Faculty
Civil Engineering and GeoSciences

Exam	**CIE3109-09 / CTB3330**
	Structural Mechanics 4
Total number of pages	**8 pages (excl cover)**
Date and time	**APR-11-2022** from **09:00-12:00**
Responsible lecturer	**J.W. Welleman**

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 4

☒ **questions may differ in weight** *(the time mentioned is an indicator for the weight)*

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with red pen or with pencils.
- Calculators with CAS and/or wifi/BT and/or PDF capabilities
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ **books** ☐ **notes** ☐ **dictionaries** ☐ **syllabus**

☐ **formula sheets (see also below under 'additional information')** ☒ **calculators**

☐ **computer** ☐ **...**

☒ **scientific (graphical)calculator** ☒ **drawing material**

Additional information (if necessary to be filled in by the examiner)

- **Use for each problem a separate examination paper**
- **The question form contains fomula sheets which can be used.**
- **Students can take the question form home after the exam.**
- **Specify the correct BSc or MSc course code on the exam paper**
- **BSc students are allowed to answer in Dutch**
- **No student leaves without delivering an exam paper with a name on it!**

Exam graded by: (the marking period is 15 working days at most)

Every suspicion of fraud is reported to the Board of Examiners.

Mobile Phone OFF.

Problem 1 : Influence Lines (approx. 40 min)

Two hinged beams with bending stiffness *EI* are shown in figure 1. The hinges are denoted with h.

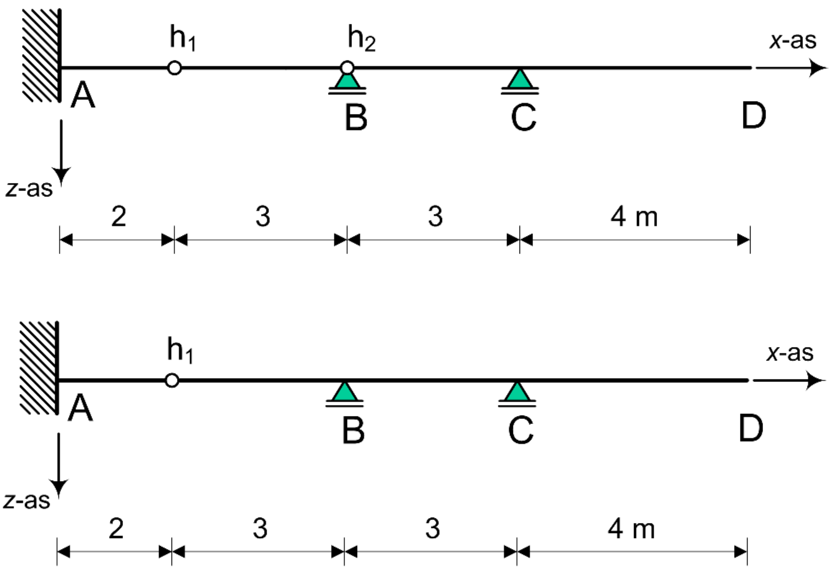


Figure 1 : Two hinged beams with different static systems

Questions:

Based on the first structure:
a) Construct the influence line for the moment at A,
b) Construct the influence line for the shear direct left of B,
c) Construct the influence line for the shear force directly to the left of C,

Based on the second structure:
d) Sketch the influence line for the support reaction at C,
e) Sketch the influence shear force direct to the left of B,
f) Sketch the influence line for the displacement of h_1.

A distributed load of 5 kN/m is placed on the first beam in such a way to obtain the maximum moment value at A.

g) Specify the location for the distributed load and the maximum value of the moment at A.

Note : "Construct" requires a correct sketch and the computed values of the influence factors at key points in the graph showing a qualitative and quantitative result. Sketching requires only a qualitative result from which it should become clear if member parts will remain straight or become curved.

Problem 2 : Work and Energy Methods (approx. 40 min)

For a hinged beam shown in figure 2, information about the deformation is asked using a work and energy method. The prismatic beam, with bending stiffness *EI* and axial stiffness *EA*, is fully clamped at A. At C a hinge is placed. Use suitable coordinate systems to solve this problem.

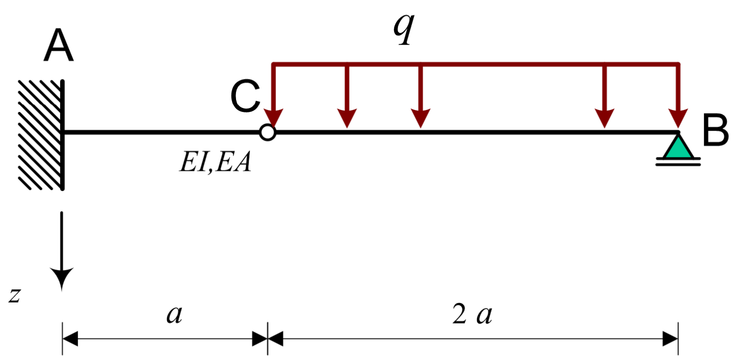


Figure 2 : Hinged beam with distributed load

Questions:

a) Explain the required steps involved to find the vertical displacement of point C due to the load q using Castigliano's theoreme.

b) Find with Castigliano's theoreme the expression for the vertical displacement at C due to the load q, expressed in the parameters used.

c) Explain the essential difference between Castigliano’s method and the work-method with unity load from Clapeyron.

d) How much additional load carrying capacity will this beam have due to the given distributed load q if we use a full plastic analysis instead of an elastic analysis assuming a shape factor 1.0. Give a clear motivation for your answer.

Note: Support your answer if needed with a clear sketch. Clearly show your positive definitions and coordinate system(s) used.

Problem 3 : Plasticity (approx. 50 min)

In figure 3, a frame is shown for which the collapse load has to be found. The frame is loaded with the two indicated concentrated loads. The parts have the denoted strengths as shown also in the figure. At D and G the connections are rigid and the right end B is fully clamped. (DUTCH : ingeklemd).

NOTE : Take care of the different strengths of the elements and the rollers A and C (DUTCH : rolopleggingen)!

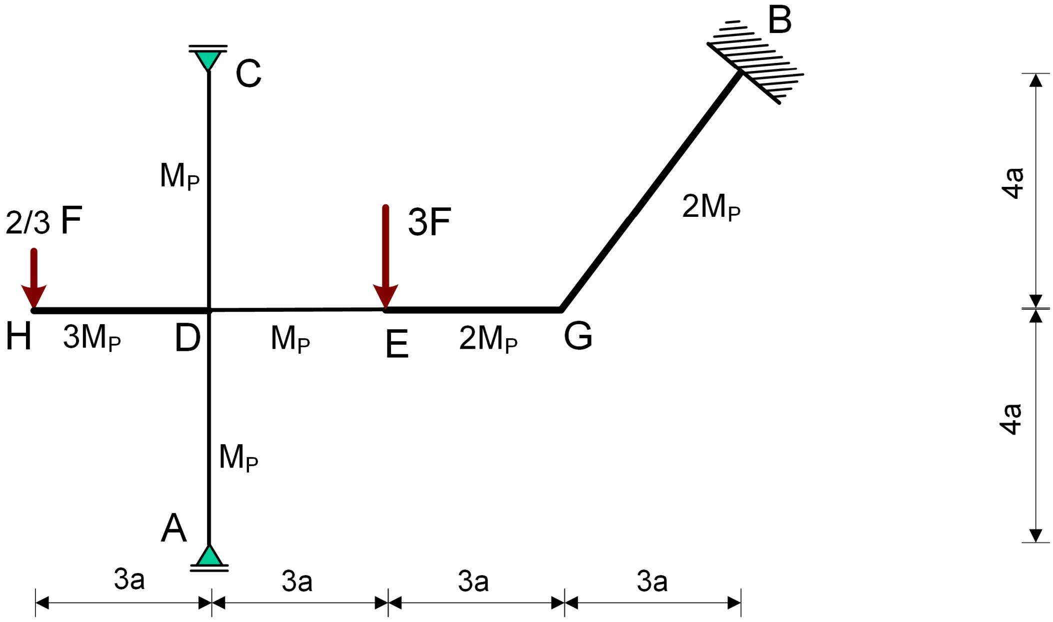


Figure 3 : Frame structure

Questions:

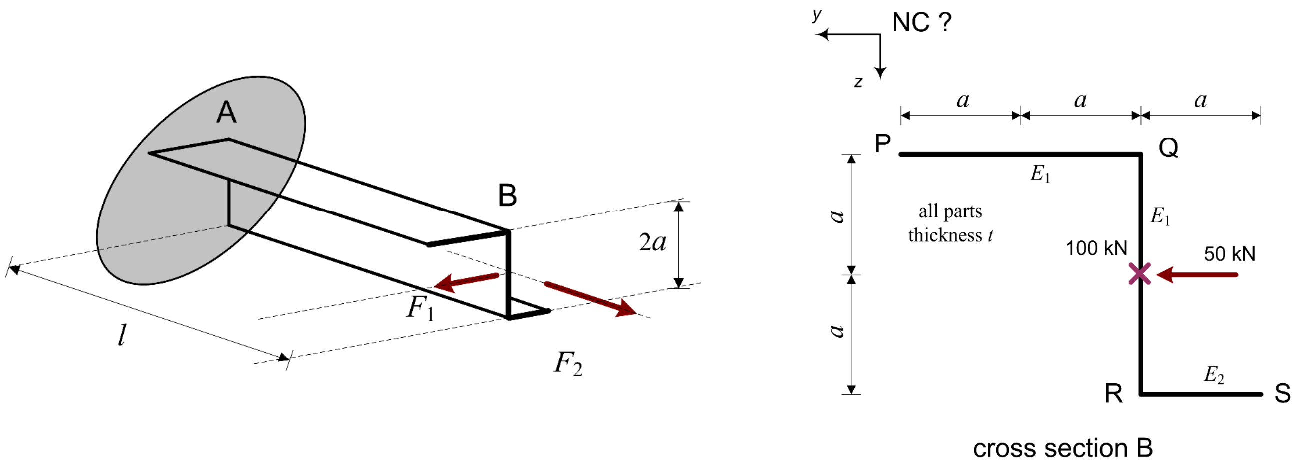
a) Copy the structure on your exam paper and clearly indicate at which points plastic hinges can **NOT** occur and mention why not.

b) Make small sketches of all possible mechanisms to consider.

c) Which mechanism is the failure mechanism? Clearly motivate you answer and give proof that you are right.

Problem 4 : Cross Section **(approx. 50 min)**

An unsymmetrical thin-walled cross section as shown in the fig. 4 (right), is used as a cantilever beam AB, see fig. 4 (left). (in Dutch: uitkragende ligger). The left end at A is fully clamped and the right end at B is free as indicated in the figure. The thickness t, is constant for the entire cross section. The Young's modulus E is different for parts of the cross section as is indicated. The beam is loaded at the free end with two concentrated loads F_1 = 50 kN and F_2 = 100 kN. The point of application is at the centre of the web. Use a x-y-z coordinate system with the origin at the normal force centre NC.



$$Given: \quad E_1 = 210000 \text{ N/mm}^2; E_2 = 70000 \text{ N/mm}^2;$$
$$L = 2000 \text{ mm}; a = 130 \text{ mm}; t = 15 \text{ mm}; F_1 = 50 \text{ kN}; F_2 = 100 \text{ kN};$$

Figure 4 : Cantilever beam with unsymmetrical cross section

To limit computations some data is given:

$$EI_{yy} = 138558875 \times 10^5 \text{ Nmm}^2; \quad EI_{zz} = 163254000 \times 10^5 \text{ Nmm}^2$$

Questions :

a) Compute all missing data and find the:
- plane of loading, plane of curvature,
- neutral axis.

Present these in a sketch of the cross section.

b) Sketch the distribution of the normal stresses for a cross section at A due to the given loading. Use a suitable presentation for this and show the values for the characteristic points P, Q, R and S of the cross section.

c) Describe how to find the shear stress distribution for the top flange. **Note** : do not compute any value, only explain all steps involved to find the distribution and support your answer with a sketch.

d) Will this beam be loaded in torsion due to the given loads? Explain this with a sketch and motivate your answer without calculations.

e) Find the position of the lower core point and present this location in a sketch of your cross section.

FORMULAS

Inhomogeous and/or asymmetrical cross sections :

$$\varepsilon(y,z) = \varepsilon + \kappa_y y + \kappa_z z \quad \text{and:} \quad \sigma(y,z) = E(y,z) \times \varepsilon(y,z)$$

$$\begin{bmatrix} M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} \kappa_y \\ \kappa_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

$$s_x^{(a)} = -\frac{V_y ES_y^{(a)}}{EI_{yy}} - \frac{V_z ES_z^{(a)}}{EI_{zz}}; \quad \text{or:} \quad s_x^{(a)} = -\frac{R_M^{(a)}}{M} V; \quad \sigma_{xt} = \frac{s_x^{(a)}}{b^{(a)}}$$

$$\tan(2\alpha) = \frac{2EI_{yz}}{\left(EI_{yy} - EI_{zz}\right)}; \quad EI_{1,2} = \tfrac{1}{2}\left(EI_{yy} + EI_{zz}\right) \pm \sqrt{\left(\tfrac{1}{2}\left(EI_{yy} - EI_{zz}\right)\right)^2 + EI_{yz}^2}$$

$$q_y^* = \frac{EI_{yy} EI_{zz} q_y - EI_{yy} EI_{yz} q_z}{EI_{yy} EI_{zz} - EI_{yz}^2}$$

$$q_z^* = \frac{-EI_{yz} EI_{zz} q_y + EI_{yy} EI_{zz} q_z}{EI_{yy} EI_{zz} - EI_{yz}^2}$$

Deformation energy:

$$E_v = \int \tfrac{1}{2} EA \varepsilon^2 \mathrm{d}x \quad \text{(extension)}$$

$$E_v = \int \tfrac{1}{2} EI \kappa^2 \mathrm{d}x \quad \text{(bending)}$$

Complementary energy:

$$E_c = \int \frac{N^2}{2EA} \mathrm{d}x \quad \text{(extension)}$$

$$E_c = \int \frac{M^2}{2EI} \mathrm{d}x \quad \text{(bending)}$$

Castigliano's theorema's:

$$F_i = \frac{\partial E_v}{\partial u_i} \qquad u_i = \frac{\partial E_c}{\partial F_i}$$

Kinematic relations:

$$\varepsilon = \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\kappa = -\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

Constitutive relations:

$$N = EA.\varepsilon$$

$$M = EI.\kappa$$

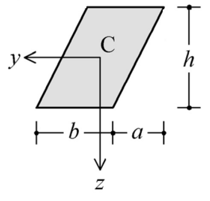
Work method with unit load:

$$u = \int \frac{M(x)m(x)dx}{EI}$$

Rayleigh:

$$F_k = \frac{E_v}{\int \tfrac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2 \mathrm{d}x}$$

Math tools:



$$I_{yy} = \tfrac{1}{12}(a^2 + b^2)bh$$

$$I_{zz} = \tfrac{1}{12}bh^3$$

$$I_{yz} = \tfrac{1}{12}abh^2$$

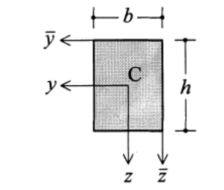
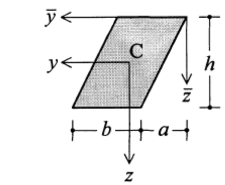
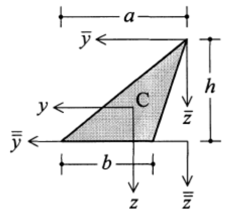
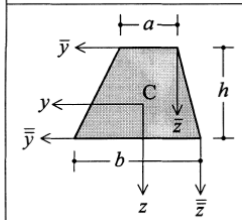
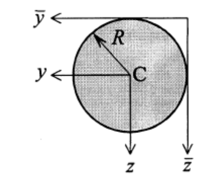
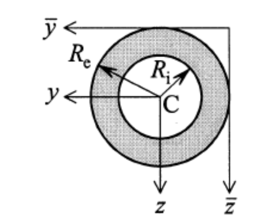
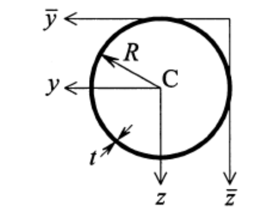
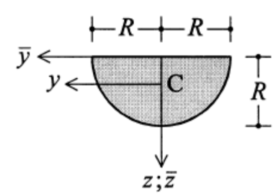
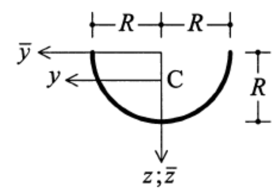
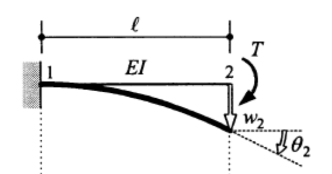
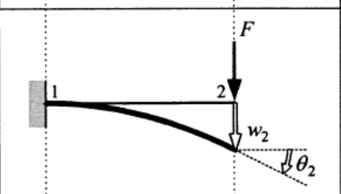
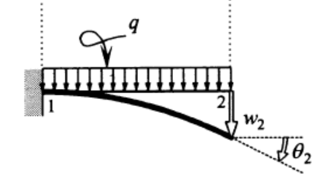
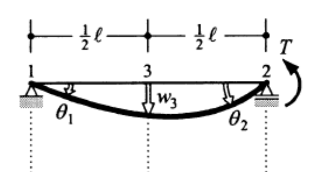
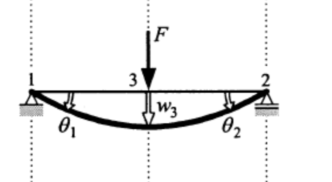
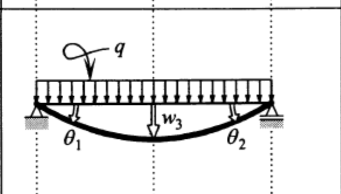
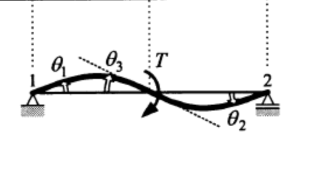
Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Rectangle $A = bh$ $\bar{y}_C = \frac{1}{2}b$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}b^3h$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = \frac{1}{3}b^3h$ $I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{4}b^2h^2$
	Parallelogram $A = bh$ $\bar{y}_C = \frac{1}{2}(a+b)$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}(a^2+b^2)bh$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = \frac{1}{12}abh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$
	Triangle $A = \frac{1}{2}bh$ $\bar{y}_C = \frac{1}{3}(2a-b)$ $\bar{z}_C = \frac{2}{3}h$	$I_{yy} = \frac{1}{36}(a^2-ab+b^2)bh$ $I_{zz} = \frac{1}{36}bh^3$ $I_{yz} = \frac{1}{72}(2a-b)bh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{4}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{8}(2a-b)bh^2$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}bh^3$
	Trapezium $A = \frac{1}{2}(a+b)h$ $\bar{z}_C = \frac{1}{3}\frac{a+2b}{a+b}h$	$I_{zz} = \frac{1}{36}\frac{a^2+4ab+b^2}{a+b}h^3$	$I_{\bar{z}\bar{z}} = \frac{1}{12}(a+3b)h^3$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}(3a+b)h^3$
	Circle $A = \pi R^2$	$I_{yy} = I_{zz} = \frac{1}{4}\pi R^4$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi R^4$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{5}{4}\pi R^4$ $I_{\bar{y}\bar{z}} = \pi R^4$

Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Thick-walled ring $A = \pi(R_e^2 - R_i^2)$	$I_{yy} = I_{zz} = \frac{1}{4}\pi(R_e^4 - R_i^4)$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi(R_e^4 - R_i^4)$	
	Thin-walled ring $A = 2\pi Rt$	$I_{yy} = I_{zz} = \pi R^3 t$ $I_{yz} = 0$ $I_p = 2\pi R^3 t$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = 3\pi R^3 t$
	Semicircle $A = \frac{1}{2}\pi R^2$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{4}{3\pi}R$ $= 0.424R$	$I_{yy} = \frac{1}{8}\pi R^4 = 0.393R^4$ $I_{zz} = (\frac{\pi}{8} - \frac{8}{9\pi})R^4$ $= 0.110R^4$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{8}\pi R^4$ $I_{\bar{y}\bar{z}} = 0$
	Semicircular ring $A = \pi Rt$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{2}{\pi}R$ $= 0.637R$	$I_{yy} = \frac{1}{2}\pi R^3 t$ $I_{zz} = (\frac{\pi}{2} - \frac{4}{\pi})R^3 t$ $= 0.298R^3 t$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{2}\pi R^3 t$ $I_{\bar{y}\bar{z}} = 0$

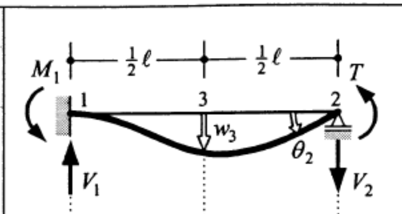
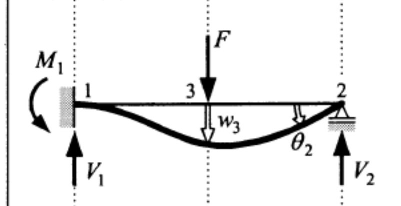
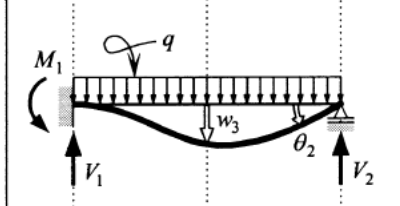
forget-me-nots

(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

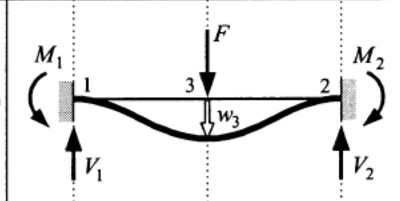
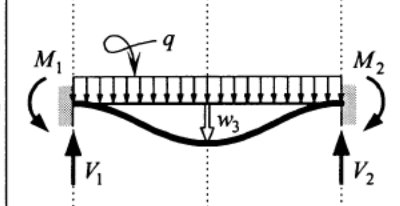
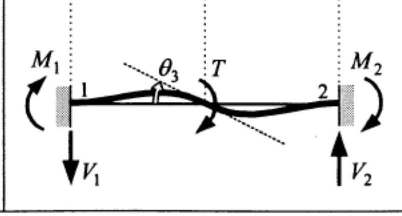
simply supported beam (statically determinate)

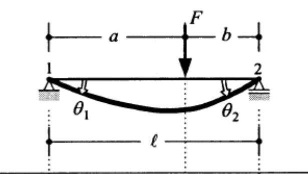
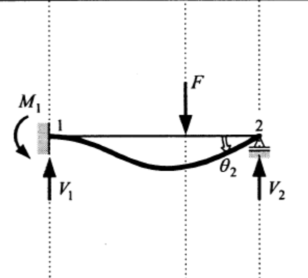
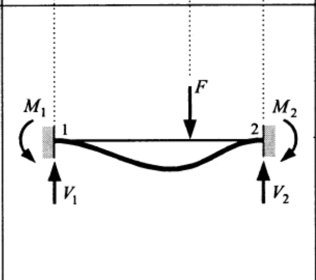
(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

statically indeterminate beam (one fixed end)

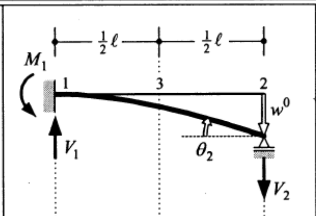
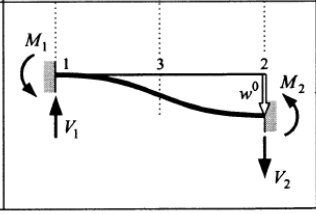
(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statically indeterminate beam (two fixed ends)

(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

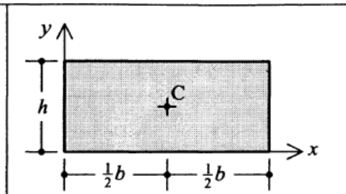
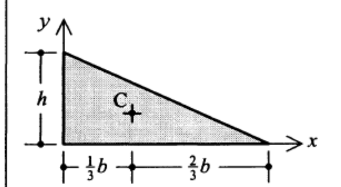
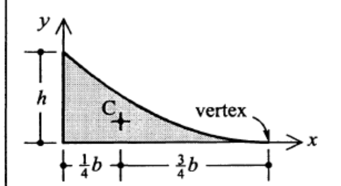
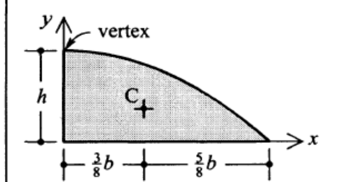
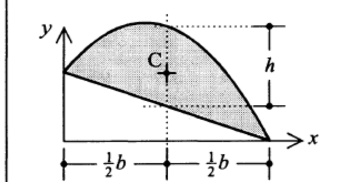
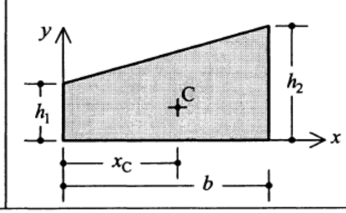
(c)		$\theta_1 = \dfrac{Fab(\ell+b)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(2\dfrac{a}{\ell} - 3\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fab(\ell+a)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(\dfrac{a}{\ell} - \dfrac{a^3}{\ell^3}\right)$
(d)		$M_1 = \dfrac{Fb(\ell^2-b^2)}{2\ell^2} = F\ell\left(\dfrac{a}{\ell} - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb(3\ell^2-b^2)}{2\ell^3} = F\left(1 - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(3\ell-a)}{2\ell^3} = F\left(\dfrac{3}{2}\dfrac{a^2}{\ell^2} - \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fa^2b}{4EI\ell} = \dfrac{F\ell^2}{4EI}\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$
(e)		$M_1 = \dfrac{Fab^2}{\ell^2} = F\ell\left(\dfrac{a}{\ell} - 2\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb^2(\ell+2a)}{\ell^3} = F\left(1 - 3\dfrac{a^2}{\ell^2} + 2\dfrac{a^3}{\ell^3}\right)$ $M_2 = \dfrac{Fa^2b}{\ell^2} = F\ell\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(\ell+2b)}{\ell^3} = F\ell\left(3\dfrac{a^2}{\ell^2} - 2\dfrac{a^3}{\ell^3}\right)$

support reactions and rotations at the beam ends

(f)		$M_1 = \dfrac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{3EI}{\ell^3}w^0$ $\theta_2 = \dfrac{3}{2}\dfrac{w^0}{\ell}$ $\theta_3 = \dfrac{9}{8}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{5}{16}w^0$
(g)		$M_1 = M_2 = \dfrac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{12EI}{\ell^3}w^0$ $\theta_3 = \dfrac{3}{2}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{1}{2}w^0$

settlements

properties of plane figures to be used for the moment-area theorems

(1)		rectangle: $y = h$ $A = bh$ $x_C = \frac{1}{2}b$
(2)		triangle: $y = h\left\{1 - \dfrac{x}{b}\right\}$ $A = \frac{1}{2}bh$ $x_C = \frac{1}{3}b$
(3)		parabola: $y = h\left\{1 - \dfrac{x}{b}\right\}^2$ $A = \frac{1}{3}bh$ $x_C = \frac{1}{4}b$
(4)		parabola: $y = h\left\{1 - \left(\dfrac{x}{b}\right)^2\right\}$ $A = \frac{2}{3}bh$ $x_C = \frac{3}{8}b$
(5)		parabola: $A = \frac{2}{3}bh$ $x_C = \frac{1}{2}b$
(6)		trapezium: $y = h_1 + (h_2 - h_1)\dfrac{x}{b}$ $A = \frac{1}{2}b(h_1 + h_2)$ $x_C = \frac{1}{3}b\dfrac{h_1 + 2h_2}{h_1 + h_2}$