Faculty
Civil Engineering and GeoSciences

Exam	**CIE3109-09 / CTB3330**
	Structural Mechanics 4
Total number of pages	**8 pages (excl cover)**
Date and time	**JUN-25-2019** from **09:00-12:00**
Responsible lecturer	**J.W. Welleman**

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Exam questions (to be filled in by course examiner)

Total number of questions: 4

☒ **questions may differ in weight** *(the time mentioned is an indicator for the weight)*

Use of tools and sources of information (to be filled in by course examiner)

Not allowed:

- Mobile phone, smart Phone or devices with similar functions.
- Answers written with red pen or with pencils.
- Calculators with CAS and/or wifi/BT and/or PDF capabilities
- Tools and/or sources of information unless otherwise specified below.

Allowed:

☐ **books** ☐ **notes** ☐ **dictionaries** ☐ **syllabus**

☐ **formula sheets (see also below under 'additional information')** ☒ **calculators**

☐ **computer** ☐ **...**

☒ **scientific (graphical)calculator** ☒ **drawing material**

Additional information (if necessary to be filled in by the examiner)

- **Use for each problem a separate examination paper**
- **The question form contains fomula sheets which can be used.**
- **Students can take the question form home after the exam.**
- **Specify the correct BSc or MSc course code on the exam paper**
- **BSc students are allowed to answer in Dutch**
- **No student leaves without delivering an exam paper with a name on it!**

Exam graded by: (the marking period is 15 working days at most)

Every suspicion of fraud is reported to the Board of Examiners.

Mobile Phone OFF.

Problem 1 : Influence Lines (approx. 40 min)

Two hinged beam structures are shown in figure 1. The hinges are denoted with h. The influence of axial deformation is neglected.

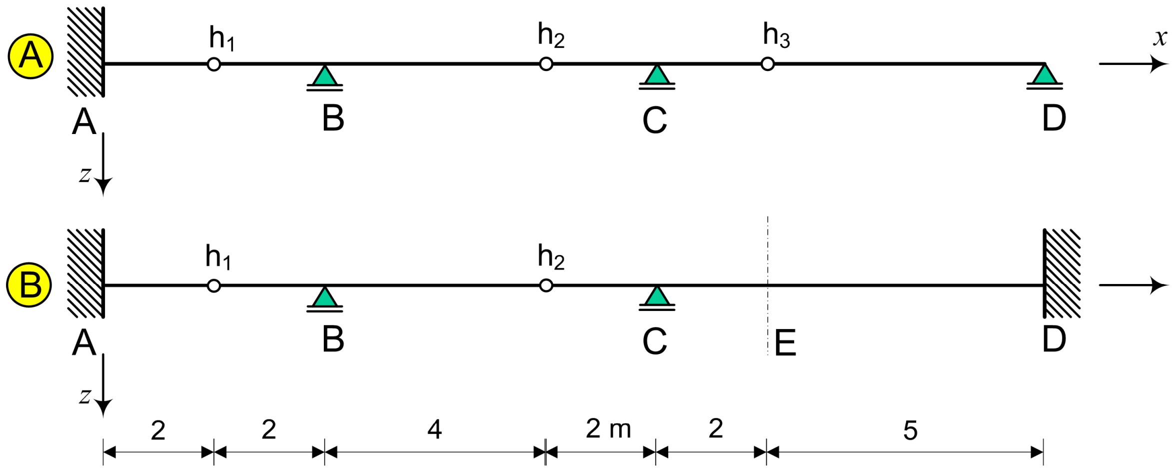


Figure 1 : Hinged beams **A** and **B**

Questions:

Answer the following questions in combination with beam (A) :

a) Construct the influence line for the moment at D,
b) Construct the influence line for the shear direct left of B,
c) Construct the influence line for the shear force directly to the left of hinge 2,
d) Construct the influence line for the support reaction at C,
e) Sketch the influence line for the rotation at B,
f) Sketch the influence line for the displacement of h_3.

Answer the following questions in combination with beam (B) :

g) Sketch the influence line for the shear force at E.
h) Sketch the influence line for the moment at D.

Note : "Construct" requires a correct sketch and the computed values of the influence factors at key points in the graph showing a qualitative and quantitative result. Sketching requires only a qualitative result from which it should become clear if member parts will remain straight or become curved.

Problem 2 : Work and Energy Methods (approx. 50 min)

For a frame structure shown in figure 2, information about the deformation is asked using a work and energy method. The prismatic beam, with bending stiffness *EI* is fully clamped at A and supported at B with a horizontal roller. A vertical concentrated load *F* acts at C as is indicated in fig. 2. Only bending deformation is considered. Use suitable local coordinate systems to solve this problem.

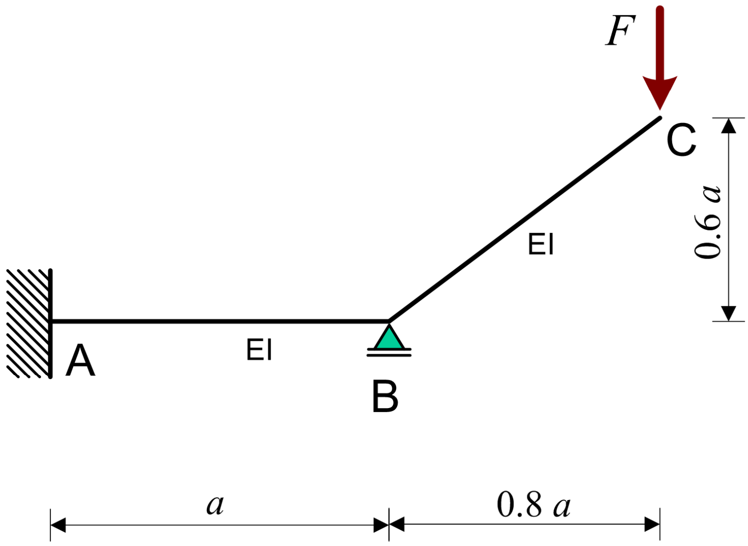


Figure 2 : Cantilever beam with concentrated load

Questions:

a) Explain the required steps involved to find the vertical displacement of point C due to the load *F* using Castigliano's theoreme.

b) Find with Castigliano's theoreme the expression for the vertical displacement at C expressed in the parameters used.

Note: Support your answer if needed with a clear sketch. Clearly show your positive definitions and coordinate system(s) used.

Problem 3 : Plasticity (approx. 45 min)

In figure 3, a frame is shown for which the collapse load has to be found. The frame is loaded with one concentrated load as indicated in the figure. The parts have the denoted strengths as also shown.

NOTE : Take care of the different strengths of the elements!

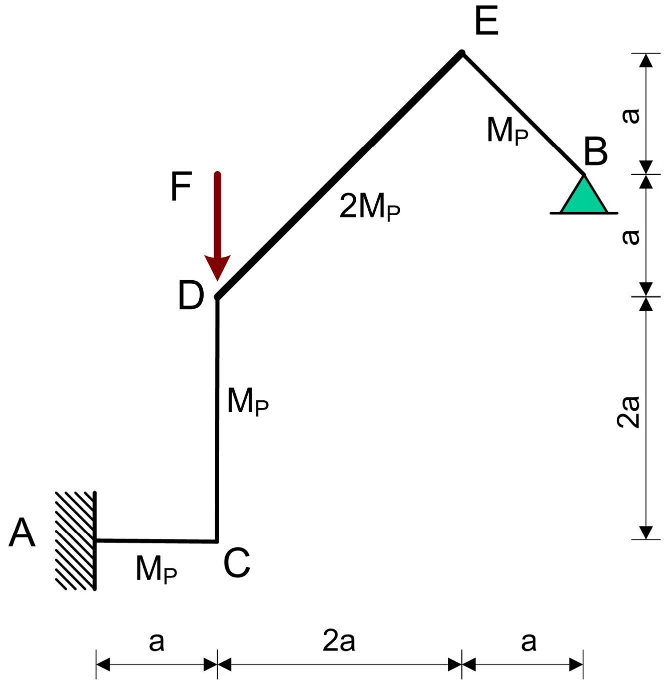


Figure 3 : Frame structure

Questions:

a) Comment on the following statement:

The failure load found by the upper bound theoreme results in a save value.

b) Comment on the following statement:

The additional load carrying capacity beyond the elastic limit is due to the structural level (static system) only.

c) Analyse the structure and determine the possible collapse mechanisms and show these with small sketches.

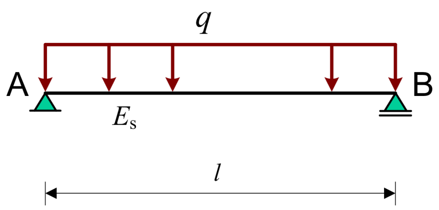
d) Find the failure mechanism and the corresponding failure load F_p .

e) How can you check your result?

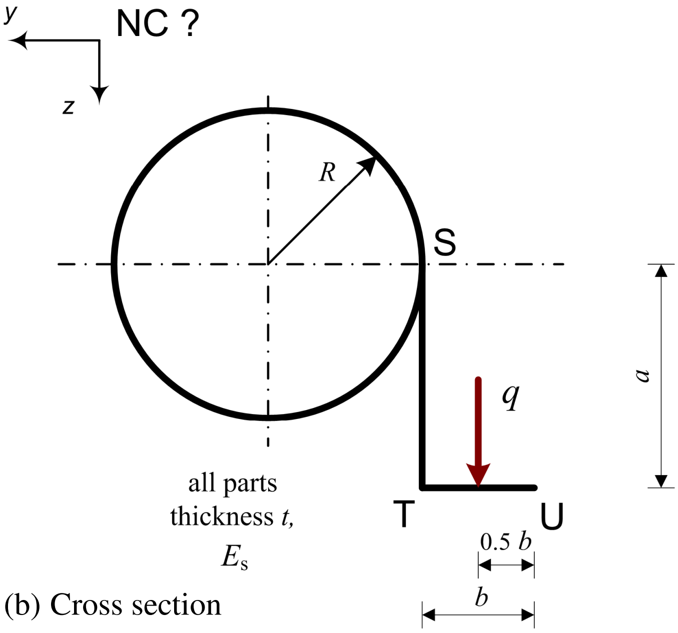
Problem 4 : Educational Cross Section (approx. 45 min)

The unsymmetrical thin-walled cross section at mid span of a simply supported beam is shown in fig. 4 (a). The span of the beam l is 3.5 m. The beam consists of a thin-walled tube with a diameter of 275 mm to which a L-section STU is welded at point S which supports the distributed load q centered between T and U as indicated in fig. 4 (b).

The thickness t and the Young's modulus E_s are constant for the entire cross section. The beam is loaded with a distributed load q = 24 kN/m. Use a x-y-z coordinate system with the origin at the normal force centre NC.



(a) Static system



(b) Cross section

Given:

$R = 137{,}5\ mm;\ l = 3500$ mm;

$a = 200\ mm;\ b = 100\ mm;\ t = 8\ mm;$

$E_s = 210000\ \text{N/mm}^2;\ q = 24$ kN/m;

thin walled :

$A_{tube} = 2\pi Rt = 69.115 \times 10^2\ \text{mm}^2$

$I_{zz-tube} = \pi R^3 t = 6533.530 \times 10^4\ \text{mm}^4$

Figure 4 : Unsymmetrical cross section of a simply supported beam

Questions :

a) Some data based on the thin-walled theory is given which can be used:

$$I_{yy} = 10967.47 \times 10^4\ \text{mm}^4;\ I_{zz} = 10767.15 \times 10^4\ \text{mm}^4;$$

Compute all missing data and find the:
- (1) plane of loading, (2) plane of curvature and (3) neutral axis.

Present these in a sketch of the cross section with maximum normal stresses.

b) Sketch the distribution of the normal stresses for part STU due to the given loading for the cross section with the highest normal stresses. Use a suitable presentation for this and show the values at the characteristic points.

c) Find the shear stress at S in part STU for the governing cross section for shear. Clearly describe how you obtained your result.

d) Explain how you can find the percentage of the shear force which will be taken by the tube.

FORMULAS

Inhomogeous and/or asymmetrical cross sections :

$$\varepsilon(y,z)=\varepsilon+\kappa_y y+\kappa_z z \qquad \text{and:} \quad \sigma(y,z)=E(y,z)\times\varepsilon(y,z)$$

$$\begin{bmatrix} M_y \\ M_z \end{bmatrix}=\begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix}\begin{bmatrix} \kappa_y \\ \kappa_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e_y \\ e_z \end{bmatrix}=\frac{1}{EA}\begin{bmatrix} EI_{yy} & EI_{yz} \\ EI_{yz} & EI_{zz} \end{bmatrix}\begin{bmatrix} -1/y_1 \\ -1/z_1 \end{bmatrix}$$

$$s_x^{(a)}=-\frac{V_y ES_y^{(a)}}{EI_{yy}}-\frac{V_z ES_z^{(a)}}{EI_{zz}}; \quad \text{or:} \quad s_x^{(a)}=-\frac{R_M^{(a)}}{M}V; \quad \sigma_{xt}=\frac{s_x^{(a)}}{b^{(a)}}$$

$$\tan(2\alpha)=\frac{2EI_{yz}}{\left(EI_{yy}-EI_{zz}\right)}; \quad EI_{1,2}=\tfrac{1}{2}\left(EI_{yy}+EI_{zz}\right)\pm\sqrt{\left(\tfrac{1}{2}\left(EI_{yy}-EI_{zz}\right)\right)^2+EI_{yz}^2}$$

$$q_y^*=\frac{EI_{yy}EI_{zz}q_y-EI_{yy}EI_{yz}q_z}{EI_{yy}EI_{zz}-EI_{yz}^2}$$

$$q_z^*=\frac{-EI_{yz}EI_{zz}q_y+EI_{yy}EI_{zz}q_z}{EI_{yy}EI_{zz}-EI_{yz}^2}$$

Deformation energy:

$$E_v=\int \tfrac{1}{2}EA\varepsilon^2\mathrm{d}x \quad \text{(extension)}$$

$$E_v=\int \tfrac{1}{2}EI\kappa^2\mathrm{d}x \quad \text{(bending)}$$

Complementary energy:

$$E_c=\int \frac{N^2}{2EA}\mathrm{d}x \quad \text{(extension)}$$

$$E_c=\int \frac{M^2}{2EI}\mathrm{d}x \quad \text{(bending)}$$

Castigliano's theorema's:

$$F_i=\frac{\partial E_v}{\partial u_i} \qquad u_i=\frac{\partial E_c}{\partial F_i}$$

Kinematic relations:

$$\varepsilon=\frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\kappa=-\frac{\mathrm{d}^2w}{\mathrm{d}x^2}$$

Constitutive relations:

$$N=EA.\varepsilon$$

$$M=EI.\kappa$$

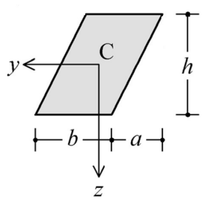
Work method with unit load:

$$u=\int \frac{M(x)m(x)dx}{EI}$$

Rayleigh:

$$F_k=\frac{E_v}{\int \frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2\mathrm{d}x}$$

Math tools:



$$I_{yy}=\tfrac{1}{12}(a^2+b^2)bh$$

$$I_{zz}=\tfrac{1}{12}bh^3$$

$$I_{yz}=\tfrac{1}{12}abh^2$$

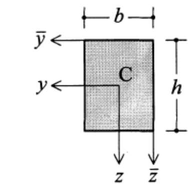
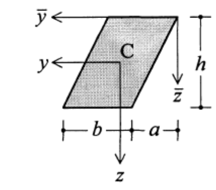
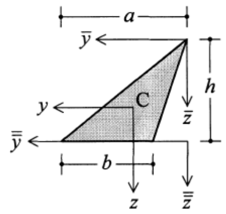
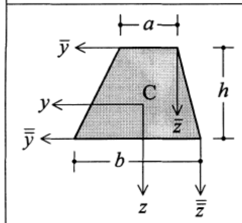
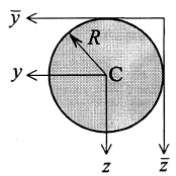
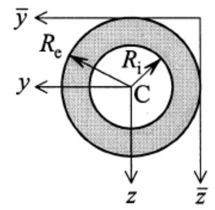
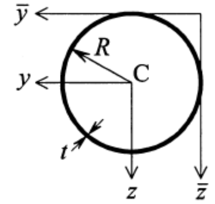
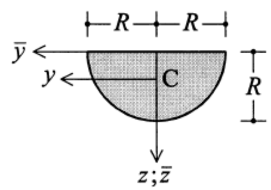
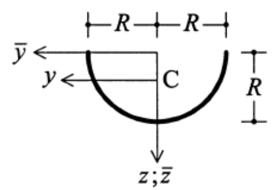
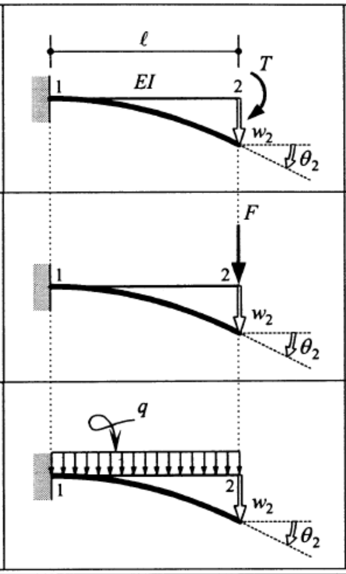
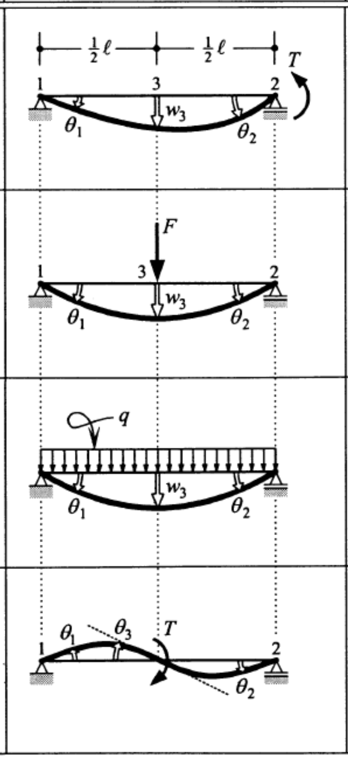
Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Rectangle $A = bh$ $\bar{y}_C = \frac{1}{2}b$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}b^3h$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = \frac{1}{3}b^3h$ $I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{4}b^2h^2$
	Parallelogram $A = bh$ $\bar{y}_C = \frac{1}{2}(a+b)$ $\bar{z}_C = \frac{1}{2}h$	$I_{yy} = \frac{1}{12}(a^2+b^2)bh$ $I_{zz} = \frac{1}{12}bh^3$ $I_{yz} = \frac{1}{12}abh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{3}bh^3$
	Triangle $A = \frac{1}{2}bh$ $\bar{y}_C = \frac{1}{3}(2a-b)$ $\bar{z}_C = \frac{2}{3}h$	$I_{yy} = \frac{1}{36}(a^2-ab+b^2)bh$ $I_{zz} = \frac{1}{36}bh^3$ $I_{yz} = \frac{1}{72}(2a-b)bh^2$	$I_{\bar{z}\bar{z}} = \frac{1}{4}bh^3$ $I_{\bar{y}\bar{z}} = \frac{1}{8}(2a-b)bh^2$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}bh^3$
	Trapezium $A = \frac{1}{2}(a+b)h$ $\bar{z}_C = \frac{1}{3}\frac{a+2b}{a+b}h$	$I_{zz} = \frac{1}{36}\frac{a^2+4ab+b^2}{a+b}h^3$	$I_{\bar{z}\bar{z}} = \frac{1}{12}(a+3b)h^3$ $I_{\bar{\bar{z}}\bar{\bar{z}}} = \frac{1}{12}(3a+b)h^3$
	Circle $A = \pi R^2$	$I_{yy} = I_{zz} = \frac{1}{4}\pi R^4$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi R^4$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{5}{4}\pi R^4$ $I_{\bar{y}\bar{z}} = \pi R^4$

Figure	Area, coordinates centroid C	Second moments of area: centroidal	Second moments of area: other
	Thick-walled ring $A = \pi(R_e^2 - R_i^2)$	$I_{yy} = I_{zz} = \frac{1}{4}\pi(R_e^4 - R_i^4)$ $I_{yz} = 0$ $I_p = \frac{1}{2}\pi(R_e^4 - R_i^4)$	
	Thin-walled ring $A = 2\pi Rt$	$I_{yy} = I_{zz} = \pi R^3 t$ $I_{yz} = 0$ $I_p = 2\pi R^3 t$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = 3\pi R^3 t$
	Semicircle $A = \frac{1}{2}\pi R^2$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{4}{3\pi}R$ $= 0.424R$	$I_{yy} = \frac{1}{8}\pi R^4 = 0.393R^4$ $I_{zz} = (\frac{\pi}{8} - \frac{8}{9\pi})R^4$ $= 0.110R^4$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{8}\pi R^4$ $I_{\bar{y}\bar{z}} = 0$
	Semicircular ring $A = \pi Rt$ $\bar{y}_C = 0$ $\bar{z}_C = \frac{2}{\pi}R$ $= 0.637R$	$I_{yy} = \frac{1}{2}\pi R^3 t$ $I_{zz} = (\frac{\pi}{2} - \frac{4}{\pi})R^3 t$ $= 0.298R^3 t$ $I_{yz} = 0$	$I_{\bar{y}\bar{y}} = I_{\bar{z}\bar{z}} = \frac{1}{2}\pi R^3 t$ $I_{\bar{y}\bar{z}} = 0$



(1)	$\theta_2 = \frac{T\ell}{EI}$; $w_2 = \frac{T\ell^2}{2EI}$
(2)	$\theta_2 = \frac{F\ell^2}{2EI}$; $w_2 = \frac{F\ell^3}{3EI}$
(3)	$\theta_2 = \frac{q\ell^3}{6EI}$; $w_2 = \frac{q\ell^4}{8EI}$

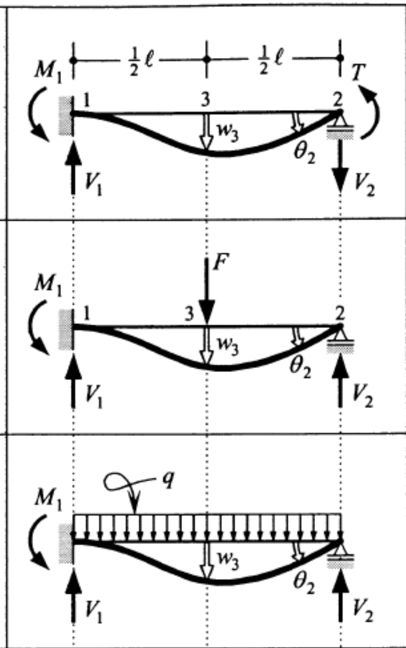
forget-me-nots



(4)	$\theta_1 = \frac{1}{6}\frac{T\ell}{EI}$; $\theta_2 = \frac{1}{3}\frac{T\ell}{EI}$; $w_3 = \frac{1}{16}\frac{T\ell^2}{EI}$
(5)	$\theta_1 = \theta_2 = \frac{1}{16}\frac{F\ell^2}{EI}$; $w_3 = \frac{1}{48}\frac{F\ell^3}{EI}$
(6)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{q\ell^3}{EI}$; $w_3 = \frac{5}{384}\frac{q\ell^4}{EI}$
(a)	$\theta_1 = \theta_2 = \frac{1}{24}\frac{T\ell}{EI}$; $\theta_3 = \frac{1}{12}\frac{T\ell}{EI}$; $w_3 = 0$

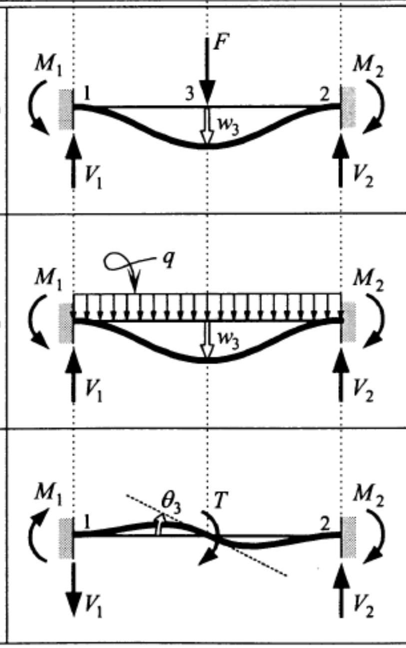
simply supported beam (statically determinate)

statically indeterminate beam (one fixed end)



(7)	$\theta_2 = \frac{1}{4}\frac{T\ell}{EI}$; $w_3 = \frac{1}{32}\frac{T\ell^2}{EI}$ $M_1 = \frac{1}{2}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$
(8)	$\theta_2 = \frac{1}{32}\frac{F\ell^2}{EI}$; $w_3 = \frac{7}{768}\frac{F\ell^3}{EI}$ $M_1 = \frac{3}{16}F\ell$; $V_1 = \frac{11}{16}F$; $V_2 = \frac{5}{16}F$
(9)	$\theta_2 = \frac{1}{48}\frac{q\ell^3}{EI}$; $w_3 = \frac{1}{192}\frac{q\ell^4}{EI}$ $M_1 = \frac{1}{8}q\ell^2$; $V_1 = \frac{5}{8}q\ell$; $V_2 = \frac{3}{8}q\ell$

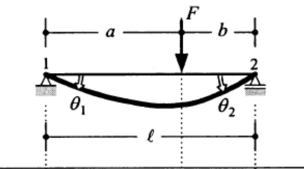
statically indeterminate beam (two fixed ends)



(10)	$w_3 = \frac{1}{192}\frac{F\ell^3}{EI}$ $M_1 = M_2 = \frac{1}{8}F\ell$; $V_1 = V_2 = \frac{1}{2}F$
(11)	$w_3 = \frac{1}{384}\frac{q\ell^4}{EI}$ $M_1 = M_2 = \frac{1}{12}q\ell^2$; $V_1 = V_2 = \frac{1}{2}q\ell$
(b)	$\theta_3 = \frac{1}{16}\frac{T\ell}{EI}$; $w_3 = 0$ $M_1 = M_2 = \frac{1}{4}T$; $V_1 = V_2 = \frac{3}{2}\frac{T}{\ell}$

support reactions and rotations at the beam ends

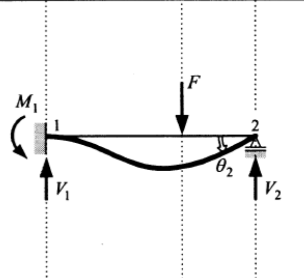
(c)



$$\theta_1 = \frac{Fab(\ell+b)}{6EI\ell} = \frac{F\ell^2}{6EI}\left(2\frac{a}{\ell} - 3\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3}\right)$$

$$\theta_2 = \frac{Fab(\ell+a)}{6EI\ell} = \frac{F\ell^2}{6EI}\left(\frac{a}{\ell} - \frac{a^3}{\ell^3}\right)$$

(d)



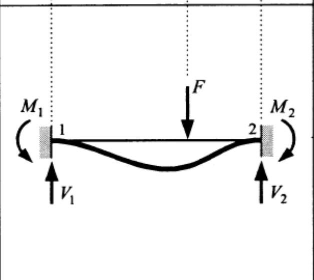
$$M_1 = \frac{Fb(\ell^2-b^2)}{2\ell^2} = F\ell\left(\frac{a}{\ell} - \frac{3}{2}\frac{a^2}{\ell^2} + \frac{1}{2}\frac{a^3}{\ell^3}\right)$$

$$V_1 = \frac{Fb(3\ell^2-b^2)}{2\ell^3} = F\left(1 - \frac{3}{2}\frac{a^2}{\ell^2} + \frac{1}{2}\frac{a^3}{\ell^3}\right)$$

$$V_2 = \frac{Fa^2(3\ell-a)}{2\ell^3} = F\left(\frac{3}{2}\frac{a^2}{\ell^2} - \frac{1}{2}\frac{a^3}{\ell^3}\right)$$

$$\theta_2 = \frac{Fa^2b}{4EI\ell} = \frac{F\ell^2}{4EI}\left(\frac{a^2}{\ell^2} - \frac{a^3}{\ell^3}\right)$$

(e)



$$M_1 = \frac{Fab^2}{\ell^2} = F\ell\left(\frac{a}{\ell} - 2\frac{a^2}{\ell^2} + \frac{a^3}{\ell^3}\right)$$

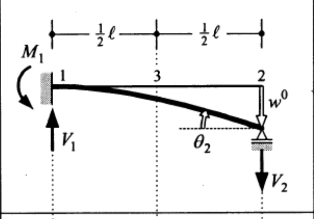
$$V_1 = \frac{Fb^2(\ell+2a)}{\ell^3} = F\left(1 - 3\frac{a^2}{\ell^2} + 2\frac{a^3}{\ell^3}\right)$$

$$M_2 = \frac{Fa^2b}{\ell^2} = F\ell\left(\frac{a^2}{\ell^2} - \frac{a^3}{\ell^3}\right)$$

$$V_2 = \frac{Fa^2(\ell+2b)}{\ell^3} = F\ell\left(3\frac{a^2}{\ell^2} - 2\frac{a^3}{\ell^3}\right)$$

settlements

(f)

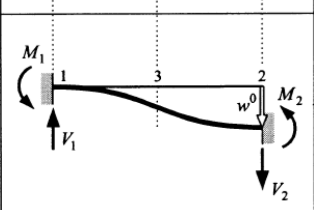


$$M_1 = \frac{3EI}{\ell^2}w^0;\quad V_1 = V_2 = \frac{3EI}{\ell^3}w^0$$

$$\theta_2 = \frac{3}{2}\frac{w^0}{\ell}$$

$$\theta_3 = \frac{9}{8}\frac{w^0}{\ell};\quad w_3 = \frac{5}{16}w^0$$

(g)

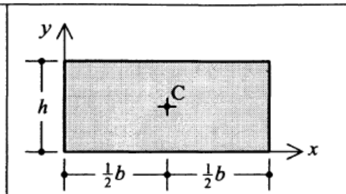


$$M_1 = M_2 = \frac{6EI}{\ell^2}w^0;\quad V_1 = V_2 = \frac{12EI}{\ell^3}w^0$$

$$\theta_3 = \frac{3}{2}\frac{w^0}{\ell};\quad w_3 = \frac{1}{2}w^0$$

properties of plane figures to be used for the moment-area theorems

(1)

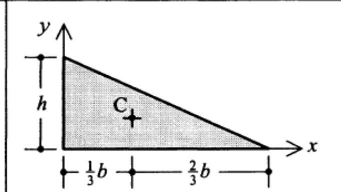


rectangle: $y = h$

$A = bh$

$x_C = \frac{1}{2}b$

(2)

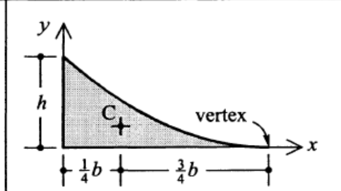


triangle: $y = h\left\{1 - \frac{x}{b}\right\}$

$A = \frac{1}{2}bh$

$x_C = \frac{1}{3}b$

(3)

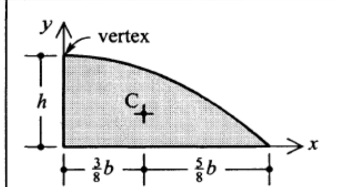


parabola: $y = h\left\{1 - \frac{x}{b}\right\}^2$

$A = \frac{1}{3}bh$

$x_C = \frac{1}{4}b$

(4)

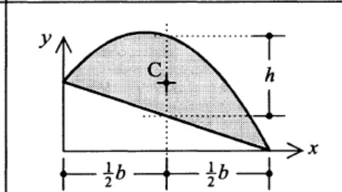


parabola: $y = h\left\{1 - \left(\frac{x}{b}\right)^2\right\}$

$A = \frac{2}{3}bh$

$x_C = \frac{3}{8}b$

(5)

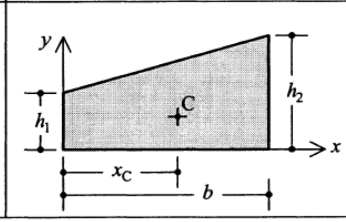


parabola:

$A = \frac{2}{3}bh$

$x_C = \frac{1}{2}b$

(6)



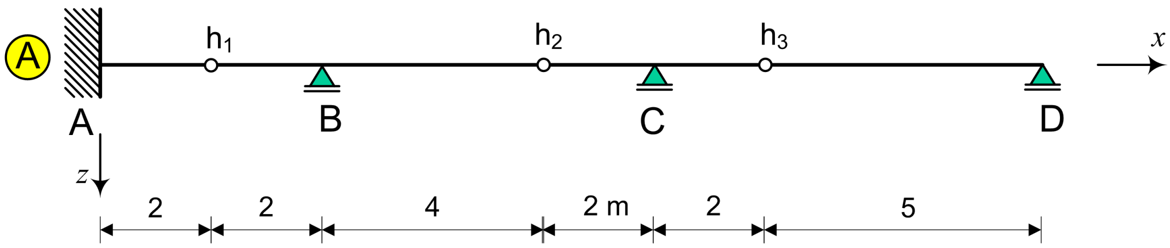
trapezium: $y = h_1 + (h_2 - h_1)\frac{x}{b}$

$A = \frac{1}{2}b(h_1 + h_2)$

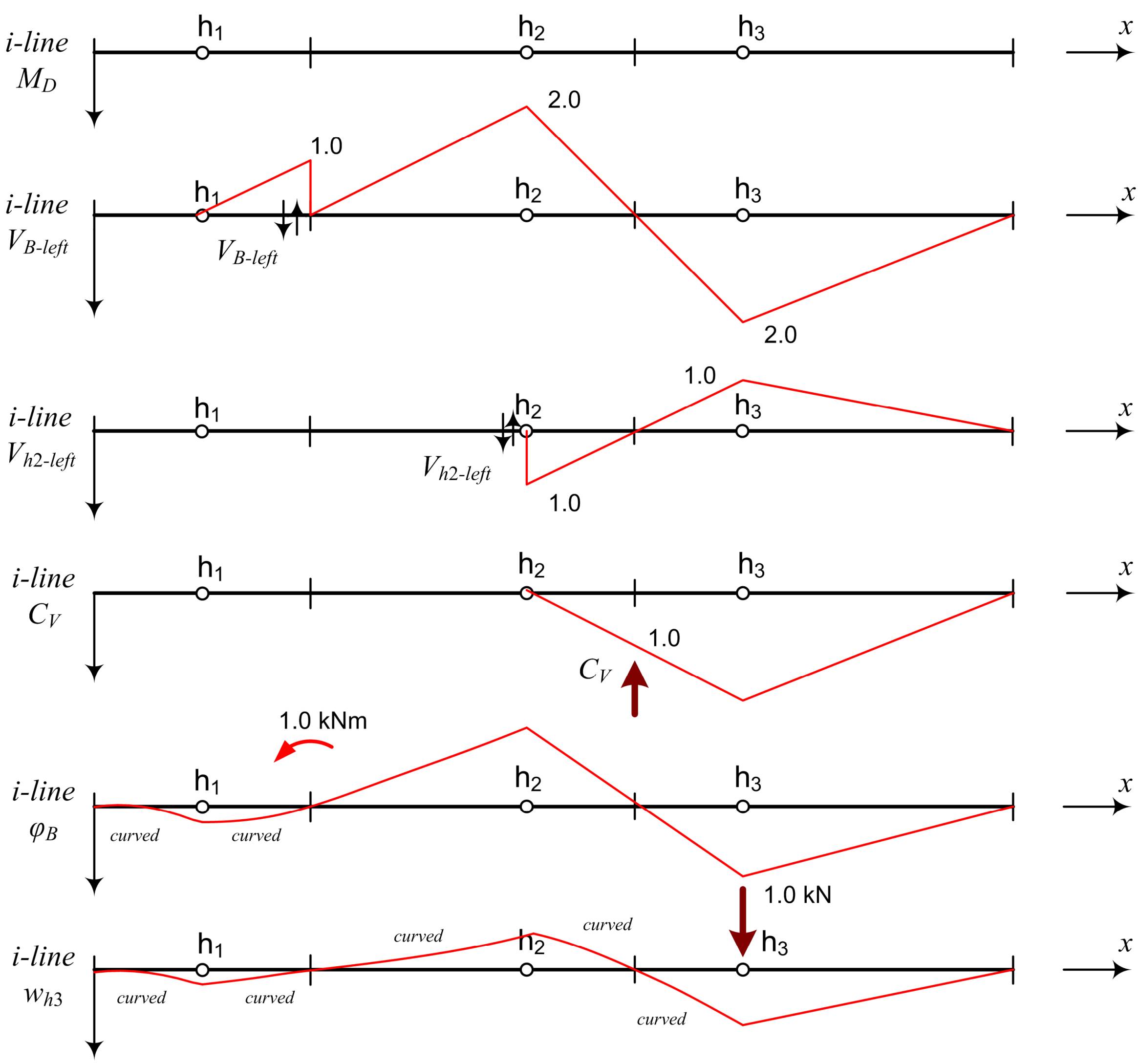
$x_C = \frac{1}{3}b\frac{h_1 + 2h_2}{h_1 + h_2}$

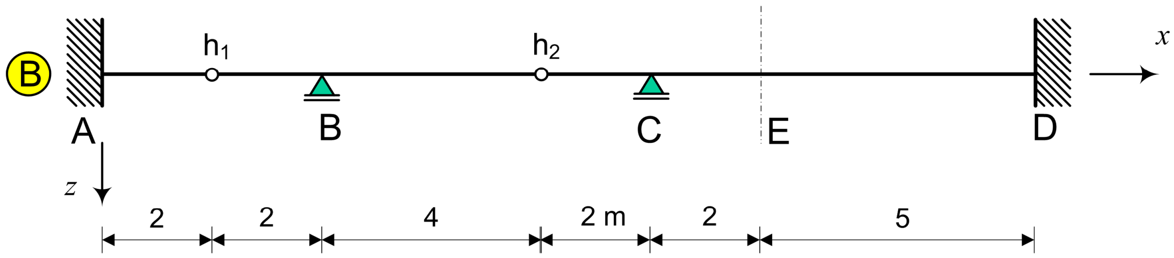
ANSWERS

Problem 1

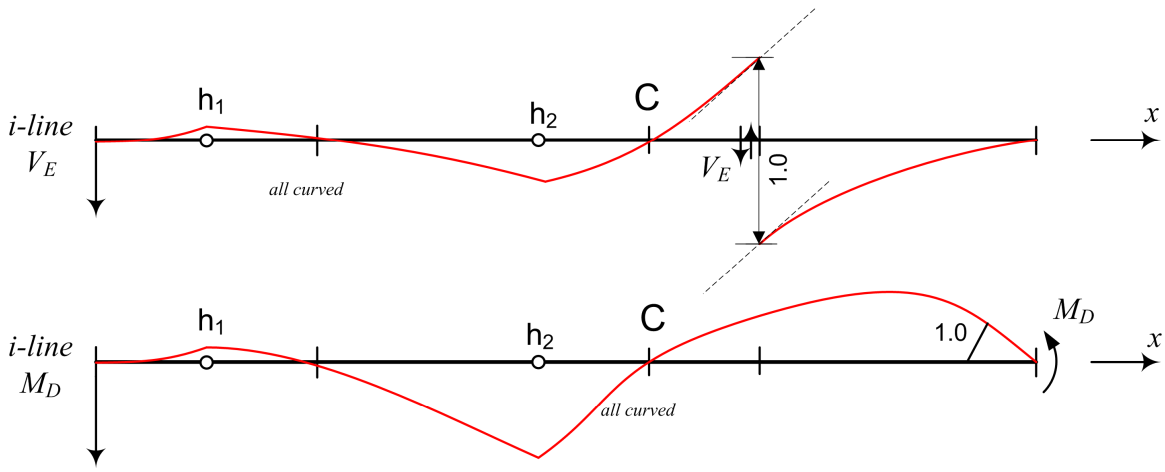


Answers beam (A):





Answers beam (B) :



Problem 2

a) In order to find the displacements, first of all the load distribution has to be obtained.
Since the structure is statically indeterminate (to the degree of 1) we could use the *force method* to solve the static unknown (also known as static redundant) with a deformation condition. This condition can be evaluated with an energy method. For instance take the moment at A as static redundant and use Castigliano's second theorem:

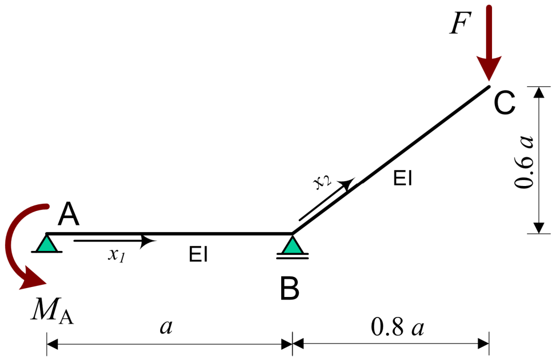
$$\frac{dE_c}{dM_A} = \varphi_A = 0$$

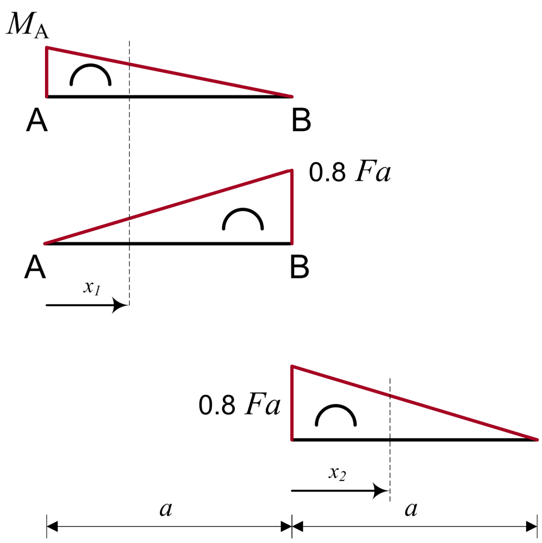
With the static redundant solved, the force distribution is known and based on this distribution with again the second theorem of Castigliano the vertical displacement at point C can be obtained:

$$w_C = \frac{dE_c}{dF}$$

b) The computation based upon the above sketched strategy results in the following steps:

Step 1: **(obtain the force distribution)**
Assume an unknown moment M_A at A (tension on the upper side of the beam).





The moment distribution for beam AB and BC with local axis along the beam axis results in:

beam AB $M^{\mathrm{AB}}(x_1) = -M_A\left(1-\frac{x_1}{a}\right) - \frac{4Fx_1}{5} \quad \frac{\mathrm{d}}{\mathrm{d}M_A}M^{\mathrm{AB}}(x_1) = -\left(1-\frac{x_1}{a}\right)$

beam BC $M^{\mathrm{BC}}(x_2) = -\frac{4Fa\left(1-\frac{x_2}{a}\right)}{5} \quad \frac{\mathrm{d}}{\mathrm{d}M_A}M^{\mathrm{BC}}(x_2) = 0$

Evaluating the deformation condition yields to:

$$\frac{\mathrm{d}E_c}{\mathrm{d}M_A} = \frac{\mathrm{d}}{\mathrm{d}M_A}\int_{ABC}\frac{M^2(x)}{2EI}\mathrm{d}x = \int_{ABC}\frac{M(x)\frac{\mathrm{d}M(x)}{\mathrm{d}M_A}}{EI}\mathrm{d}x =$$

$$= \int_{x_1=0}^{a}\frac{M^{\mathrm{AB}}(x_1)\frac{\mathrm{d}M^{\mathrm{AB}}(x_1)}{\mathrm{d}M_A}}{EI}\mathrm{d}x_1 + \int_{x_2=0}^{a}\frac{M^{\mathrm{BC}}(x_2)\frac{\mathrm{d}M^{\mathrm{BC}}(x_2)}{\mathrm{d}M_A}}{EI}\mathrm{d}x_2 =$$

$$= \int_{x_1=0}^{a}\frac{M^{\mathrm{AB}}(x_1)\frac{\mathrm{d}M^{\mathrm{AB}}(x_1)}{\mathrm{d}M_A}}{EI}\mathrm{d}x_1 = \int_{x_1=0}^{a}\frac{\left(-M_A\left(1-\frac{x_1}{a}\right)-\frac{4Fx_1}{5}\right)\left(-\left(1-\frac{x_1}{a}\right)\right)}{EI}\mathrm{d}x_1 =$$

$$= \frac{5aM_A + 2Fa^2}{15EI} = 0 \quad \Leftrightarrow M_A = -\tfrac{2}{5}Fa$$

Step 2 : (obtain displacement)

The obtained force distribution (moment distribution) can be written as:

beam AB $M^{\mathrm{AB}}(x_1) = \tfrac{2}{5}Fa\left(1-\frac{x_1}{a}\right) - \tfrac{4}{5}Fx_1 \quad \frac{\mathrm{d}}{\mathrm{d}F}M^{\mathrm{AB}}(x_1) = \tfrac{2}{5}a\left(1-\frac{x_1}{a}\right) - \tfrac{4}{5}x_1$

beam BC $M^{\mathrm{BC}}(x_2) = -\tfrac{4}{5}Fa\left(1-\frac{x_2}{a}\right) \quad \frac{\mathrm{d}}{\mathrm{d}F}M^{\mathrm{BC}}(x_2) = -\tfrac{4}{5}a\left(1-\frac{x_2}{a}\right)$

The vertical displacement at C can be found with:

$$\frac{\mathrm{d}E_c}{\mathrm{d}F} = \frac{\mathrm{d}}{\mathrm{d}F}\int_{ABC}\frac{M^2(x)}{2EI}\mathrm{d}x = \int_{ABC}\frac{M(x)\frac{\mathrm{d}M(x)}{\mathrm{d}F}}{EI}\mathrm{d}x =$$

$$= \int_{x_1=0}^{a}\frac{M^{\mathrm{AB}}(x_1)\frac{\mathrm{d}M^{\mathrm{AB}}(x_1)}{\mathrm{d}F}}{EI}\mathrm{d}x_1 + \int_{x_2=0}^{a}\frac{M^{\mathrm{BC}}(x_2)\frac{\mathrm{d}M^{\mathrm{BC}}(x_2)}{\mathrm{d}F}}{EI}\mathrm{d}x_2 =$$

$$= \frac{28Fa^3}{75EI}$$

Problem 3

a) See reader
b) See reader
c) The structure is two-fold statically indeterminate. The maximum number of hinges to obtain a mechanism is three. These hinges may occur at four positions. Therefor a maximum of four mechanisms is to investigated:

1 hinges at CDE, the load will not produce work, infinite failure load
2 hinges at ACD
3 hinges at ACE
4 hinges at ADE

So only the last three mechanisms have to be examined.

d) The failure load has to be found based on a correct mechanism with correct virtual rotations of all elements. The correct direction of the full plastic moments is also essential and finally a correct expression for the virtual work. This results in:

ACD $2{,}33M_p/a$ *failure load*

ACE $2{,}667M_p/a$

ADE $4M_p/a$

e) The check is to construct the moment diagram for the failure load and check if Prager's uniqueness condition is met. At no position in the structure the moment can exceed the full plastic capacity.

Problem 4

a) This cross section is loaded in bending in the $x - z$ plane only. The plane of loading (*m-m*) follows from this. To find the plane of curvature first the position of the normal force centre NC and all cross sectional properties with respect to this NC have to be computed. With this results, the constitutive relation for the cross section can be obtained. With this relation and the cross sectional loading, the deformation at the cross section can be found. This results in the plane of curvature (*k-k*). From this also the position of the neutral axis *n.a.* can be found. According to the theory, the neutral axis must be perpendicular to the plane of curvature and will run through the NC.

For the cross section yields:

$$EA = 210\times10^3 \times\left(A_{tube} + a\times t + b\times t\right) = 210\times10^3 \times 9311.50 = 1955.42\times10^6 \text{ N}$$

Finding the NC: (with reference to the tube centre)

$$y_{NC} = \frac{210\times10^3(a\times t\times(-R) + b\times t\times(-R - \tfrac{1}{2}b)}{EA} = -39.74 \text{ mm}$$

$$z_{NC} = \frac{210\times10^3(a\times t\times\tfrac{1}{2}a + b\times t\times a)}{EA} = 34.67 \text{ mm}$$

The cross sectional properties (*double letter symbols*) for the composite cross section can be found as:

$$EI_{yy} = 210\times10^3\times10967.47\times10^4 \text{ Nmm}^2$$
$$EI_{zz} = 210\times10^3\times10767.15\times10^4 \text{ Nmm}^2$$
$$EI_{yz} = -210\times10^3\times3928.45\times10^4 \text{ Nmm}^2$$

The cross section at mid span is loaded in bending only:

$$N = 0 \text{ N}$$
$$M_y = 0 \text{ Nmm}$$
$$M_z = 36.75\times10^6 \text{ Nmm}$$

And the constitutive relation:

$$\begin{bmatrix} 0 \\ 0 \\ 36.75\times10^6 \end{bmatrix} = 210\times10^3 \begin{bmatrix} 93.1150\times10^2 & 0 & 0 \\ 0 & 10967.47\times10^4 & -3928.45\times10^4 \\ 0 & -3928.45\times10^4 & 10767.15\times10^4 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

The deformation can be found:

$$\varepsilon = 0; \quad \kappa_y = 0.66969\times10^{-6}; \quad \kappa_z = 0.18696560\times10^{-5};$$

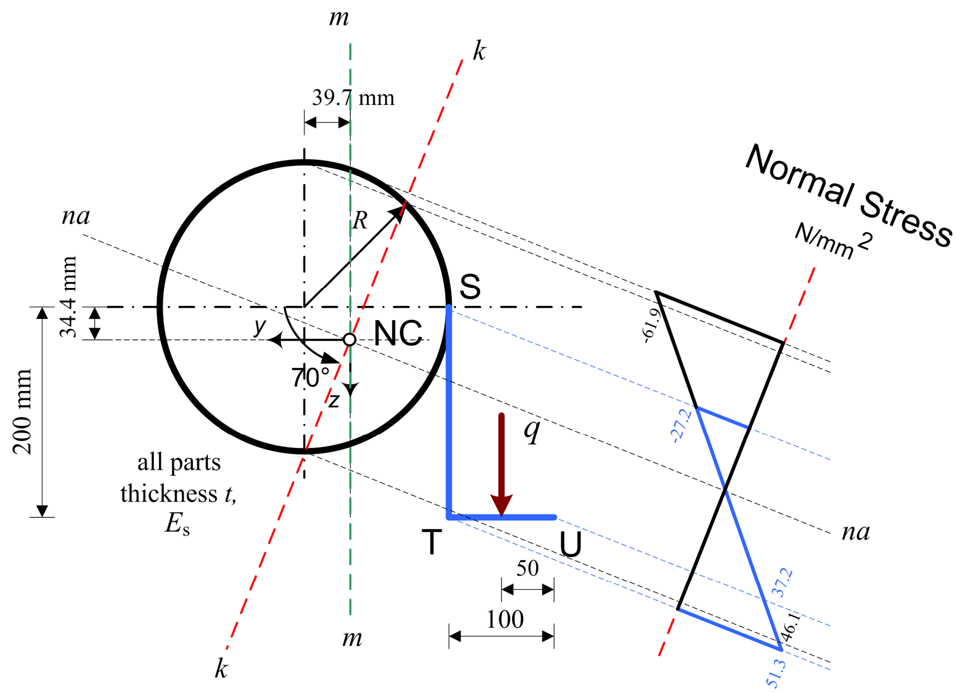
From this the directions of the *plane of loading* and the *plane of curvature* are:

$$\tan\alpha_k = \frac{\kappa_z}{\kappa_y} \quad \Rightarrow \quad \alpha_k = 70.3^{\text{o}}$$

$$\tan\alpha_m = \frac{M_z}{M_y} \Rightarrow \quad \alpha_m = 90.0^{\text{o}}$$

In this case the plane of loading *m-m* and the plane of curvature *k-k* differ by 20 degrees. The neutral axis *n.a.* can be expressed as:

$$0.6697\,y - 1.8697\,z = 0$$



Resulting *k-k*, *m-m* and *n.a.* due to given loads and normal stress distribution

b) The stresses can be found with the deformations:

$$\sigma(y,z) = E\times\left(\varepsilon + y\times\kappa_y + z\times\kappa_z\right)$$

With (y, z) as location with respect to the NC this results per point in:

point	y [mm]	z [mm]	E [N/mm^2]	σ [N/mm^2]
O top tube	39.74	-171.87	210000	-61.9
P lower tube	39.74	103.13	210000	46.1
S tube right	-97.76	-34.67	210000	-27.2
T lower corner	-97.76	165.63	210000	51.3
U free end	165.63	240	210000	37.2

These stresses are presented in the sketch above where the *k-k* line has been used to present the distribution of the normal stresses in the cross section for both materials.

c) The maximum shear stress at S in the L-section can be found with the resultant normal force $R_M^{(a)}$ based on the normal stress distribution due to bending only for the released part $A^{(a)}$ STU based on:

$$\tau_{\text{S-L}} = \frac{s_x^{(a)}}{t} = -\frac{R_{\text{M}}^{(a)}}{M}\frac{V}{t}$$

Where V is the maximum shear force of 42 kN at one of the supports and M is the resulting bending moment needed to find the resulting normal force R_M for a sliding part. For this we take either a unit moment (and subsequently the normal stress distribution due to this unit moment) or a cross section we already know, e.g. mid span.

Computing results in:

$$R_{\text{M}}^{(a)} = \tfrac{1}{2} b \times t \times (\sigma_{\text{U}} + \sigma_{\text{T}}) + \tfrac{1}{2} a \times t \times (\sigma_{\text{T}} + \sigma_{\text{S}}) = 54634.08 \text{ N}$$

$$\tau_{\text{S-L}} = -7.8 \text{ N/mm}^2$$

Note: The shear force at mid span is not significant in this case, maximum shear occurs at the supports.

d) First determine the total shear force taken by the L-section. From this, the part taken by the tube can be found. For the vertical shear force at the L-section, the shear stress distribution of the vertical part ST has to be integrated and multiplied by the thickness t. In order to do so the function of the normal stress for ST depending on the vertical position in the cross section has to be specified. Use a temporary local axis to express this function.

NOT ASKED FOR:
This computation reveals that the tube takes almost 70 % of the vertical shear.