Faculty
Civil Engineering and GeoSciences

Exam	**CIE3109-09 / CTB3330**
	Structural Mechanics 4
Total number of pages	**8 pages (excl cover)**
Date and time	**JUN-20-2023** from **09:00-12:00**
Responsible lecturer	**J.W. Welleman**

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 4

☒ **questions may differ in weight** *(the time mentioned is an indicator for the weight)*

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with red pen or with pencils.
- Calculators with CAS and/or wifi/BT and/or PDF capabilities
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ **books** ☐ **notes** ☐ **dictionaries** ☐ **syllabus**

☐ **formula sheets (see also below under 'additional information')** ☒ **calculators**

☐ **computer** ☐ **...**

☒ **scientific (graphical)calculator** ☒ **drawing material**

Additional information (if necessary to be filled in by the examiner)

- **Use for each problem a separate examination paper**
- **The question form contains fomula sheets which can be used.**
- **Students can take the question form home after the exam.**
- **Specify the correct BSc or MSc course code on the exam paper**
- **BSc students are allowed to answer in Dutch**
- **No student leaves without delivering an exam paper with a name on it!**

Exam graded by: (the marking period is 15 working days at most)

Every suspicion of fraud is reported to the Board of Examiners.

Mobile Phone OFF.

Problem 1 : Influence Lines (approx. 40 min)

Two hinged beams with bending stiffness *EI* are shown in figure 1. The hinges are denoted with h.

“Construct” requires a correct sketch and the computed values of the influence factors at key points in the graph showing a qualitative and quantitative result from which it should become clear if member parts will remain straight or become curved and or if kinks are present. Unit displacements, rotations or loads must be specified.

“Sketching” requires only a qualitative result from which it should become clear if member parts will remain straight or become curved and or if kinks are present. Unit displacements, rotations or loads must be specified.

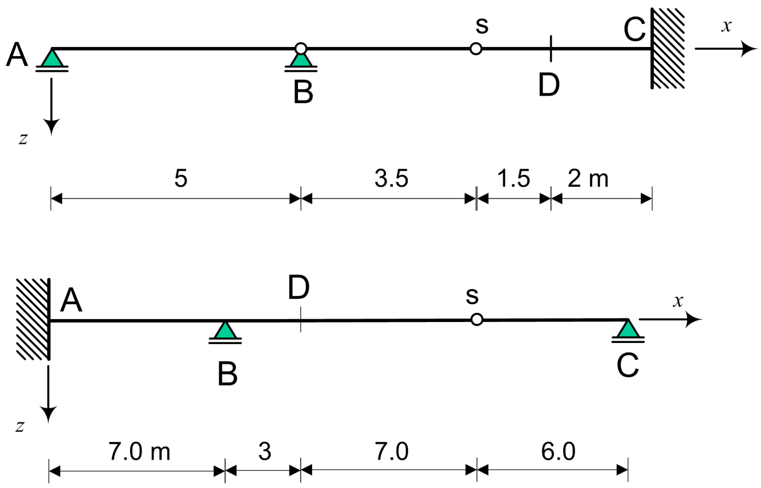


Figure 1 : Two hinged beams with different static systems

Questions:

Based on the first structure:
a) Construct the influence line for the moment at B
b) Construct the influence line for the displacement at D.
c) Construct the influence line for the moment at support C.

Based on the second structure:
d) Sketch the influence line for the support reaction at A. Show the exact value for $x = 0$
e) Sketch the influence line for the shear force at D.

Problem 2 : Work and Energy Methods (approx. 40 min)

In fig. 2, a cantilever beam is given which can be loaded with a unit vertical force at B and/or a unit couple at C. Both are applied in the positive direction with respect to the coordinate system.

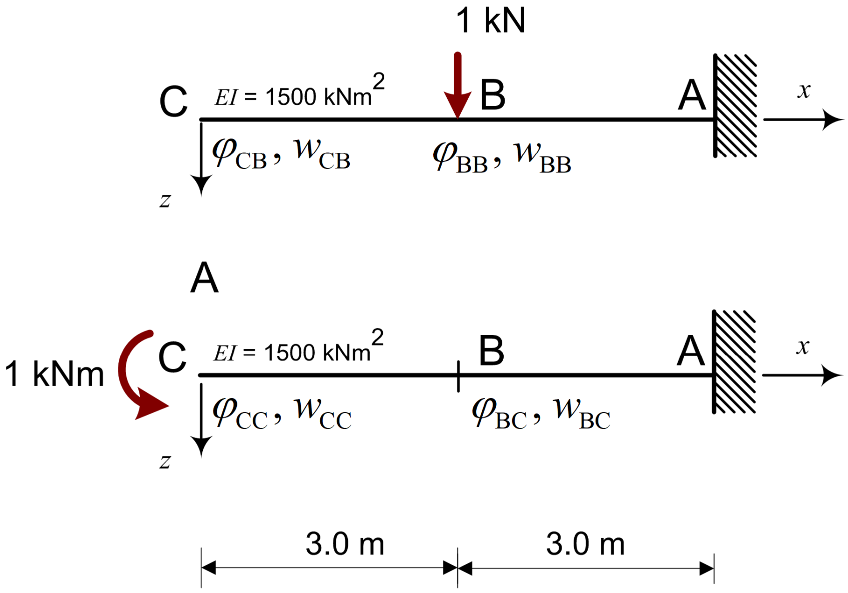


Figure 2 : Cantilever beam loaded with unit load and or unit couple.

Questions:

a) Prove for the general case, using work, the Maxwell's reciprocal theorem which states that in general $w_{BC} = \varphi_{CB}$.

Constraint:
You cannot make use of the method of superposition (*vergeet-mij-nietjes*), the differential equation or the moment-area method (*momentenvlakstelling*). Also, do not calculate the actual displacement and do not make use of the length and stiffness of the beam.

b) Calculate the displacement w_{BC} or φ_{CB} using a work or energy method of your own choice. For intermediate steps such as finding support reactions or moment distributions, you can use any method of your choice.

Problem 3 : Plasticity (approx. 50 min)

In figure 3, a frame is shown for which the collapse load has to be found. The frame is loaded with the indicated concentrated load at E. The parts have the denoted strengths and bending stiffnesses. Connections between parts in A, C, E and D are rigid. B is a hinged connection between part AB and BD. Hinged supports are used with a horizontal roller at B. Any axial deformation can be neglected in this problem.

NOTE : Take care of the different strengths and bending stiffnesses of the elements.

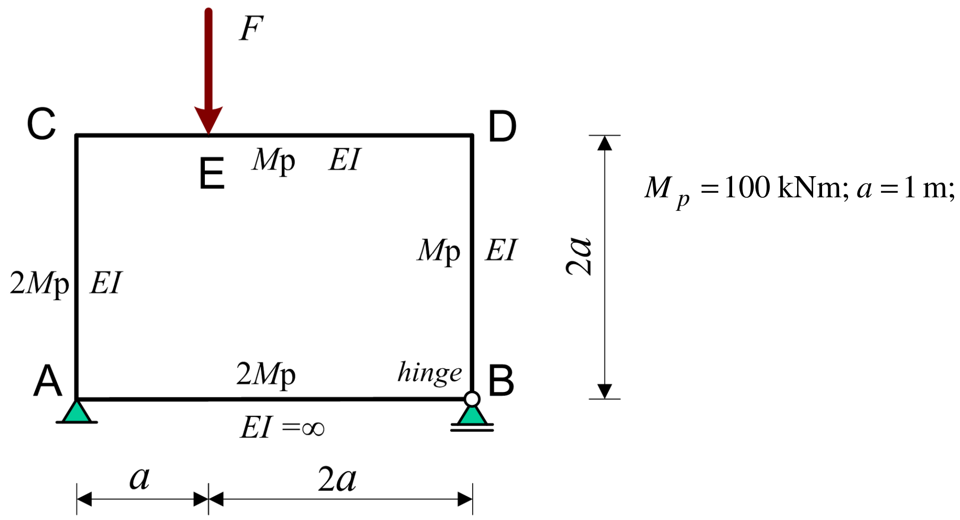


Figure 3 : Frame structure

Questions:

a) Analyse this structure and explain the **upper-bound theorem** and its consequence of finding the failure load. Include small sketches to substantiate your answer.

b) Find the failure load and the failure mechanism which belongs to it.

c) Explain how to check that your mechanism results indeed to the correct failure load and perform this check.

d) The upper-bound theorem assumes a shape factor 1.0. Explain the meaning of this and also discuss briefly how the redundancy in moment capacity of structures in plasticity can be described.

Problem 4 : Cross Section **(approx. 50 min)**

An unsymmetrical cross section as shown in the fig. 4, is loaded in the vertical plane with a positive moment of 30 kNm, in the horizontal plane with a negative moment of 15 kNm. The shear force with a magnitude of 160 kN is applied in this plane of loading. The cross section is a composite based on two different materials. Focus in this problem is at the interface between the materials.

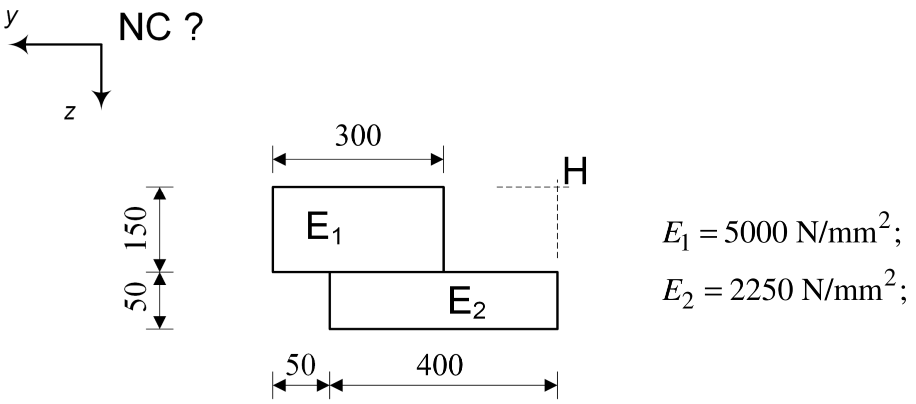


Fig. 4 : Composite Cross section

The two rectangular blocks are glued together in such a way that perfect bonding exist and an inhomogeneous and unsymmetrical cross-sectional approach can be followed.

Questions :

a) Find the location of the normal force centre with respect to point H.
b) Find the bending stiffness EI_{yz} of the composite cross section in Nmm^2.

To avoid too much calculus some of the cross-sectional properties are listed as:
$EI_{yy} = 26625 \times 10^8$ Nmm^2; $EI_{zz} = 8062.5 \times 10^8$ Nmm^2;

c) Find the plane of loading, the plane of curvature and the neutral axis of this cross section and present these in a clear sketch of the cross section.
d) Explain how to find the magnitude of the sliding force at the interface between the two blocks and calculate the magnitude of the shear stress in N/mm^2 at this interface.
e) Explain how to find the core of this cross section and support your answer with a clear sketch.

FORMULAS

Inhomogeneous and/or unsymmetrical cross sections :

$$\varepsilon(y,z)=\varepsilon+\kappa_y y+\kappa_z z \qquad \text{and:} \quad \sigma(y,z)=E(y,z)\times\varepsilon(y,z)$$

$$\begin{bmatrix} M_y \\ M_z \end{bmatrix}=\begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix}\begin{bmatrix} \kappa_y \\ \kappa_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e_y \\ e_z \end{bmatrix}=\frac{1}{EA}\begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix}\begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

$$s_x^{(a)}=-\frac{V_y ES_y^{(a)}}{EI_{yy}}-\frac{V_z ES_z^{(a)}}{EI_{zz}}; \quad \text{or:} \quad s_x^{(a)}=-\frac{R_M^{(a)}}{M}V; \quad \sigma_{xt}=\frac{s_x^{(a)}}{b^{(a)}}$$

$$\tan(2\alpha)=\frac{2EI_{yz}}{\left(EI_{yy}-EI_{zz}\right)}; \quad EI_{1,2}=\tfrac{1}{2}\left(EI_{yy}+EI_{zz}\right)\pm\sqrt{\left(\tfrac{1}{2}\left(EI_{yy}-EI_{zz}\right)\right)^2+EI_{yz}^2}$$

$$q_y^*=\frac{EI_{yy}EI_{zz}q_y-EI_{yy}EI_{yz}q_z}{EI_{yy}EI_{zz}-EI_{yz}^2}$$

$$q_z^*=\frac{-EI_{yz}EI_{zz}q_y+EI_{yy}EI_{zz}q_z}{EI_{yy}EI_{zz}-EI_{yz}^2}$$

Deformation energy:

$$E_v=\int\tfrac{1}{2}EA\varepsilon^2\mathrm{d}x \quad \text{(extension)}$$

$$E_v=\int\tfrac{1}{2}EI\kappa^2\mathrm{d}x \quad \text{(bending)}$$

Kinematic relations:

$$\varepsilon=\frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\kappa=-\frac{\mathrm{d}^2w}{\mathrm{d}x^2}$$

Complementary energy:

$$E_c=\int\frac{N^2}{2EA}\mathrm{d}x \quad \text{(extension)}$$

$$E_c=\int\frac{M^2}{2EI}\mathrm{d}x \quad \text{(bending)}$$

Constitutive relations:

$$N=EA.\varepsilon$$

$$M=EI.\kappa$$

Castigliano's theorem's:

$$F_i=\frac{\partial E_v}{\partial u_i} \qquad u_i=\frac{\partial E_c}{\partial F_i}$$

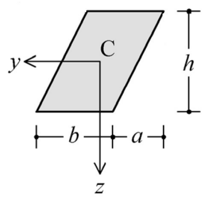
Work method with unit load:

$$u=\int\frac{M(x)m(x)dx}{EI}$$

Rayleigh:

$$F_k=\frac{E_v}{\int\frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2\mathrm{d}x}$$

Math tools:



$$I_{yy}=\tfrac{1}{12}(a^2+b^2)bh$$

$$I_{zz}=\tfrac{1}{12}bh^3$$

$$I_{yz}=\tfrac{1}{12}abh^2$$

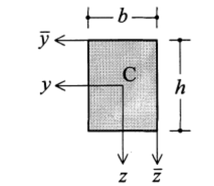
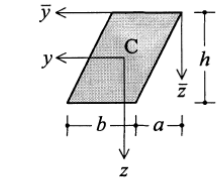
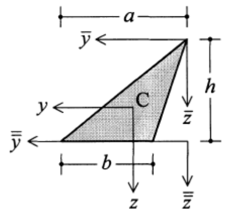
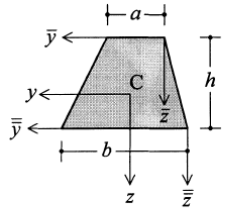
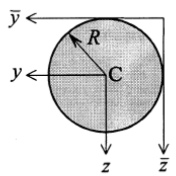
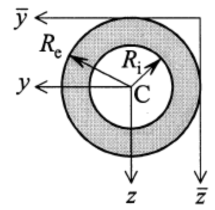
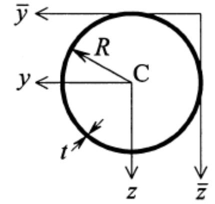
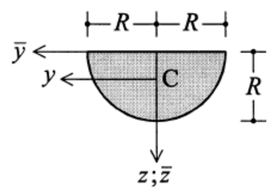
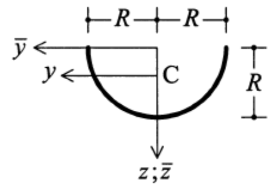
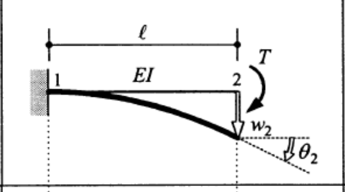
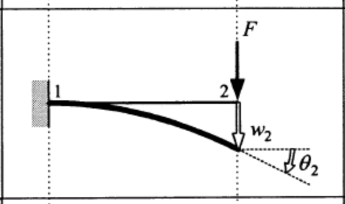
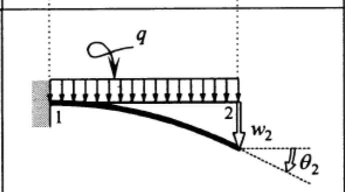
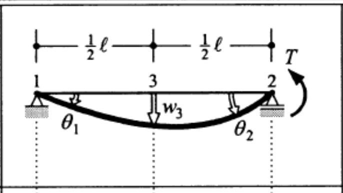
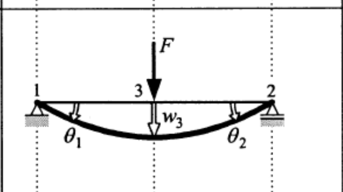
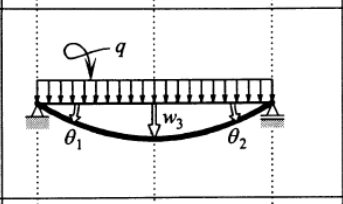
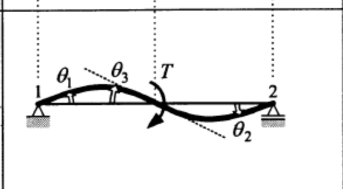
Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Rectangle $A = bh$ $\bar{y}_C = \frac{1}{2}b$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}b^3h$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = \frac{1}{3}b^3h$ $I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{4}b^2h^2$
	Parallelogram $A = bh$ $\bar{y}_C = \frac{1}{2}(a+b)$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}(a^2+b^2)bh$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = \frac{1}{12}abh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$
	Triangle $A = \frac{1}{2}bh$ $\bar{y}_C = \frac{1}{3}(2a-b)$ $\bar{z}_C = \frac{2}{3}h$	$I_{yy} = \frac{1}{36}(a^2-ab+b^2)bh$ $I_{zz} = \frac{1}{36}bh^3$ $I_{yz} = \frac{1}{72}(2a-b)bh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{4}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{8}(2a-b)bh^2$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}bh^3$
	Trapezium $A = \frac{1}{2}(a+b)h$ $\bar{z}_C = \frac{1}{3}\frac{a+2b}{a+b}h$	$I_{zz} = \frac{1}{36}\frac{a^2+4ab+b^2}{a+b}h^3$	$I_{\bar{z}\bar{z}} = \frac{1}{12}(a+3b)h^3$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}(3a+b)h^3$
	Circle $A = \pi R^2$	$I_{yy} = I_{zz} = \frac{1}{4}\pi R^4$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi R^4$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{5}{4}\pi R^4$ $I_{\bar{y}\bar{z}} = \pi R^4$

Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Thick-walled ring $A = \pi(R_e^2 - R_i^2)$	$I_{yy} = I_{zz} = \frac{1}{4}\pi(R_e^4 - R_i^4)$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi(R_e^4 - R_i^4)$	
	Thin-walled ring $A = 2\pi Rt$	$I_{yy} = I_{zz} = \pi R^3 t$ $I_{yz} = 0$ $I_p = 2\pi R^3 t$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = 3\pi R^3 t$
	Semicircle $A = \frac{1}{2}\pi R^2$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{4}{3\pi}R$ $= 0.424R$	$I_{yy} = \frac{1}{8}\pi R^4 = 0.393R^4$ $I_{zz} = (\frac{\pi}{8} - \frac{8}{9\pi})R^4$ $= 0.110R^4$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{8}\pi R^4$ $I_{\bar{y}\bar{z}} = 0$
	Semicircular ring $A = \pi Rt$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{2}{\pi}R$ $= 0.637R$	$I_{yy} = \frac{1}{2}\pi R^3 t$ $I_{zz} = (\frac{\pi}{2} - \frac{4}{\pi})R^3 t$ $= 0.298R^3 t$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{2}\pi R^3 t$ $I_{\bar{y}\bar{z}} = 0$

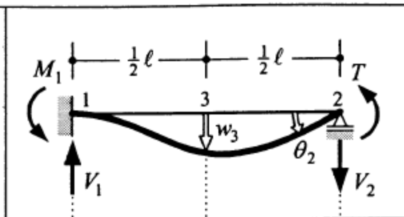
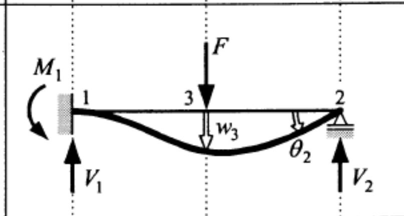
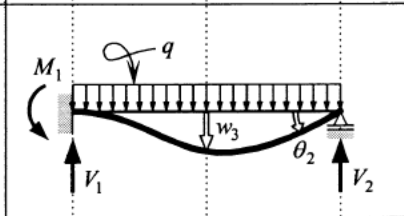
(1)		$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)		$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)		$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

forget-me-nots

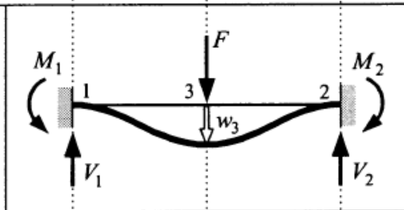
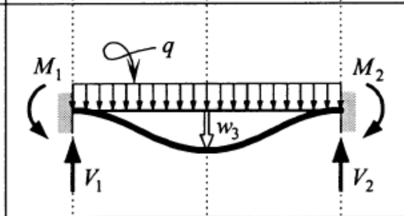
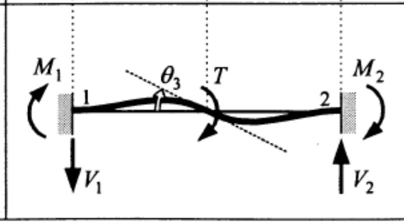
(4)		$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)		$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)		$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

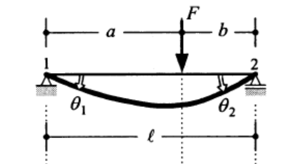
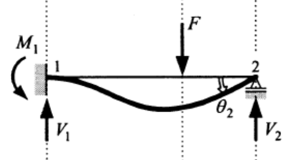
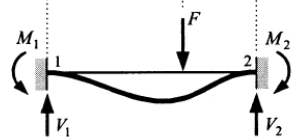
simply supported beam (statically determinate)

statically indeterminate beam (one fixed end)

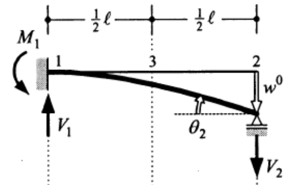
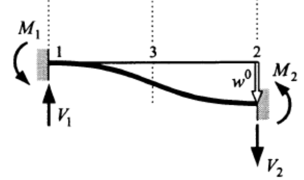
(7)		$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)		$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)		$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

statically indeterminate beam (two fixed ends)

(10)		$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)		$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)		$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

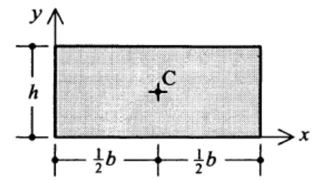
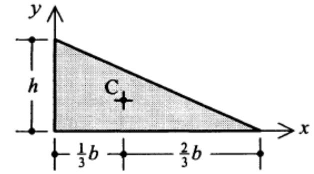
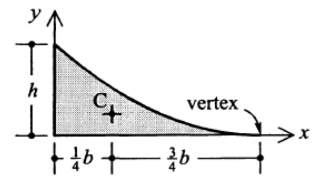
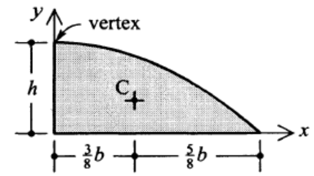
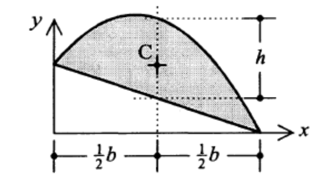
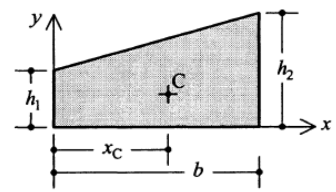
(c)		$\theta_1 = \dfrac{Fab(\ell+b)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(2\dfrac{a}{\ell} - 3\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fab(\ell+a)}{6EI\ell} = \dfrac{F\ell^2}{6EI}\left(\dfrac{a}{\ell} - \dfrac{a^3}{\ell^3}\right)$
(d)		$M_1 = \dfrac{Fb(\ell^2-b^2)}{2\ell^2} = F\ell\left(\dfrac{a}{\ell} - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb(3\ell^2-b^2)}{2\ell^3} = F\left(1 - \dfrac{3}{2}\dfrac{a^2}{\ell^2} + \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(3\ell-a)}{2\ell^3} = F\left(\dfrac{3}{2}\dfrac{a^2}{\ell^2} - \dfrac{1}{2}\dfrac{a^3}{\ell^3}\right)$ $\theta_2 = \dfrac{Fa^2b}{4EI\ell} = \dfrac{F\ell^2}{4EI}\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$
(e)		$M_1 = \dfrac{Fab^2}{\ell^2} = F\ell\left(\dfrac{a}{\ell} - 2\dfrac{a^2}{\ell^2} + \dfrac{a^3}{\ell^3}\right)$ $V_1 = \dfrac{Fb^2(\ell+2a)}{\ell^3} = F\left(1 - 3\dfrac{a^2}{\ell^2} + 2\dfrac{a^3}{\ell^3}\right)$ $M_2 = \dfrac{Fa^2b}{\ell^2} = F\ell\left(\dfrac{a^2}{\ell^2} - \dfrac{a^3}{\ell^3}\right)$ $V_2 = \dfrac{Fa^2(\ell+2b)}{\ell^3} = F\ell\left(3\dfrac{a^2}{\ell^2} - 2\dfrac{a^3}{\ell^3}\right)$

support reactions and rotations at the beam ends

(f)		$M_1 = \dfrac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{3EI}{\ell^3}w^0$ $\theta_2 = \dfrac{3}{2}\dfrac{w^0}{\ell}$ $\theta_3 = \dfrac{9}{8}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{5}{16}w^0$
(g)		$M_1 = M_2 = \dfrac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \dfrac{12EI}{\ell^3}w^0$ $\theta_3 = \dfrac{3}{2}\dfrac{w^0}{\ell};\quad w_3 = \dfrac{1}{2}w^0$

settlements

properties of plane figures to be used for the moment-area theorems

(1)		rectangle: $y = h$ $A = bh$ $x_C = \frac{1}{2}b$
(2)		triangle: $y = h\left\{1 - \dfrac{x}{b}\right\}$ $A = \frac{1}{2}bh$ $x_C = \frac{1}{3}b$
(3)		parabola: $y = h\left\{1 - \dfrac{x}{b}\right\}^2$ $A = \frac{1}{3}bh$ $x_C = \frac{1}{4}b$
(4)		parabola: $y = h\left\{1 - \left(\dfrac{x}{b}\right)^2\right\}$ $A = \frac{2}{3}bh$ $x_C = \frac{3}{8}b$
(5)		parabola: $A = \frac{2}{3}bh$ $x_C = \frac{1}{2}b$
(6)		trapezium: $y = h_1 + (h_2 - h_1)\dfrac{x}{b}$ $A = \frac{1}{2}b(h_1 + h_2)$ $x_C = \frac{1}{3}b\dfrac{h_1 + 2h_2}{h_1 + h_2}$