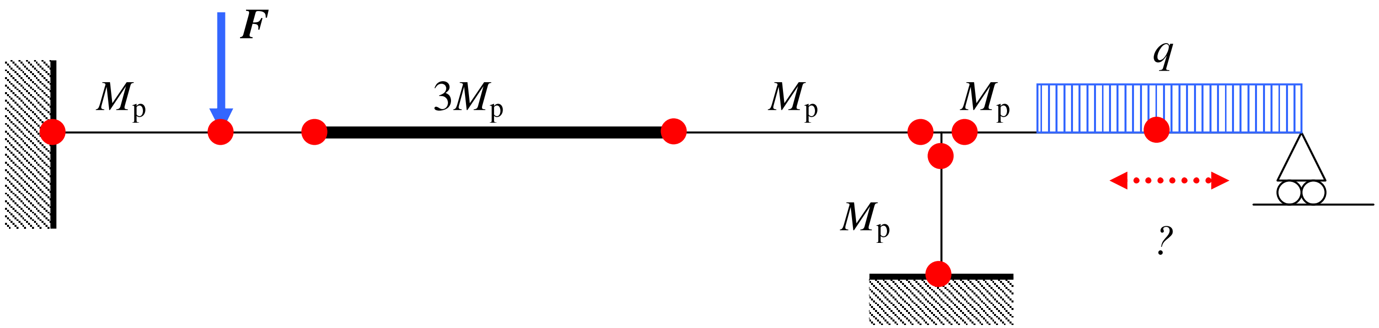
$$F_p = M_p \left(\frac{2}{a} + \frac{1}{b} \right)$$

$$\theta_1 = \frac{w}{a}$$

$$\theta_2 = \frac{w}{b}$$

POSSIBLE POSITIONS FOR PLASTIC HINGES

- At the bar ends
- At the supports (fixed supports)
- Under concentrated loads and distributed loads
- At changes in strength (M_p changes in magnitude)



Example : 9 possible positions (**p** = 9)
max 4+1 = 5 hinges needed (**s** = 5)

nr of possible mechanisms?

NUMBER OF MECHANISM

- DETERMINE THE DEGREE OF STATICALLY INDETERMINANCY (**n**)
- DETERMINE THE MAXIMUM REQUIRED NUMBER OF HINGES FOR A MECHANISM (**s = n+1**)

STATICALLY INDETERMINATE	**n > 0**
STATICALLY DETERMINATE	**n = 0**
MECHANISM	**n = -1**

- DETERMINE THE NR OF POSSIBLE **p** HINGES
- COMPUTE THE MAXIMUM NUMBER OF COMBINATIONS

NUMBER OF POSSIBLE MECHANISMS ?

PERMUTATIONS AND COMBINATIONS

(STATISTICS)

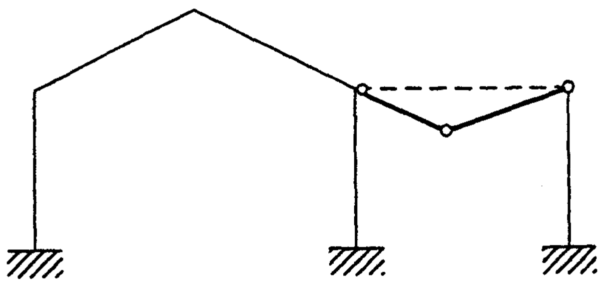
THE NUMBER OF COMBINATIONS OF *p* FROM *s* ELEMENTS ($s \le p$) IF THE ORDER DOES NOT MATTER:

$$C = \binom{p}{s} = \frac{p!}{(p-s)!\,s!}$$

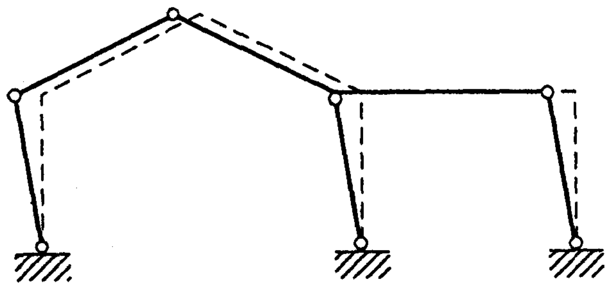
“*p* over s”

PRACTICAL SOLUTIONS

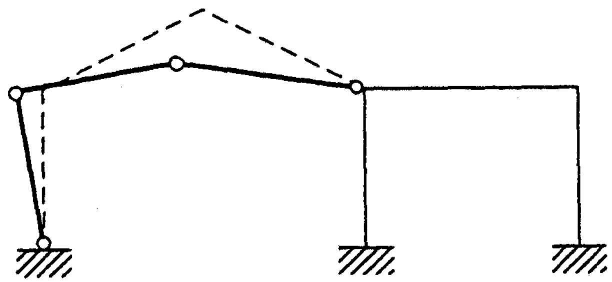
USE BASIC or DEFAULT MECHANISMS (see NL book §5.1, UK book §4.3.2)



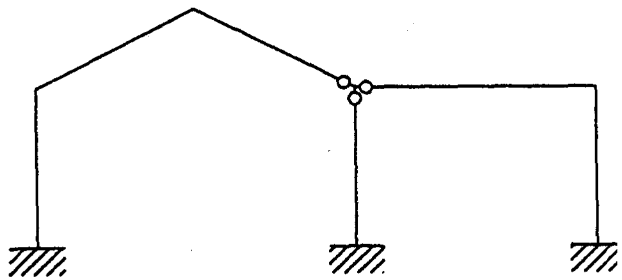
partial beam mechanism



global sway mechanism

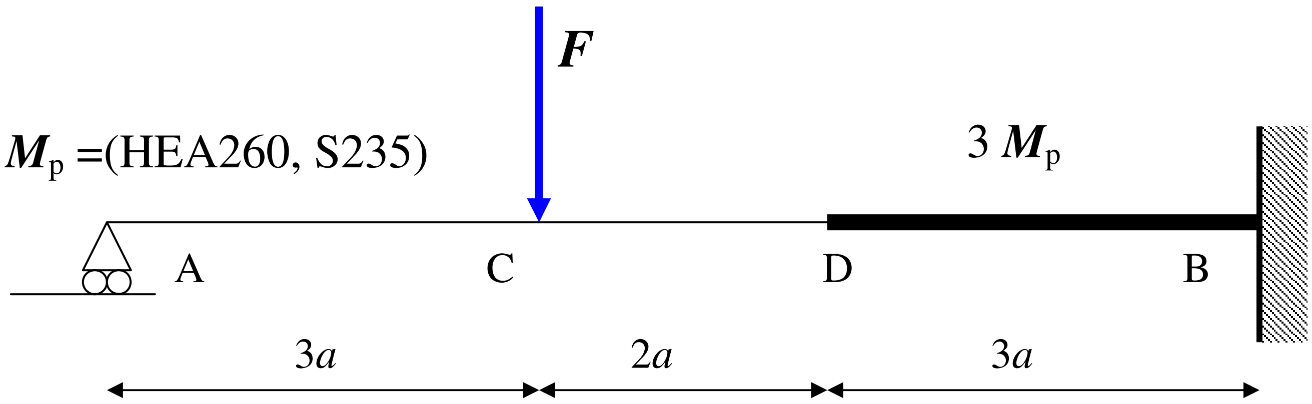


partial mechanism



partial rotational node mechanism

EXAMPLE



DETERMINE THE COLLAPSE LOAD F_p ?

given :

$$a = 1,0 \text{ m}$$

$$M_p = W_p \times f_y = 919,8 \times 10^3 \times 235 \times 10^{-6} = 216 \text{ kNm}$$

Steel sections: Staalprofielen uit de serie (Over) Spannend Staal

SYSTEMATIC APPROACH (upper bound method)

- Determine the maximum required *nr of hinges* for a mechanism (**s**)
- Determine the *nr of possible positions* for hinges (**p**)
- Compute the nr of possible mechanisms $C = \binom{p}{s} = \dfrac{p!}{(p-s)!\,s!}$
- Determine for each mechanism the ultimate load with *virtual work*
- The mechanism with the smallest load is the collapse mechanism
- CHECK : The moment distribution at collapse must remain between the plastic boundaries of each cross section (uniqueness)