Subfaculty of Civil Engineering
Structural Mechanics

Mark each page with your:
STUDENT NUMBER :
NAME :

Exam CIE3109

STRUCTURAL MECHANICS 4

11 April 2016,
09:00 – 12:00 hours

- This exam consists of 4 **problems**.
- Use for each problem a separate sheet of paper.
- Do not forget to mention your name and number on each paper.
- Work neat and tidy, the quality of the presentation can be used in the grading.
- The use of Phone's or computers, PDA's and /or Wifi or Blue Tooth equipment is not allowed. Turn off the equipment and remove it from your table.
- A scientific (programmable) calculator is allowed
- All required formulas can be found on the last pages of this exam
- Keep an eye on the clock and use the specified times per problem as guidance.

Problem 1 : Influence lines (40 min)

A hinged beam as shown in figure 1 is partially loaded with a distributed load q. The position of this load is yet unknown. The hinged beam has two hinges S1 and S2.

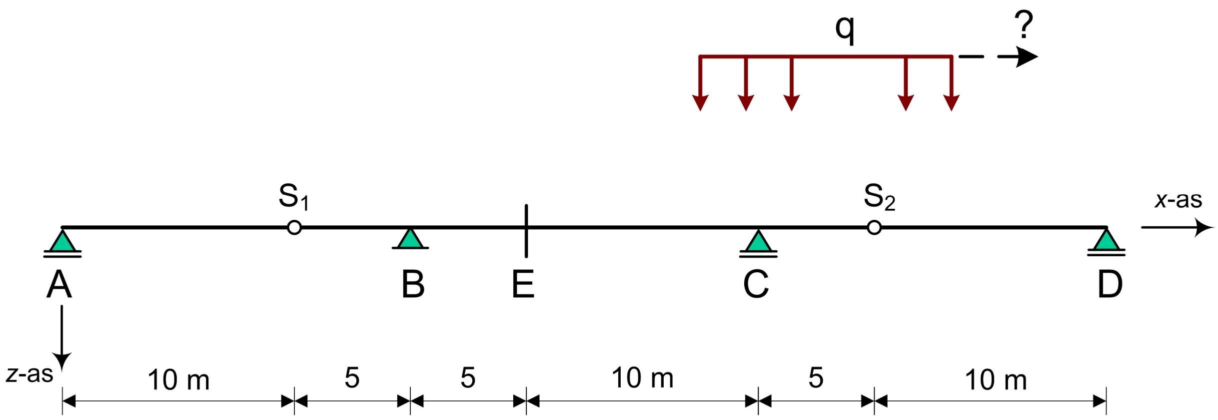


Figure 1 : Hinged beam

Questions:

a) Construct the influence line for the moment at B,
b) Construct the influence line for the shear force directly to the left of B,
c) Construct the influence line for the shear force directly to the right of B,
d) Construct the influence line for the shear force at E,
e) Construct the influence line for the support reaction at C,

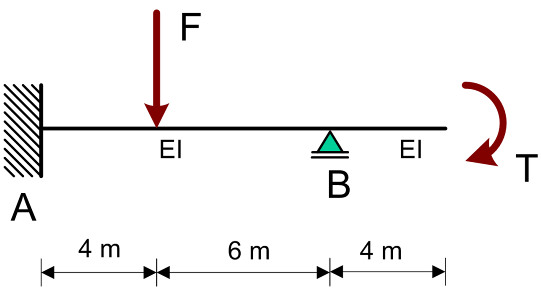
f) Sketch the influence line for the deflection at hinge S1,
g) Sketch the influence line for the rotation at C,

h) Find the most unfavorable position of the distributed load q of 5 kN/m for the moment at B and find the extreme value for this moment due to this load by using the influence line for the moment at B.

Note : “Construct” requires a correct sketch and the computed values of the influence factors at key points in the graph. Thus showing a qualitative and quantitative result. Sketching requires only a qualitative result from which it should become clear if member parts will remain straight or become curved.

Problem 2 : work and energy methods (50 min)

In figure 2 a beam structure is given loaded by a concentrated force F and couple T. The influence of any possible axial or shear deformation is neglected.



Given : F = 125 kN; T = 80 kNm; EI = 15000 kNm^2

Figure 2 : Beam structure

Questions:

a) Describe in short your approach to find the force distribution by using Castigliano's theorem.

b) Write down the required expressions to solve the problem. Support this with sketches to show the positive definition of your unknown(s).

c) Elaborate your expressions and find, following your approach, the unknown(s).

d) Draw the moment distribution of the entire structure including the deformation signs and add also the moment values at characteristic points.

Problem 3 : Plasticity (40 min)

In the following figure a frame ACDEB is shown. The concentrated load acts at C and D. The beam parts have the denoted strengths as shown in the figure. Take care of the different strengths!

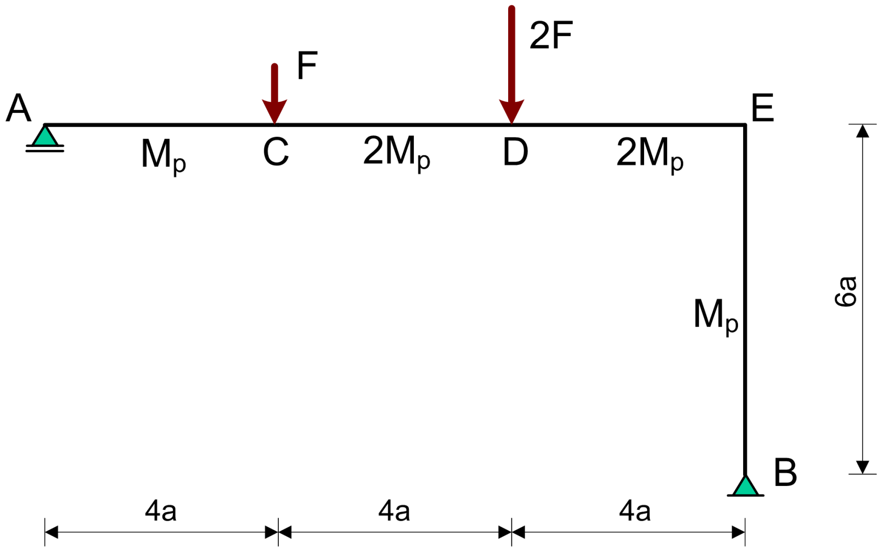


Figure 3 : Frame structure

Questions:

a) Determine the possible collapse mechanisms and show these with small sketches.

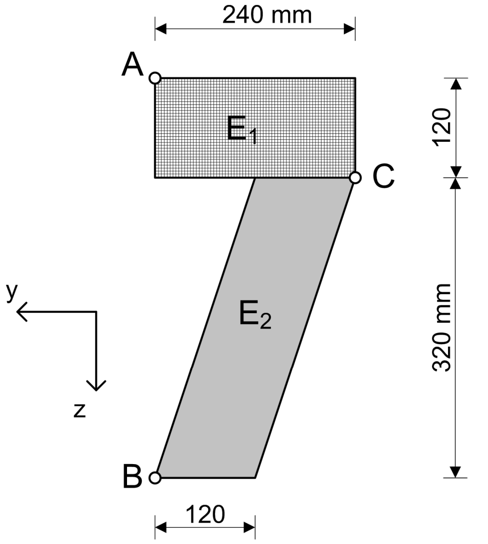
b) Compute the collapse load F_p and prove the uniqueness of your solution.

c) Show the moment distribution at collapse.

d) What does the uniqueness theorem of Prager mean?

Problem 4 (50 min)

A unsymmetrical and inhomogeneous cross section consists of two materials which are perfectly bonded together. In figure 4 the cross section is shown. At three points normal stresses are given; at point A (material 1), at point B (material 2) and at point C (material 2). Each material has its own Young's modulus denoted by E_1 and E_2. The shear force is regarded to be applied at the shear force centre of the cross section. The normal force acts at the normal force centre. The influence of the deformation due to a possible normal force is neglected.



$E_1 = 3000$ N/mm^2; $E_2 = 750$ N/mm^2;
materiaal 1 : $\sigma_A = 48$ N/mm^2;
materiaal 2 : $\sigma_B = 48$ N/mm^2; $\sigma_C = -30$ N/mm^2;

Figure 4 : Inhomogeneous and unsymmetrical cross section

Questions :

a) Your colleague argues that due to the given normal stress distribution the cross section must be in bending in the *x-y* plane only. How do you respond to this?

b) Find for the cross section the cross-sectional stiffness quantities and give the constitutive relation. As a check:
$|EI_{yy}| = 4838400 \times 10^5$ Nmm^2; $|EI_{yz}| = 921600 \times 10^5$ Nmm^2; $|EI_{zz}| = 13948800 \times 10^5$ Nmm^2

c) Describe in short how you can find the cross sectional forces for this problem.

d) Find the cross sectional forces. You can make use of your graphical calculator. In case of a hand calculation you may assume a strain at the bar axis of $-3/500$.

e) Sketch the position (and slope) of the neutral axis in the cross section.

f) Sketch of the shape of the kernel and find the position of the utmost left kernel point.

g) Describe briefly the essential steps needed to compute the shear flow in the interface between material 1 and 2. Do <u>not</u> compute this value!

FORMULAS

Inhomogeous and/or asymmetrical cross sections :

$$\varepsilon(y,z) = \varepsilon + \kappa_y y + \kappa_z z \qquad \text{and:} \quad \sigma(y,z) = E(y,z)\times\varepsilon(y,z)$$

$$\begin{bmatrix} M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} \kappa_y \\ \kappa_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

$$s_x^{(a)} = -\frac{V_y ES_y^{(a)}}{EI_{yy}} - \frac{V_z ES_z^{(a)}}{EI_{zz}}; \quad \text{or:} \quad s_x^{(a)} = -\frac{R_M^{(a)}}{M} V; \quad \sigma_{xt} = \frac{s_x^{(a)}}{b^{(a)}}$$

$$\tan(2\alpha) = \frac{2EI_{yz}}{\left(EI_{yy} - EI_{zz}\right)}; \quad EI_{1,2} = \tfrac{1}{2}\left(EI_{yy} + EI_{zz}\right) \pm \sqrt{\left(\tfrac{1}{2}\left(EI_{yy} - EI_{zz}\right)\right)^2 + EI_{yz}^2}$$

$$q_y^* = \frac{EI_{yy}EI_{zz}q_y - EI_{yy}EI_{yz}q_z}{EI_{yy}EI_{zz} - EI_{yz}^2}$$

$$q_z^* = \frac{-EI_{yz}EI_{zz}q_y + EI_{yy}EI_{zz}q_z}{EI_{yy}EI_{zz} - EI_{yz}^2}$$

Deformation energy:

$$E_v = \int \tfrac{1}{2} EA\varepsilon^2 \mathrm{d}x \quad \text{(extension)}$$

$$E_v = \int \tfrac{1}{2} EI\kappa^2 \mathrm{d}x \quad \text{(bending)}$$

Kinematic relations:

$$\varepsilon = \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\kappa = -\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

Complementary energy:

$$E_c = \int \frac{N^2}{2EA} \mathrm{d}x \quad \text{(extension)}$$

$$E_c = \int \frac{M^2}{2EI} \mathrm{d}x \quad \text{(bending)}$$

Constitutive relations:

$$N = EA.\varepsilon$$

$$M = EI.\kappa$$

Castigliano's theorema's:

$$F_i = \frac{\partial E_v}{\partial u_i} \qquad u_i = \frac{\partial E_c}{\partial F_i}$$

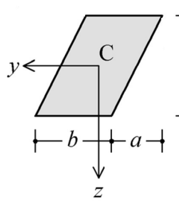
Work method with unit load:

$$u = \int \frac{M(x)m(x)dx}{EI}$$

Rayleigh:

$$F_k = \frac{E_v}{\int \tfrac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2 \mathrm{d}x}$$

Math tools:

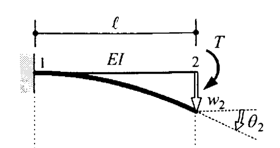
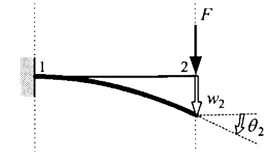
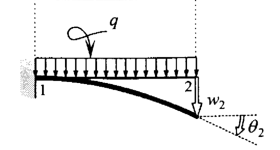


$$I_{yy} = \tfrac{1}{12}(a^2 + b^2)bh$$

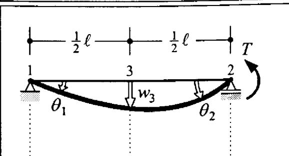
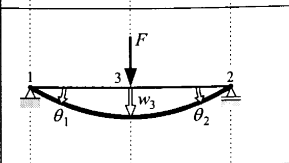
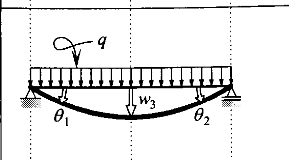
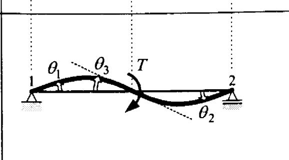
$$I_{zz} = \tfrac{1}{12}bh^3$$

$$I_{yz} = \tfrac{1}{12}abh^2$$

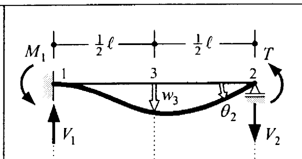
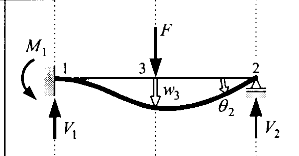
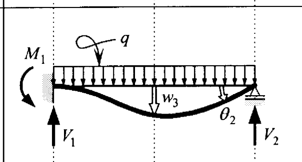
vergeet-mij-nietjes

(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

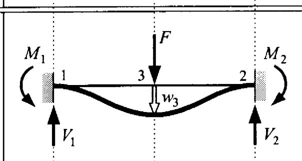
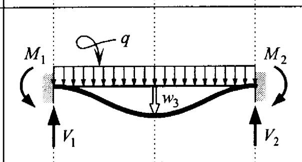
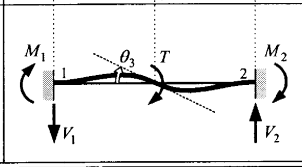
vrij opgelegde ligger (statisch bepaald)

(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

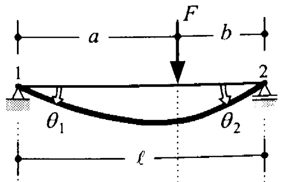
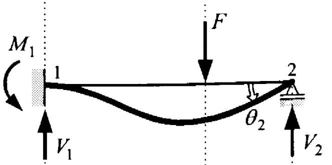
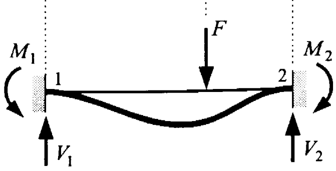
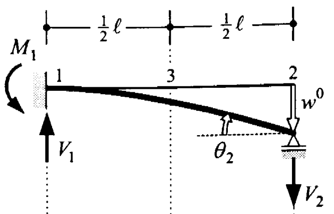
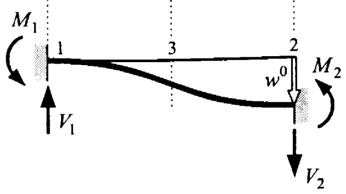
statisch onbepaalde ligger (enkelzijdig ingeklemd)

(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statisch onbepaalde ligger (tweezijdig ingeklemd)

(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

Enkele formules voor prismatische liggers met buigstijfheid EI.
T, F en q zijn belastingen door resp. een koppel, kracht en gelijkmatig verdeelde belasting.
M_i en V_i zijn het buigend moment en de dwarskracht op einddoorsnede i van de ligger ten gevolge van de oplegreacties.

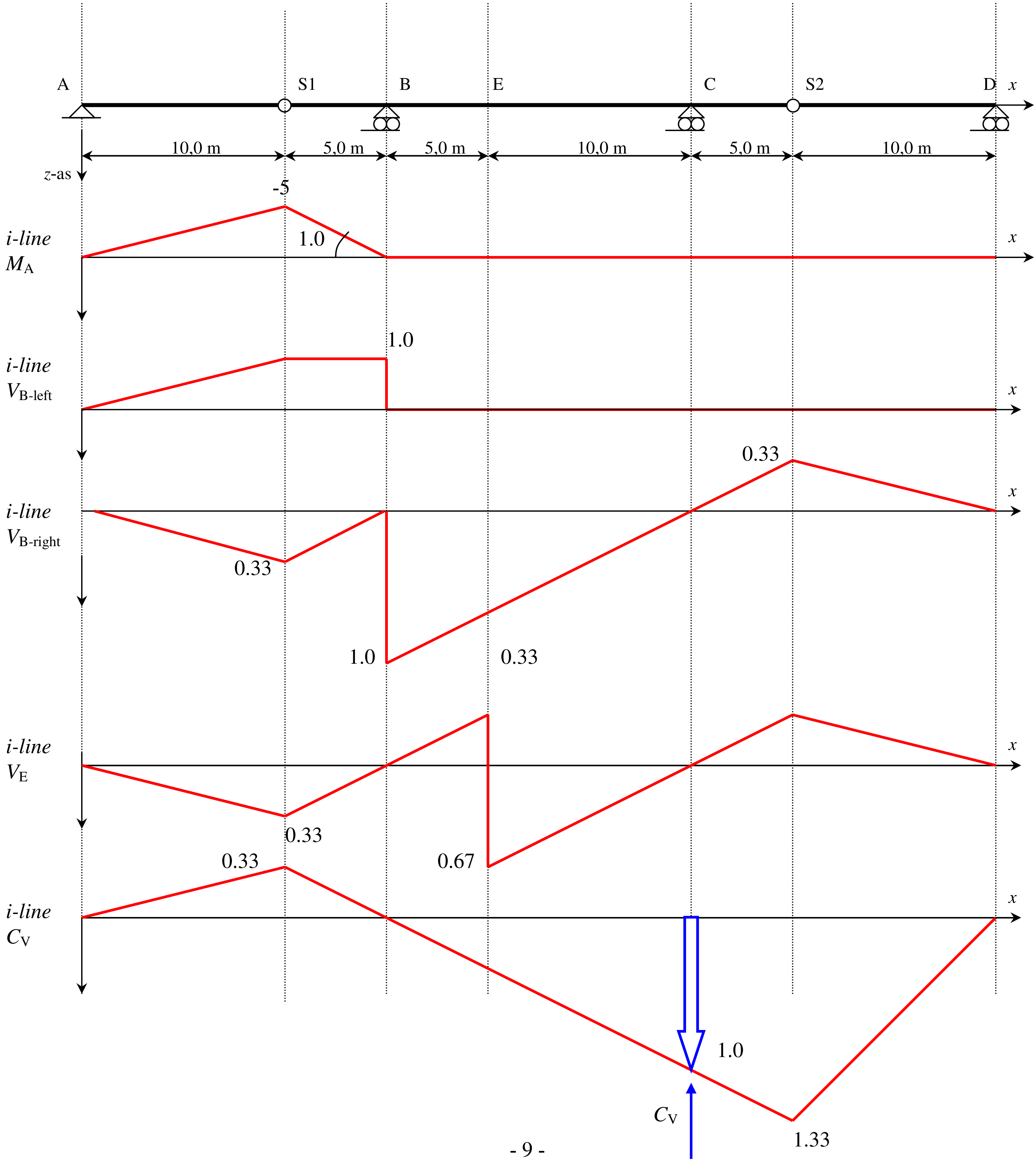
(c)		$\theta_1 = \frac{Fab(\ell+b)}{6EI\ell} = \frac{F\ell^2}{6EI}\left(2\frac{a}{\ell} - 3\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3}\right)$ $\theta_2 = \frac{Fab(\ell+a)}{6EI\ell} = \frac{F\ell^2}{6EI}\left(\frac{a}{\ell} - \frac{a^3}{\ell^3}\right)$
(d)		$M_1 = \frac{Fb(\ell^2 - b^2)}{2\ell^2} = F\ell\left(\frac{a}{\ell} - \frac{3}{2}\frac{a^2}{\ell^2} + \frac{1}{2}\frac{a^3}{\ell^3}\right)$ $V_1 = \frac{Fb(3\ell^2 - b^2)}{2\ell^3} = F\left(1 - \frac{3}{2}\frac{a^2}{\ell^2} + \frac{1}{2}\frac{a^3}{\ell^3}\right)$ $V_2 = \frac{Fa^2(3\ell - a)}{2\ell^3} = F\left(\frac{3}{2}\frac{a^2}{\ell^2} - \frac{1}{2}\frac{a^3}{\ell^3}\right)$ $\theta_2 = \frac{Fa^2b}{4EI\ell} = \frac{F\ell^2}{4EI}\left(\frac{a^2}{\ell^2} - \frac{a^3}{\ell^3}\right)$
(e)		$M_1 = \frac{Fab^2}{\ell^2} = F\ell\left(\frac{a}{\ell} - 2\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3}\right)$ $V_1 = \frac{Fb^2(\ell + 2a)}{\ell^3} = F\left(1 - 3\frac{a^2}{\ell^2} + 2\frac{a^3}{\ell^3}\right)$ $M_2 = \frac{Fa^2b}{\ell^2} = F\ell\left(\frac{a^2}{\ell^2} - \frac{a^3}{\ell^3}\right)$ $V_2 = \frac{Fa^2(\ell + 2b)}{\ell^3} = F\ell\left(3\frac{a^2}{\ell^2} - 2\frac{a^3}{\ell^3}\right)$
(f)		$M_1 = \frac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \frac{3EI}{\ell^3}w^0$ $\theta_2 = \frac{3}{2}\frac{w^0}{\ell}$ $\theta_3 = \frac{9}{8}\frac{w^0}{\ell};\quad w_3 = \frac{5}{16}w^0$
(g)		$M_1 = M_2 = \frac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \frac{12EI}{\ell^3}w^0$ $\theta_3 = \frac{3}{2}\frac{w^0}{\ell};\quad w_3 = \frac{1}{2}w^0$

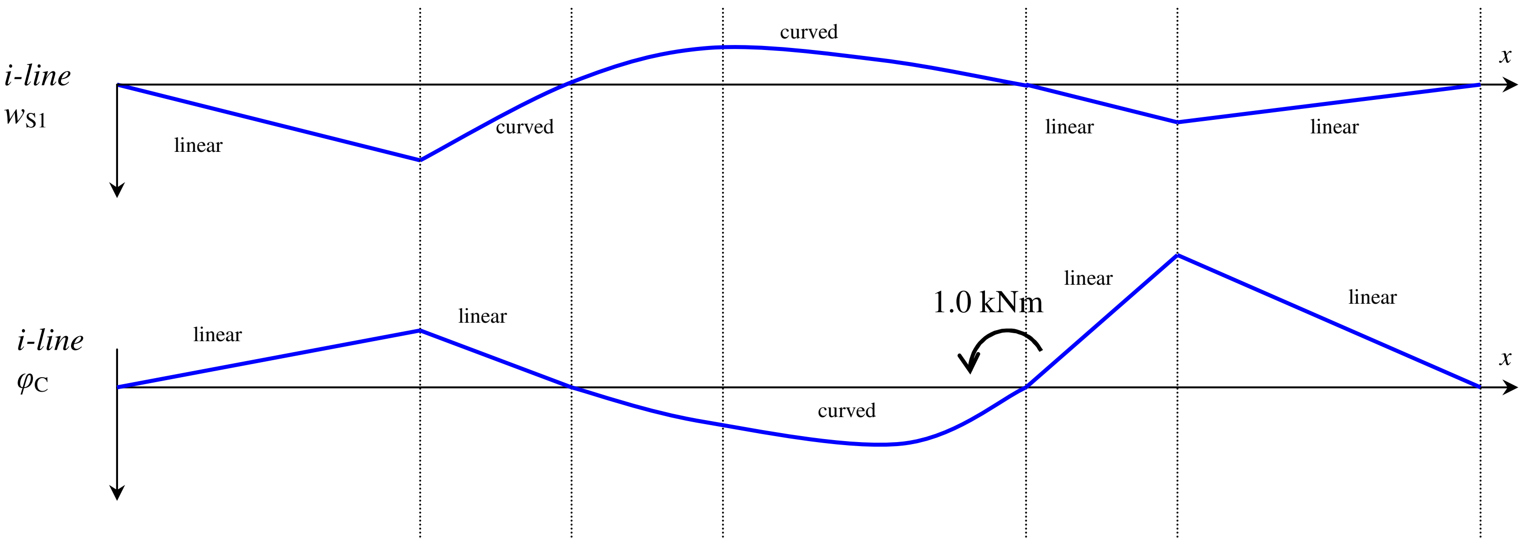
drie bij-de handjes

zettingen

ANSWERS

Problem 1
The constructed influence lines a) through e) and sketched lines f) and g). The general used directions for positive quantities have been used.





h) The constant distributed load should be placed on part AB which results in a maximum moment at A of: $M_A = \frac{1}{2} \times 15 \times (-5.0) \times 5 = -187.5 \text{ kNm}$.

Problem 2

a) Simply static indeterminate structure. The redunant can be a force/moment for which the associated degree of freedom has a zero value. For this requirement Castigliano's second theoreme can be used. In this case the redundant is the support reaction at B with its associated zero displacement at B.

b) To express the strain energy in terms of the redundant the moment distribution is required in terms of all the elements of loading as a function of *x* including the redundant B_V:

$$M_1(x) = -F(4-x) \qquad 0 \le x \le 4,0$$
$$M_2(x) = B_V(10-x) - T \qquad 0 \le x \le 10,0$$
$$M_3(x) = -T \qquad 10 \le x \le 14,0$$

$$E_v = \int_0^{4,0} \frac{\left(M_1(x) + M_2(x)\right)^2 dx}{2EI} + \int_{4,0}^{10,0} \frac{M_2^2(x)dx}{2EI} + \int_{10}^{14} \frac{M_3^2(x)dx}{2EI}$$

The deformation condition yields:

$$w_B = 0 \Rightarrow \quad w_B = \frac{\mathrm{d}E_V}{\mathrm{d}B_V}$$

Since the moment distribution from 10.0 to 14.0 m is constant and not depends on the static unknown, this part can be left out.

c) Use Castigliano on the distributions AB:

$$w_B = \frac{\mathrm{d}}{\mathrm{d}B_V} E_v = \int_0^4 \frac{\mathrm{d}}{\mathrm{d}B_V}\left[\frac{(-4F + Fx + 10B_V - B_V x - T)^2}{2EI}\right]\mathrm{d}x + \int_4^{10} \frac{\mathrm{d}}{\mathrm{d}B_V}\left[\frac{(10B_V - B_V x - T)^2}{2EI}\right]\mathrm{d}x$$

$$= \int_0^4 \frac{(-4F + Fx + 10B_V - B_V x - T)(10-x)}{EI}\,\mathrm{d}x + \int_4^{10} \frac{(10B_V - B_V x - T)(10-x)}{EI}\mathrm{d}x =$$

$$= \int_0^4 \frac{(-40F + 14Fx - Fx^2 + 100B_V - 20B_V x + B_V x^2 - 10T + Tx)}{EI}\,\mathrm{d}x +$$

$$\int_4^{10} \frac{(100B_V - 20B_V x + B_V x^2 - 10T + Tx)}{EI}\mathrm{d}x =$$

$$= -\frac{26F}{5625} + \frac{B_V}{45} - \frac{T}{300} = 0$$

Using F = 125 kN and T = 80 kNm results in:

$$B_V = 38 \text{ kN} \uparrow$$

d) Correct moment distribution required, no mistakes in shape. Moment at the support is 200 kNm, at the concentrated load 148 kNm and at the support 80 kNm.

Problem 3

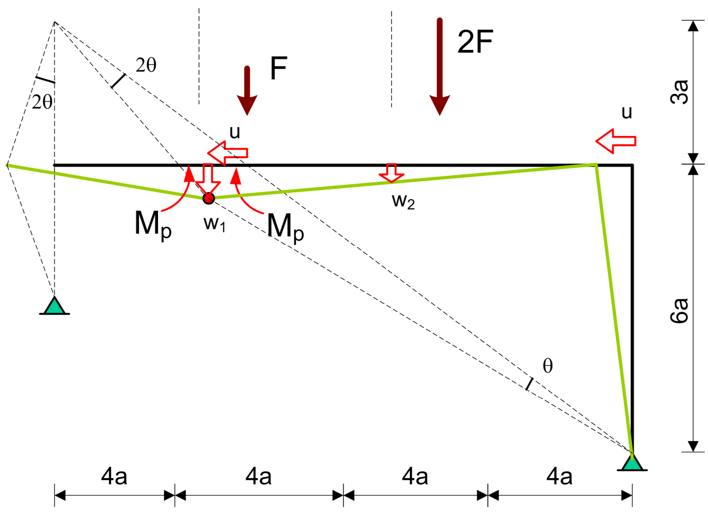
a) This is a static determinate structure (!) and thus only one additional hinge is needed to create a mechanism. For this plastic hinge three positions are possible which results in three possible mechanisms.

Hinges at C, D or E. In this latter case no work is generated by the load which results in an infinite load. This is certainly not the mechanism with the lowest failure load. So only hinges at C and D have to be discussed.

b) Present a complete mechanism:
1. Proper visualisation of the mechanism including the rotations of each element in relation to the others
2. Correct direction of the plastic moments at the hinges.
3. Correct virtual work equation.

Errors in the kinematics are fatal, the mechanism will than not be taken into account for grading since the kinematics is an essential learning objective. Hint: use the *u* and the *w* to find all correct rotations without gambling.

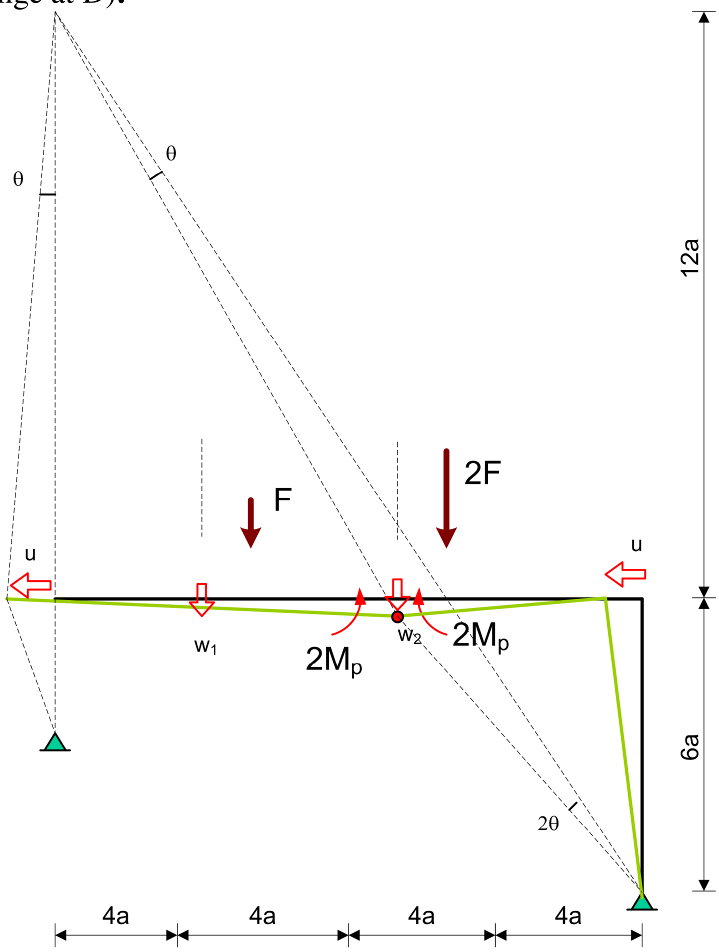
Mechanism 1 (hinge at C)**:**



$$\delta A = -M_p 2\delta\theta - M_p \delta\theta + F \times 8a \times \delta\theta + 2F \times 4a \times \delta\theta = 0$$

$$F_p = \frac{3M_p}{16a} = 0.1875\frac{M_p}{a}$$ **LOWEST FAILURE LOAD**

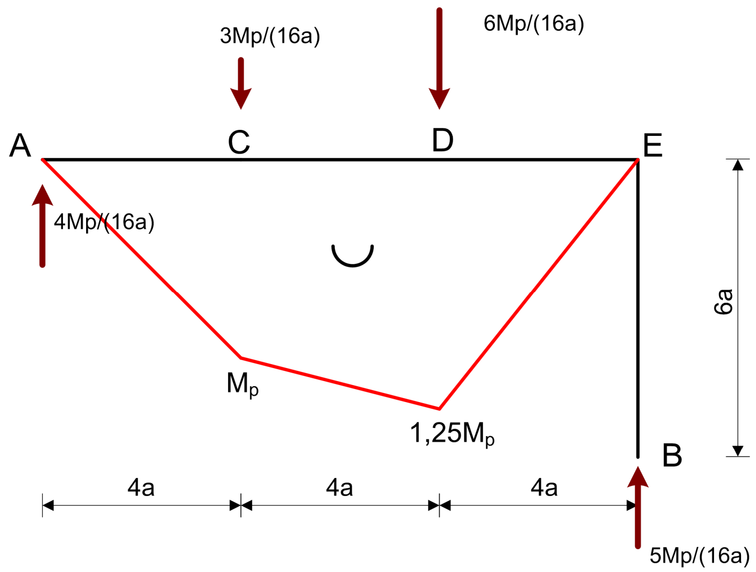
Mechanism 2 (hinge at D)**:**



$$\delta A = -2M_p \times \delta\theta - 2M_p \times 2\delta\theta + F \times 4a \times \delta\theta + 2F \times 4a \times 2\delta\theta = 0$$

$$F_p = \frac{3M_p}{10a} = 0.3\frac{M_p}{a}$$

c) The moment distribution for mechanism 1 is shown below. The moment at D has a magnitude of $1.25M_\mathrm{p}$ and this is smaller than the strenght $2M_\mathrm{p}$ at D.



Note that the horizontal support reaction at B must be zero due to the roller at A. So no moment distribution in the column BE please!

d) Prager's uniqueness theoreme states that the moment distribution at no point exceeds the strength of the structure for the failure mechanism with the smallest failure or collapse load.

Problem 4

a) Although the stresses at A and B are equal, the strains are not due to the difference in Young's modulus for material 1 and 2. This results in a neutral axis which will not run parallel to AB. The plane of curvature can therefore not be a horizontal plane.

b) The axial stiffness can be found with:

$$EA = E_1(240\times120)+E_2(120\times320)=1152\times10^5 \text{ Nmm}$$

The position of the normal force centre (with respect tot he upper left corner) can be found with:

$$y_{\text{NC}} = \frac{E_1(220\times120\times120)+E_2(120\times320\times120)}{EA}=120 \quad \text{mm}$$

$$z_{\text{NC}} = \frac{E_1(220\times120\times60)+E_2(120\times320\times280)}{EA}=115 \quad \text{mm}$$

The bending stiffness tensor thus becomes:

$$EI_{yy} = 3000\left(\tfrac{1}{12}\times120\times240^3\right)+$$
$$750\left(\tfrac{1}{12}\times2\times120^2\times120\times320\right)=4838400\times10^5 \text{ Nmm}^2$$

$$EI_{yz} = 750\left(\tfrac{1}{12}\times120^2\times320^2\right)=921600\times10^5 \text{ Nmm}^2$$

$$EI_{zz} = 3000\left(\tfrac{1}{12}\times240\times120^3+240\times120\times(55)^2\right)+$$
$$750\left(\tfrac{1}{12}\times120\times320^3+120\times320\times(-165)^2\right)=13948800\times10^5 \text{ Nmm}^2$$

And finally the constitutive relation of the cross section yields:

$$\begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix} = 10^5 \begin{bmatrix} 1152 & & \\ & 4838400 & 921600 \\ & 921600 & 13948800 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

c) For three points the stresses are known. This results in three equations with three unknown deformations:

$$\sigma_A = E_1(\varepsilon + \kappa_y y_A + \kappa_z z_A)$$
$$\sigma_B = E_2(\varepsilon + \kappa_y y_B + \kappa_z z_B)$$
$$\sigma_C = E_2(\varepsilon + \kappa_y y_C + \kappa_z z_C)$$

The deformations can be found en with the constitutive relation the sectional force scan be computed:

$$\begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EA & & \\ & EI_{yy} & EI_{yz} \\ & EI_{zy} & EI_{zz} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

d) Solving this set of expressions results in:

$$48 = 3000\varepsilon + 360000\kappa_y - 345000\kappa_z$$
$$48 = 750\varepsilon + 90000\kappa_y + 243750\kappa_z$$
$$-30 = 750\varepsilon - 90000\kappa_y + 3750\kappa_z$$
$$\Leftrightarrow \begin{bmatrix} 48 \\ 48 \\ -30 \end{bmatrix} = \begin{bmatrix} 3000 & 360000 & -345000 \\ 750 & 90000 & 243750 \\ 750 & -90000 & 3750 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

with the Graphical Calculator:

$$\varepsilon = -\frac{3}{500}; \quad \kappa_y = \frac{19}{66000}; \quad \kappa_z = \frac{3}{27500}; \quad (-0,006; 2,878\times10^{-4}; 1,090\times10^{-4})$$

The resulting sectional forces found are:

$$\begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix} = 10^5 \begin{bmatrix} 1152 & & \\ & 4838400 & 921600 \\ & 921600 & 13948800 \end{bmatrix} \begin{bmatrix} -3/500 \\ 19/66000 \\ 3/27500 \end{bmatrix} = \begin{bmatrix} -691,2\times10^3 \text{ N} \\ 149,34\times10^6 \text{ Nmm} \\ 178,70\times10^6 \text{ Nmm} \end{bmatrix}$$

No bending in the *x*-*y*-plane as this result reveals. The shear forces can not be found since we do not have more info on the moment distribution along the *x*-axis.

e) The expression for the neutral axis yields:

$$95y + 36z = 1980$$

The *na* intersects the *y*-axis for $z = 0$ at $y = 20.8$ mm and the *z*-axis for $y = 0$ at $z =$ 55 mm and does not run through the normal force centre due to the non-zero normal force. The plane of curvature is perpendicular to the neutral axis and the plane of loading which results from the bending moment components does not coincide with either one of the axis of the cross section. This result was to be expected since the cross section is both unsymmetrical and inhomogeneous.

f) The kernel is a pentagon since five tangent lines can be drawn at the boundaries of this cross section which do not intersect with the cross section. Each line can be regarded as a position of the *na* just running outside the cross section thus generating a force point in which an axial load should be applied to obtain the assumed *na*. Connecting the five points results in the kernel.

The left kernel point is found with a *na* which runs at the right side of the cross section:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{1152} \begin{bmatrix} 4838400 & 921600 \\ 921600 & 13948800 \end{bmatrix} \begin{bmatrix} -1/(-120) \\ 0 \end{bmatrix} = \begin{bmatrix} 35.0 \\ 6.67 \end{bmatrix}$$

g) In order to find the shear flow *s* at the interface due to a shear force *V* the resultant normal force due to bending only is required on the released part of the structure:

- Find the normal stress distribution on the released part of the structure due to bending only
- Find the resultant normal force due to this normal stress by integration of the stress distribution (linear distribution so this is very simple)
- Find the shear flow with:

  $$s_x^{(a)} = -\frac{R_M^{(a)}}{M} V;$$

  Both the moment *M* and the shear force *V* are the total moment and shear force applied at the cross section.