TU Delft

Subfaculty Civil Engineering
Strucutral Mechanics

who are you?
STUDENT NUMBER
NAME

Exam CT3109

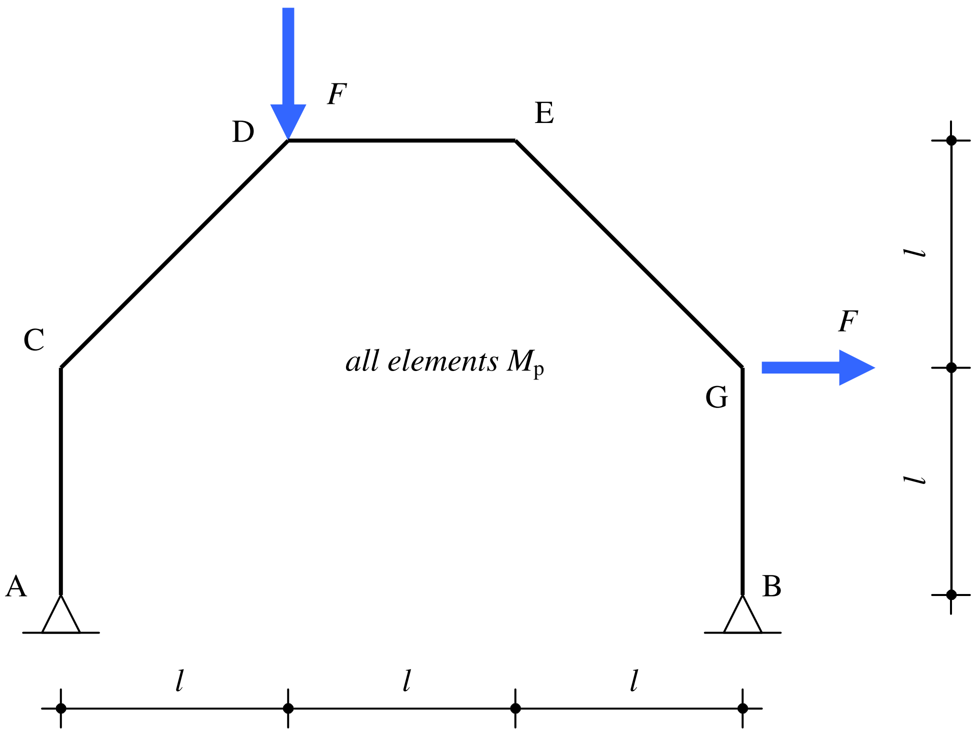
STRUCTURAL MECHANICS 4

18 jan 2010,
09:00 – 12:00 hours

- This exam contains 4 **problems**.
- Use for each problem a seperate sheet of paper.
- Do not forget to mention your name and number on each paper.
- Work neat and tidy, the quality of the presentation can be used in the grading.
- The use of Phone's or computers and PDA's is not allowd. Turn the equipment off and remove it from your desk.
- A scientific (programmable) calculater is allowed
- All required formulas can be found on the last pages of this exam
- Keep an eye on the clock and use the specified times per problem as guidance.

PROBLEM 1 : Plasticity (ca 45 min)

In the following figure a structure is shown with a simple support at A and B. The concentrated loads F are applied in D and G as shown.



Questions:

a) Determine the possible collapse mechanisms and show these with small sketches.

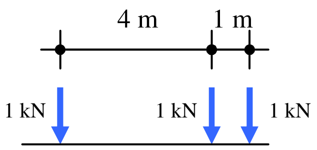
b) Of all possible mechanisms you are only allowed to investigate three. Show these three mechanism for which you think the mechanism with the lowest collapse load will be present and compute the collapse load F_p .

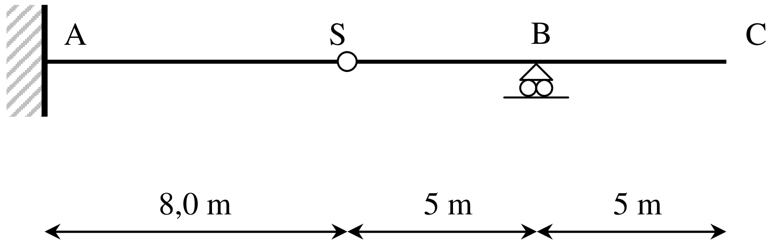
Remark : Based on sound engineering judgement you should be able to select the three mechanisms.

c) Show the moment distribution at the moment of collapse.

PROBLEM 2 : Influence lines (ca 45 min)

A system of concentrated loads (e.g. a boogy) moves along a structure. The loads have a constant distance with respect to each other as is shown in the figure below.





Questions :

Construct the following influence lines and draw these Lines in one figure directly below each other :

a) The vertical support reaction at A
b) The bending moment (in the beam) at A
c) The shear force directly to the left of the hinge
d) The shear force directly to the right of the hinge
e) The deflection of the hinge
f) The support reaction at B

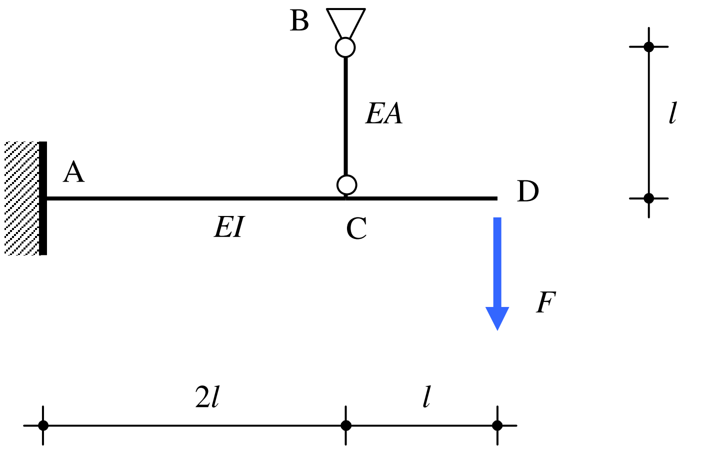
Show the most critical position of the boogy for the following quantities and compute the extreme value of these quantities:

g) The bending moment (in the beam) at A
h) The support reaction at B

> *Remark :* *Construct means a precise influence line with the correct shape and computed extreme values.*

PROBLEM 3 : Work and Energy methods (ca 45 min)

A rigidly connected beam ACD with bending stiffness *EI* is supported by a two-force-member (truss element) BC with axial stiffness *EA*. The structure is loaded with a vertical concentrated load *F* at the end of the beam.



Given : $l = 2$ m; $EI = 10000$ kNm^2; $EA = 1000$ kN; $F = 62$ kN

Questions:

a) Find with a work or energy method the moment distribution in the beam. Draw the moment distribution and write down the extreme values.

   Note : Although it is very tempting to use the force method, you must use a **Work or Energy Method.**

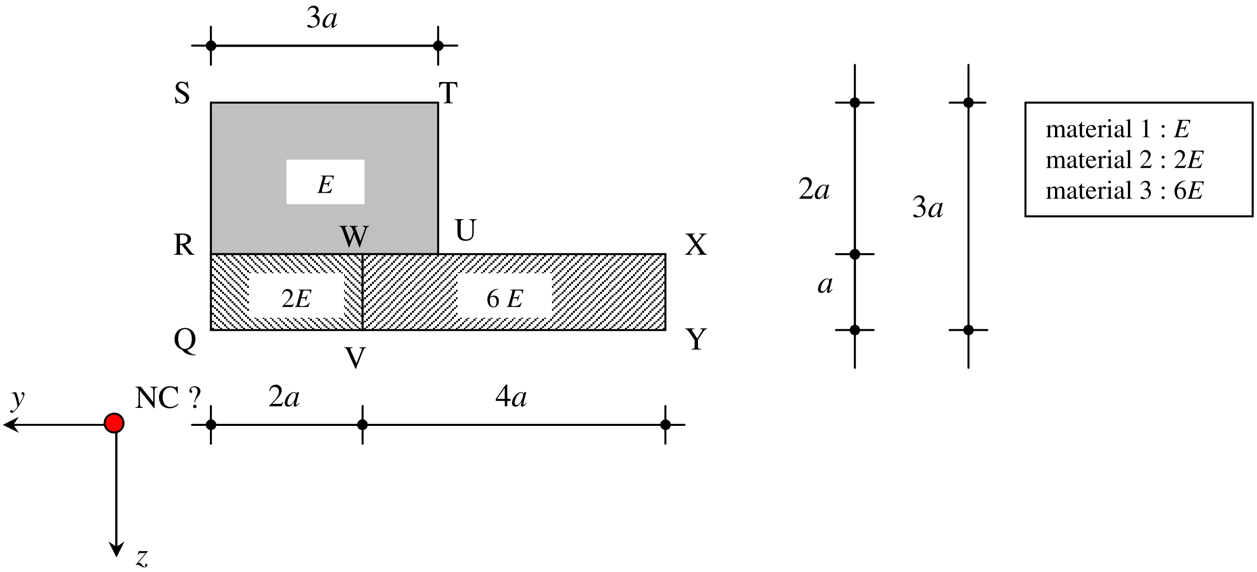
b) Find the normal force in the two-force-member

c) Find the normal force in the two-force-member in case this member is inextensible.

   Note : In this case a specific method is not prescribed. So any feasible method you like to use is accepted.

PROBLEM 4 : Unsymmetrical cross section (ca 45 min)

An element in a structure is subject to bending. The cross section is a composite of three different materials. Each material has its own modulus of elasticity as is specified in the figure below. The total composite can be regarded as a rigid cross section.



Given : E = 15000 N/mm^2; a = 50 mm;

$N = 0$ kN;

$V_y = 10$ kN; $V_z = 0$ kN

$M_y = 70,0$ kNm; $M_z = 0$ kNm

Questions:

a) Determine the position of the normal force centre NC.
b) What is the constitutive relation for a cross section? Show the quantities involved.
c) The constitutive matrix for this cross section is specified below. Check the value of the element at row 2 and column 3.

$$\frac{Ea^2}{102}\begin{bmatrix} 3468 & 0 & 0 \\ 0 & 9169a^2 & -1566a^2 \\ 0 & -1566a^2 & 1576a^2 \end{bmatrix}$$

d) Show in one figure of the cross section, the *neutral axis n.a.*, the plane of loading *m* and the plane of curvature *k*.
e) Show in a second graph the normal stress distribution for material 1 (part RSTU) and material 3 (part VWXY). Use for this stress diagram a line perpendicular to the neutral axis.
f) Determine the shear flow (in N/mm) in the horizontal interface RU.
g) Explain the shape of the kern for this cross section and explain the strategy how to compute the exact shape. **(note:** do not compute the values, explain only!)

FORMULAS

Inhomogeneous and/or unsymmetrical cross sections :

$$\varepsilon(y,z) = \varepsilon + \kappa_y y + \kappa_z z \quad \text{and:} \quad \sigma(y,z) = E(y,z)\times\varepsilon(y,z)$$

$$\begin{bmatrix} M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix}\begin{bmatrix} \kappa_y \\ \kappa_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{EA}\begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix}\begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

$$s_x^{(a)} = -\frac{V_y ES_y^{(a)}}{EI_{yy}} - \frac{V_z ES_z^{(a)}}{EI_{zz}}; \quad \text{or:} \quad s_x^{(a)} = -\frac{R_M^{(a)}}{M}V; \quad \sigma_{xt} = \frac{s_x^{(a)}}{b^{(a)}}$$

Deformation energy:

$$E_v = \int \tfrac{1}{2} EA\varepsilon^2 \mathrm{d}x \quad \text{(extension)}$$

$$E_v = \int \tfrac{1}{2} EI\kappa^2 \mathrm{d}x \quad \text{(bending)}$$

Complementaire energie:

$$E_c = \int \frac{N^2}{2EA}\mathrm{d}x \quad \text{(extension)}$$

$$E_c = \int \frac{M^2}{2EI}\mathrm{d}x \quad \text{(bending)}$$

Castigliano's theorema:

$$F_i = \frac{\partial E_v}{\partial u_i} \qquad u_i = \frac{\partial E_c}{\partial F_i}$$

Kinematic relation:

$$\varepsilon = \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\kappa = -\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

Constitutive relation:

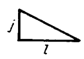
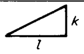
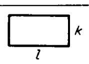
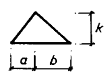
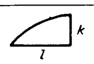
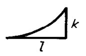
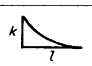
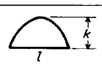
$$N = EA.\varepsilon$$

$$M = EI.\kappa$$

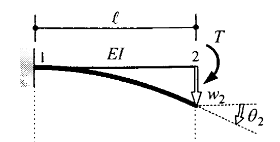
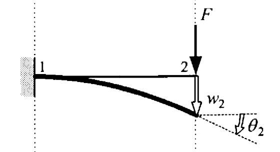
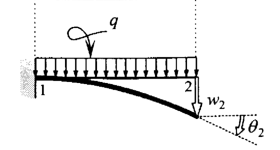
Work method with unity load:

$$u = \int \frac{M(x)m(x)dx}{EI}$$

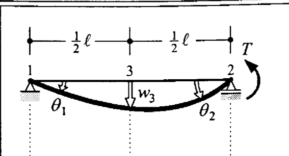
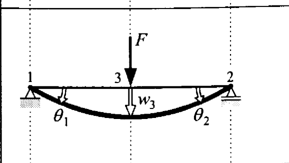
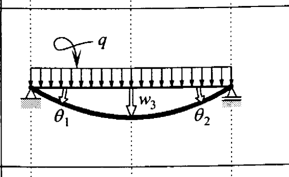
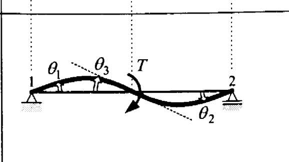
"Old style help" for integration of products of frequently used shapes :

M \ m				
	$\frac{1}{3}jkl$	$\frac{1}{6}jkl$	$\frac{1}{6}(j_1+2j_2)kl$	$\frac{1}{2}jkl$
	$\frac{1}{6}jkl$	$\frac{1}{3}jkl$	$\frac{1}{6}(2j_1+j_2)kl$	$\frac{1}{2}jkl$
	$\frac{1}{6}j(k_1+2k_2)l$	$\frac{1}{6}j(2k_1+k_2)l$	$\frac{1}{6}\{j_1(2k_1+k_2) + j_2(k_1+2k_2)\}l$	$\frac{1}{2}j(k_1+k_2)l$
	$\frac{1}{2}jkl$	$\frac{1}{2}jkl$	$\frac{1}{2}(j_1+j_2)kl$	jkl
	$\frac{1}{6}jk(l+a)$	$\frac{1}{6}jk(l+b)$	$\frac{1}{6}\{j_1(l+b) + j_2(l+a)\}k$	$\frac{1}{2}jkl$
	$\frac{5}{12}jkl$	$\frac{1}{4}jkl$	$\frac{1}{12}(3j_1+5j_2)kl$	$\frac{2}{3}jkl$
	$\frac{1}{4}jkl$	$\frac{5}{12}jkl$	$\frac{1}{12}(5j_1+3j_2)kl$	$\frac{2}{3}jkl$
	$\frac{1}{4}jkl$	$\frac{1}{12}jkl$	$\frac{1}{12}(j_1+3j_2)kl$	$\frac{1}{3}jkl$
	$\frac{1}{12}jkl$	$\frac{1}{4}jkl$	$\frac{1}{12}(3j_1+j_2)kl$	$\frac{1}{3}jkl$
	$\frac{1}{3}jkl$	$\frac{1}{3}jkl$	$\frac{1}{3}(j_1+j_2)kl$	$\frac{2}{3}jkl$

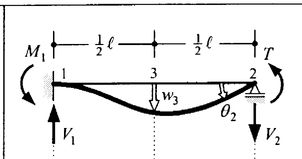
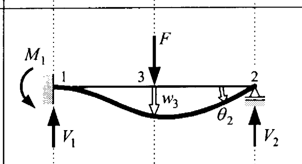
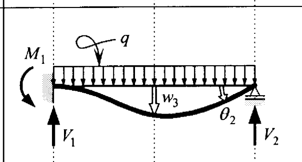
vergeet-mij-nietjes

(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

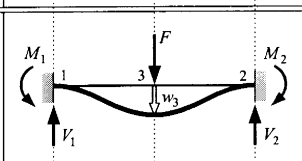
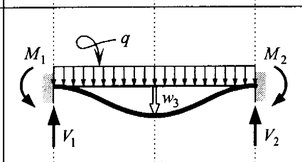
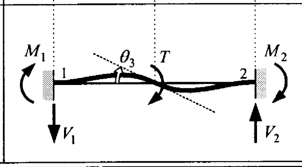
vrij opgelegde ligger (statisch bepaald)

(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

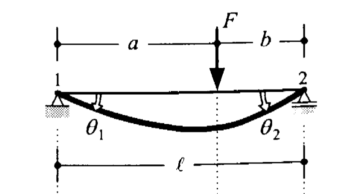
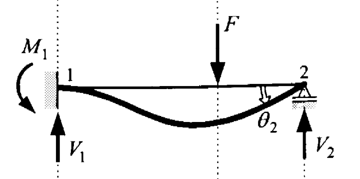
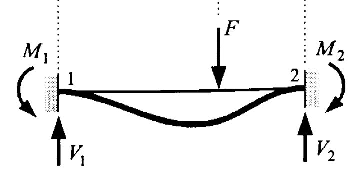
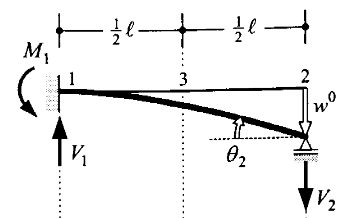
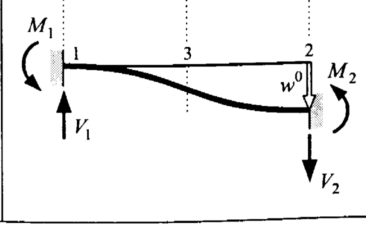
statisch onbepaalde ligger (enkelzijdig ingeklemd)

(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statisch onbepaalde ligger (tweezijdig ingeklemd)

(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

Enkele formules voor prismatische liggers met buigstijfheid EI.
T, F en q zijn belastingen door resp. een koppel, kracht en gelijkmatig verdeelde belasting.
M_i en V_i zijn het buigend moment en de dwarskracht op einddoorsnede i van de ligger ten gevolge van de oplegreacties.

(c)		$\theta_1 = \frac{Fab(\ell+b)}{6EI\ell} = \frac{F\ell^2}{6EI}\left(2\frac{a}{\ell} - 3\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3}\right)$ $\theta_2 = \frac{Fab(\ell+a)}{6EI\ell} = \frac{F\ell^2}{6EI}\left(\frac{a}{\ell} - \frac{a^3}{\ell^3}\right)$
(d)		$M_1 = \frac{Fb(\ell^2 - b^2)}{2\ell^2} = F\ell\left(\frac{a}{\ell} - \frac{3}{2}\frac{a^2}{\ell^2} + \frac{1}{2}\frac{a^3}{\ell^3}\right)$ $V_1 = \frac{Fb(3\ell^2 - b^2)}{2\ell^3} = F\left(1 - \frac{3}{2}\frac{a^2}{\ell^2} + \frac{1}{2}\frac{a^3}{\ell^3}\right)$ $V_2 = \frac{Fa^2(3\ell - a)}{2\ell^3} = F\left(\frac{3}{2}\frac{a^2}{\ell^2} - \frac{1}{2}\frac{a^3}{\ell^3}\right)$ $\theta_2 = \frac{Fa^2 b}{4EI\ell} = \frac{F\ell^2}{4EI}\left(\frac{a^2}{\ell^2} - \frac{a^3}{\ell^3}\right)$
(e)		$M_1 = \frac{Fab^2}{\ell^2} = F\ell\left(\frac{a}{\ell} - 2\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3}\right)$ $V_1 = \frac{Fb^2(\ell + 2a)}{\ell^3} = F\left(1 - 3\frac{a^2}{\ell^2} + 2\frac{a^3}{\ell^3}\right)$ $M_2 = \frac{Fa^2 b}{\ell^2} = F\ell\left(\frac{a^2}{\ell^2} - \frac{a^3}{\ell^3}\right)$ $V_2 = \frac{Fa^2(\ell + 2b)}{\ell^3} = F\ell\left(3\frac{a^2}{\ell^2} - 2\frac{a^3}{\ell^3}\right)$
(f)		$M_1 = \frac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \frac{3EI}{\ell^3}w^0$ $\theta_2 = \frac{3}{2}\frac{w^0}{\ell}$ $\theta_3 = \frac{9}{8}\frac{w^0}{\ell};\quad w_3 = \frac{5}{16}w^0$
(g)		$M_1 = M_2 = \frac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \frac{12EI}{\ell^3}w^0$ $\theta_3 = \frac{3}{2}\frac{w^0}{\ell};\quad w_3 = \frac{1}{2}w^0$

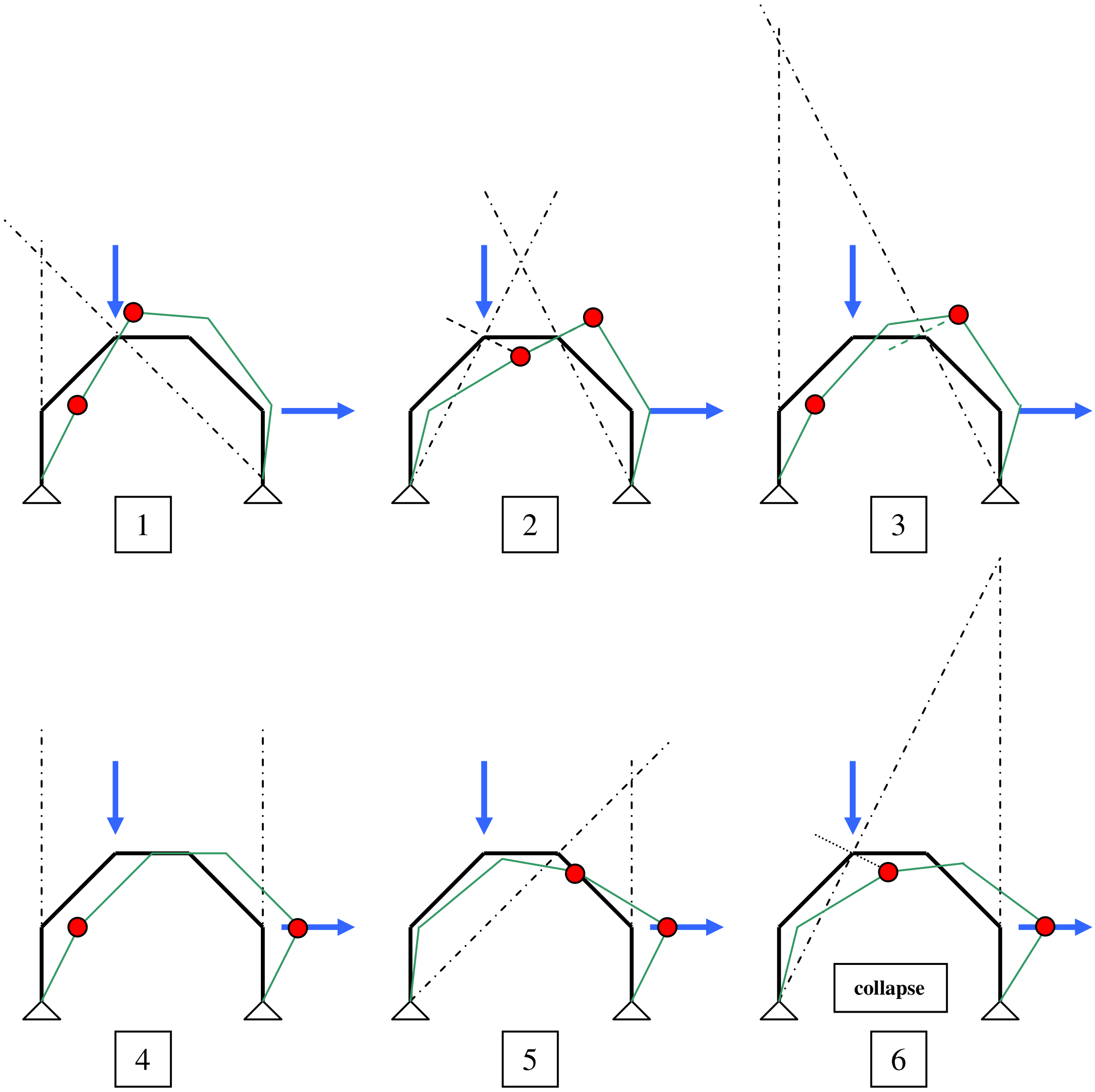
drie bij-de handjes

zettingen

ANSWERS

Problem 1 : Plasticity

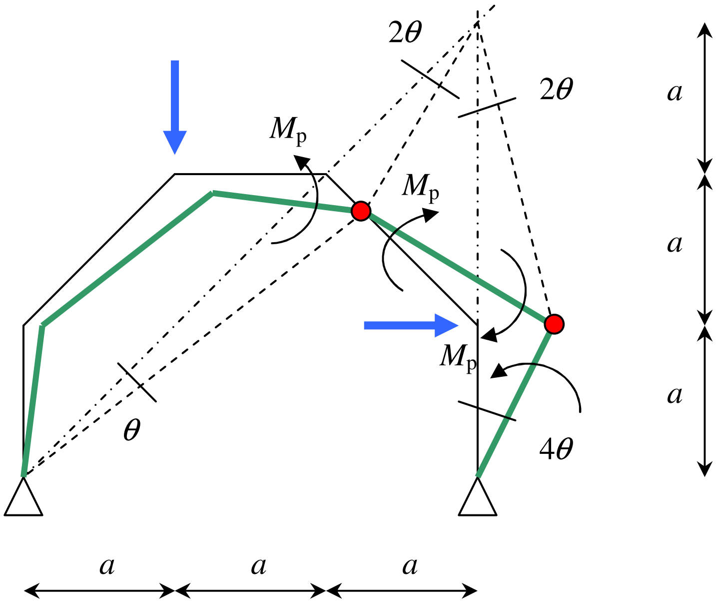
a) The structure is statically indeterminate to the degree of one. We therefore need two hinges for a (plastic) mechanism. There are four possible positions for hinges. This results in 6 possible mechanisms to investigate. Small sketches are given below.



From these sketches it is possible to deduce the most relevant mechanisms. The sway mechanism nr 4 is always very sensitive. Also mechanisms for which both the displacement and load are pointing in the same direction are most likely to occur. This results in mechanism 2,4,5 and 6. This will lead to the smallest collapse load. So we will only investigate these latter three mechanisms.

b) Mechanism 4 is very simple. From the sketch follows directly $F_p = \dfrac{2M_p}{a}$

Mechanism nr 5 is shown in detail below.

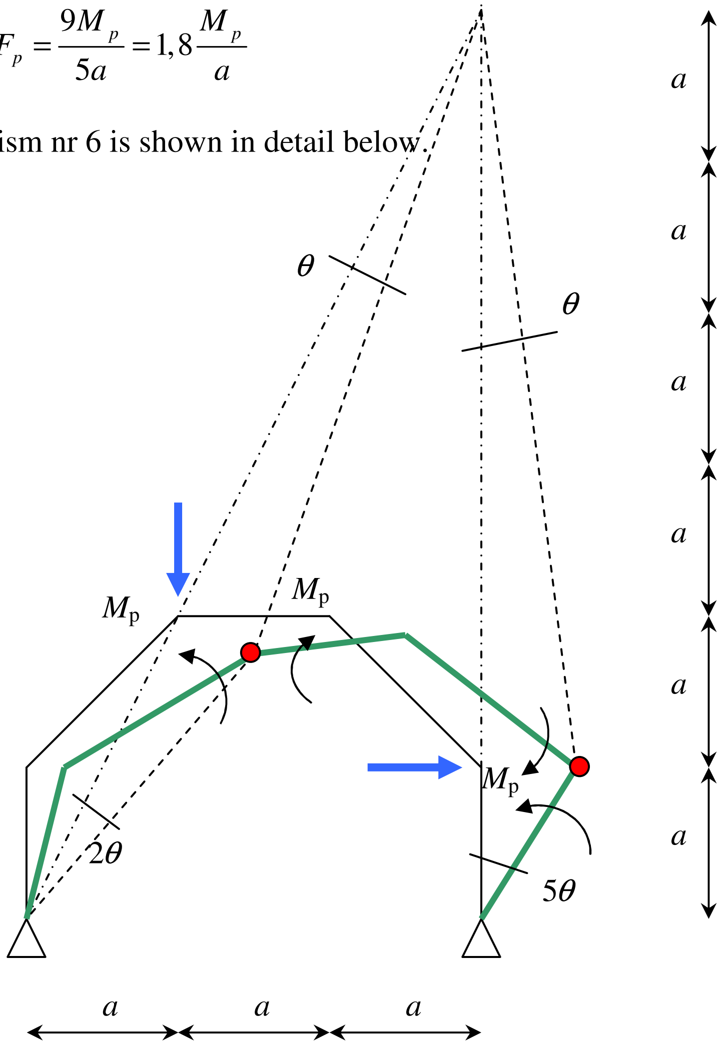


From this we can find the corresponding load using virtual work:

$$\delta A = -M_p \delta\theta - M_p 2\delta\theta - M_p 2\delta\theta - M_p 4\delta\theta + Fa\delta\theta + Fa4\delta\theta = 0$$

$$F_p = \frac{9M_p}{5a} = 1{,}8\frac{M_p}{a}$$

Mechanism nr 6 is shown in detail below.



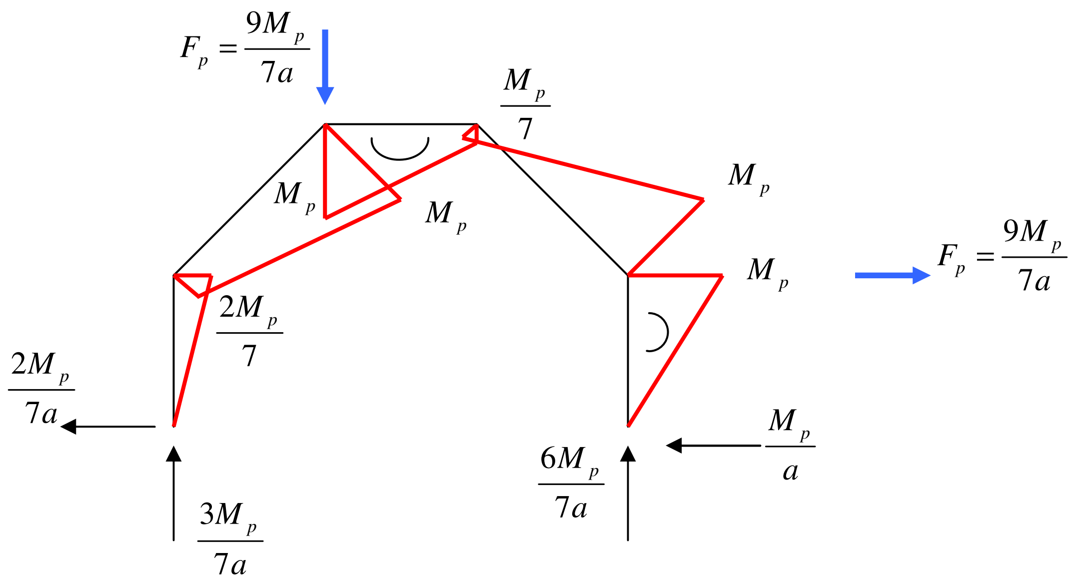
From this we can find the corresponding load using virtual work:

$$\delta A = -M_p 2\delta\theta - M_p\delta\theta - M_p\delta\theta - M_p 5\delta\theta + Fa2\delta\theta + Fa5\delta\theta = 0$$

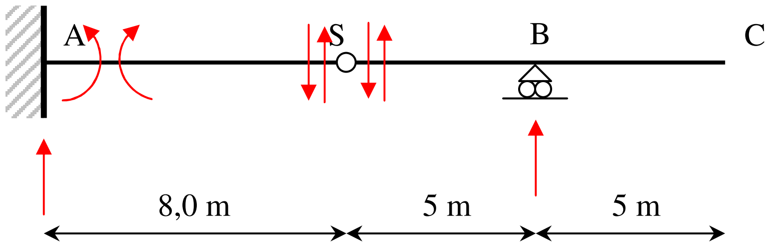
$$F_p = \frac{9M_p}{7a} = 1,28\frac{M_p}{a}$$

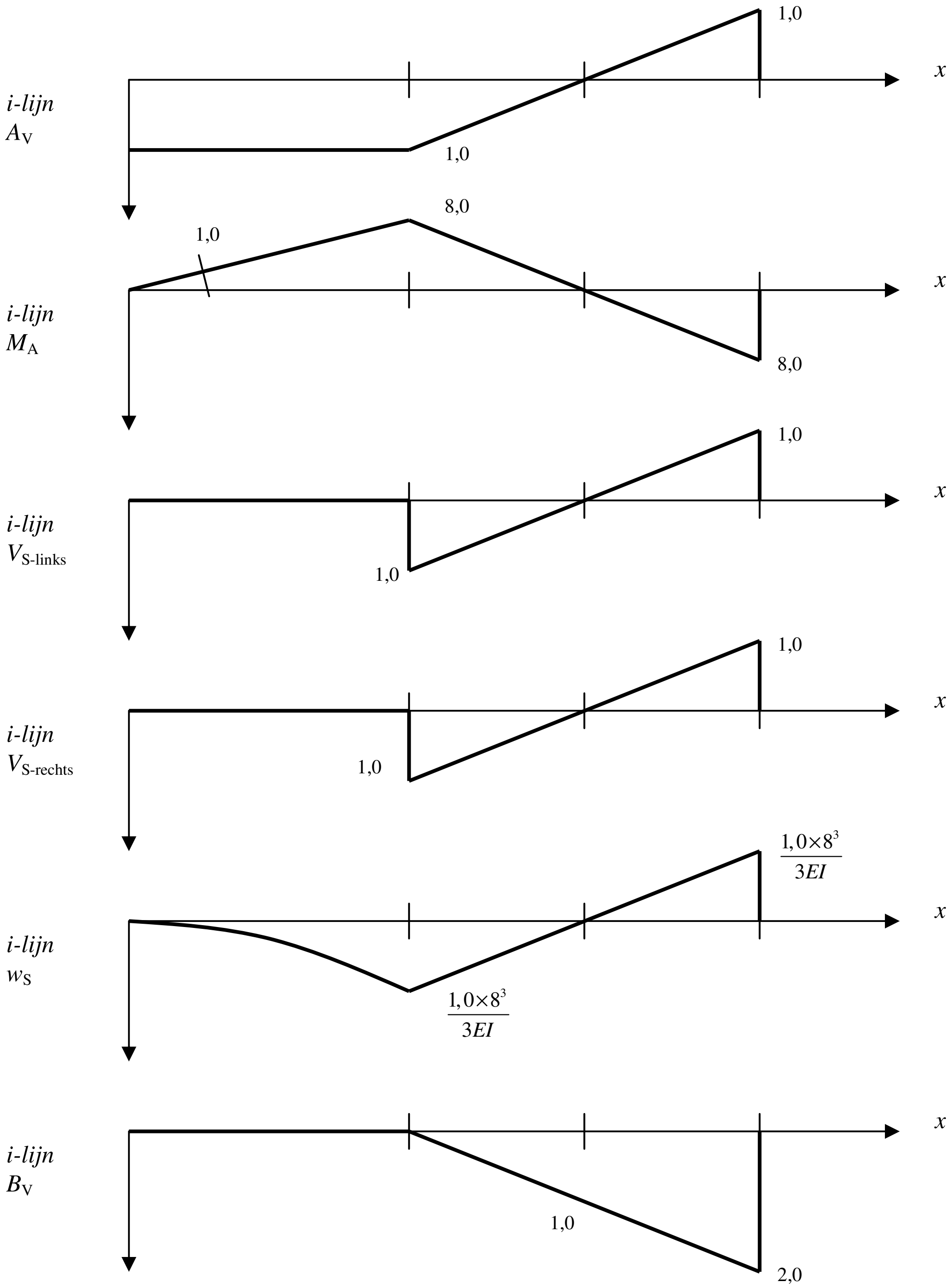
Mechanism 6 results in the smallest load for which a mechanism occurs. This load is therefore the collapse load.

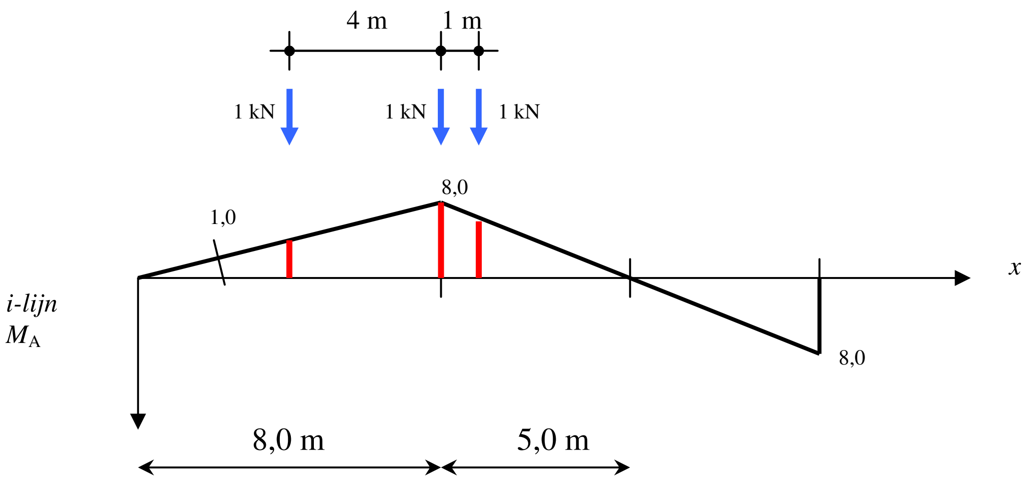
c) The moment distribution at collapse can be found based on equilibrium.



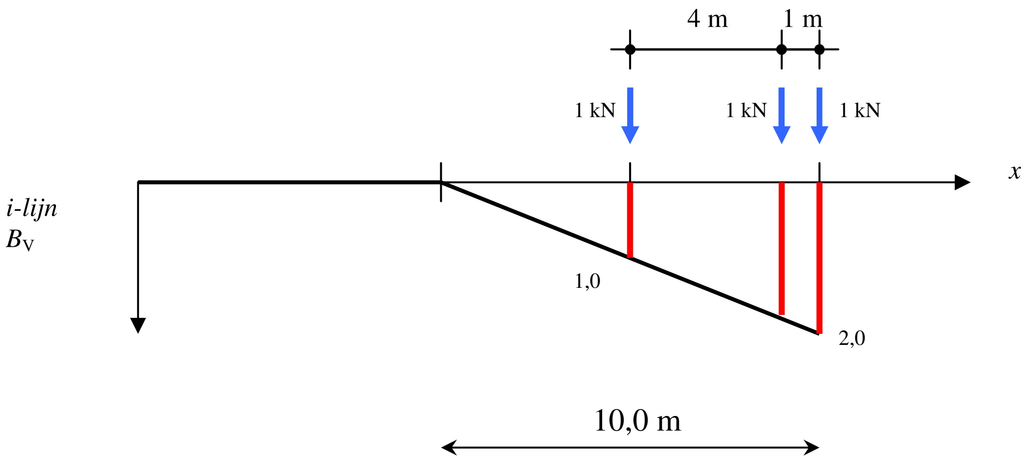
Problem 2 : Influence lines







$$M_A = -4,0 \times F + 8,0 \times F - \tfrac{4}{5} \times 8,0 \times F = -18,4F = -18,4 \quad \text{kNm}$$



$$B_V = 1,0 \times F + 1,8 \times F + 2,0 \times F = 4,8F = 4,8 \quad \text{kN}$$

Problem 3 : Work and Energy Methods

The structure is statically indeterminate (n = 1). If we consider the normal force in the two-force-member BC as the fundamental unknown, both the bending moment distribution for element ACD and the normal force distribution for BC can be computed:

$$M^{AC}(x) = \begin{cases} -F(3l-x)+N(2l-x) & 0<x<2l \\ -F(3l-x) & 2l<x<3l \end{cases}$$

$$N^{BS}(x) = N$$

Minimum work (deformation energy) requires: (one of the options ….)

$$\frac{\partial E_v}{\partial N}=0 \quad \text{with:} \quad E_v = \int_0^{3l} \frac{M^2}{2EI}dx + \int_0^{l} \frac{N^2}{2EA}dx$$

To simplify the hand calculation we use:

$$\frac{\partial E_v}{\partial N} = \frac{\partial \int_0^{3l} \frac{M^2}{2EI}dx + \int_0^{l} \frac{N^2}{2EA}dx}{\partial N} = \int_0^{3l} \frac{M}{EI}\frac{\partial M}{\partial N}dx + \int_0^{l} \frac{N}{EA}\frac{\partial N}{\partial N}dx = \int_0^{3l} \frac{M}{EI}\frac{\partial M}{\partial N}dx + \frac{Nl}{EA}$$

This results in:

$$\frac{\partial E_v}{\partial N} = \int_0^{3l} \frac{M}{EI}\frac{\partial M}{\partial N}dx + \frac{Nl}{EA} = 0; \quad \text{with:} \quad \frac{\partial M}{\partial N} = \begin{cases} 2l-x & 0<x<2l \\ 0 & 2l<x<3l \end{cases}$$

Elaborating this latter expression results in:

$$\frac{1}{EI}\int_0^{2l} \left[-F(3l-x)+N(2l-x)\right](2l-x)dx + \frac{1}{EI}\int_{2l}^{3l} \left[-F(3l-x)\right]0dx + \frac{Nl}{EA} = 0$$

$$\frac{-F}{EI}(6l^2-5lx+x^2)\Big|_0^{2l} + \frac{N}{EI}(4l^2-4lx+x^2)\Big|_0^{2l} + \frac{Nl}{EA} = 0$$

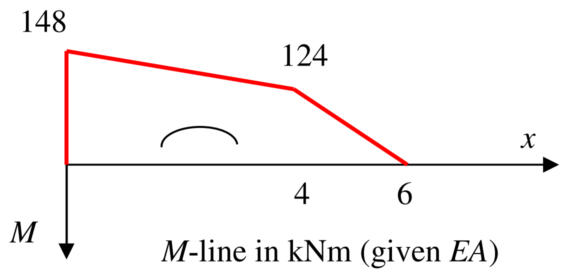
$$\frac{-F}{EI}(12l^3-10l^3+\tfrac{8}{3}l^3) + \frac{N}{EI}(8l^3-8l^3+\tfrac{8}{3}l^3) + \frac{Nl}{EA} = 0$$

$$N\left(\frac{8l^3}{3EI}+\frac{l}{EA}\right) = \frac{14Fl^3}{3EI} \quad \Leftrightarrow \quad N = \frac{\frac{14Fl^3}{3EI}}{\frac{8l^3}{3EI}+\frac{l}{EA}} = \frac{14Fl^2EA}{8l^2EA+3EI} = 56 \text{ kN}$$

For infinite EA the resulting normal force in the two-force-member becomes:

$$N = \frac{14Fl^2EA}{8l^2EA+3EI} = \frac{14Fl^2}{8l^2+\frac{3EI}{EA}}$$

$$\lim_{EA\to\infty} N = \frac{14F}{8} = 1{,}75F = 108{,}5 \text{ kN}$$



M-line in kNm (given EA)

OPGAVE 4 : Niet-symmetrische doorsnede

The axial stiffness of the cross section can be found with:

$$EA = E\times(3a\times2a)+2E\times(a\times2a)+6E\times(a\times4a)=34Ea^2$$

The origin of the coordinate system used is located at the NC. The vertical position of the NC with respect to the upper side of the cross section is:

$$\Delta z_{NC} = \frac{E\times(3a\times2a\times a)+2E\times(a\times2a\times2\tfrac{1}{2}a)+6E\times(a\times4a\times2\tfrac{1}{2}a)}{EA}=\frac{38}{17}a=111,7\ \text{ mm}$$

The horizontal position with respect to the left side of the cross section is:

$$\Delta y_{NC} = \frac{E\times(3a\times2a\times\tfrac{3}{2}a)+2E\times(a\times2a\times a)+6E\times(a\times4a\times4a)}{EA}=\frac{109}{34}a=160,3\ \text{ mm}$$

Only one bending stiffness was asked for:

$$\begin{aligned} EI_{yz} = {} & E\times\left[3a\times2a\times(y_{NC}-1,5a)\times(-z_{NC}+a)\right]+ \\ & 2E\times\left[a\times2a\times(y_{NC}-a)\times(-z_{NC}+2,5a)\right]+ \\ & 6E\times\left[a\times4a\times(y_{NC}-4a)\times(-z_{NC}+2,5a)\right]=-\frac{261Ea^4}{17}=-1566a^2\times\frac{Ea^2}{102}=-1439,3\times10^9\ \text{Nmm}^2 \end{aligned}$$

The *cross sectional constitutive relation* relates the sectional forces to the deformations of the cross section:

$$\begin{bmatrix} N \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} EA & 0 & 0 \\ 0 & EI_{yy} & EI_{yz} \\ 0 & EI_{yz} & EI_{zz} \end{bmatrix}\begin{bmatrix} \varepsilon \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

cross sectional constitutive relation:

$$\frac{Ea^2}{102}\begin{bmatrix} 3468 & 0 & 0 \\ 0 & 9169a^2 & -1566a^2 \\ 0 & -1566a^2 & 1576a^2 \end{bmatrix} = \begin{bmatrix} 1275\times10^6 & 0 & 0 \\ 0 & 8227\times10^9 & -1439\times10^9 \\ 0 & -1439\times10^9 & 1449\times10^9 \end{bmatrix}$$

Since this structure is loaded in bending only, the strain ε at the NC must be zero. The curvatures can be found with the constitutive relation:

$$\varepsilon = \frac{N}{EA}=0$$

$$\kappa_y = \frac{1}{EI_{yy}EI_{zz}-EI_{yz}^2}\left(EI_{zz}\times M_y - EI_{yz}\times M_z\right)=1,0\times10^{-5}$$

$$\kappa_z = \frac{1}{EI_{yy}EI_{zz}-EI_{yz}^2}\left(-EI_{yz}\times M_y + EI_{yy}\times M_z\right)=1,0\times10^{-5}$$

The direction of the *plane of loading* and the *plane of curvature* can be obtained with:

$$\tan\alpha_m = \frac{M_z}{M_y}=0 \quad\Rightarrow\quad \alpha_m = 0^\circ; \qquad \tan\alpha_k = \frac{\kappa_z}{\kappa_y}=1,0 \quad\Rightarrow\quad \alpha_k = 45^\circ$$

The stresses for each point of the cross section can be computed with:

$$\sigma(y,z) = E \times (\varepsilon + \kappa_y \times y + \kappa_z \times z) \quad \text{N/mm}^2$$

The neutral axis *n.a.* can also be found with this latter expression by:

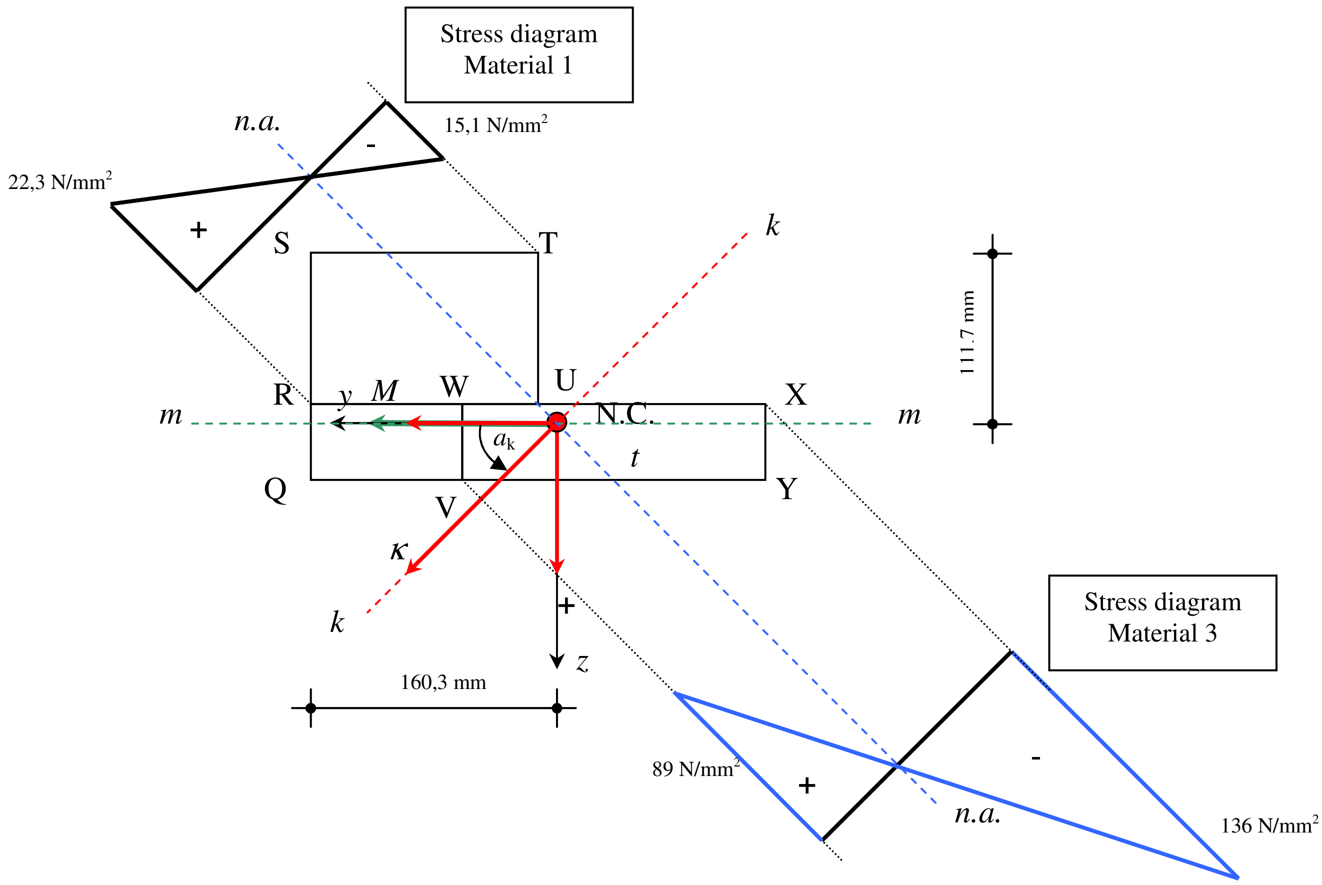
$$\varepsilon(y,z) = \varepsilon + \kappa_y \times y + \kappa_z \times z = 0 \quad \Leftrightarrow$$
$$\kappa_y \times y + \kappa_z \times z = 0$$

The stress distribution can be visualized with a few points. Only the most outer points with respect to the neutral axis are needed (material 1 : T and R; material 3 : X and V). Points S and U are added for the next question.

Tabel : Stress in the specified points

Material	point	*y* [mm]	*z* [mm]	*E* [N/mm^2]	*Stress* [N/mm^2]
1	S	160,3	-111,8	15^{e}3	7,4
	R	160,3	-11,8	15^{e}3	22,3
	U	10,3	-11,8	15^{e}3	-0,2
	T	10,3	-111,8	15^{e}3	-15,1
3	V	60,3	38,2	90^{e}3	88,5
	X	-139,7	-11,8	90^{e}3	-136,3

The graphical presentation of the stress distribution at C is given in the next graph. The neutral axis goes through the NC since the normal force *N* is zero.



The moment *M* and thus the load *F* acts in the *x*-*m* plane. The curvature κ acts in the *x*-*k* plane. Due to the unsymmetrical cross section these planes do not coincide.

The shear flow in the interface RU can be obtained with the resulting force at the sliding element (a) with:

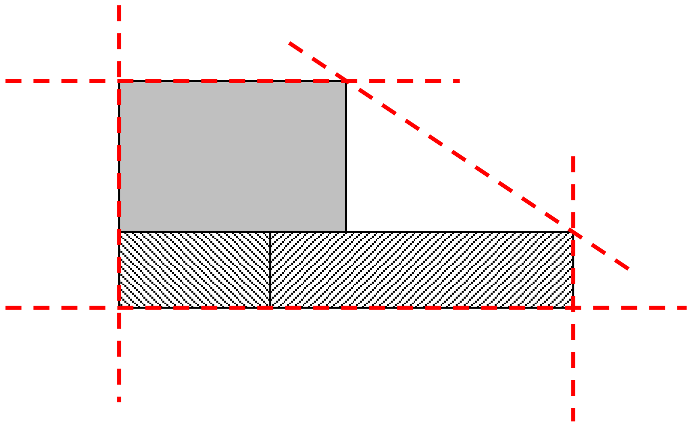
$$s_x = -\frac{R_M^{(a)}}{M} \times V = -\frac{\frac{1}{4}(\sigma_{\mathrm{R}} + \sigma_{\mathrm{S}} + \sigma_{\mathrm{T}} + \sigma_{\mathrm{U}}) \times 2a \times 3a}{M_y} \times V_y$$

$$s_x = -7,69 \text{ N/mm}$$

The kern of the cross section can be obtained with:

$$\begin{bmatrix} e_y \\ e_z \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix} \begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

For each *n.a.* which is just outside the cross section, one corner point of the kern can be found. For this cross section we have to investigate five positions of the *n.a.* as can be seen from the figure below.



The exact location of the (corner) points of the kern were not asked for.