

Remarks: See §2.6.4, pages 40 till 42

Hint:

Calculate and draw the ε -diagram

Answers:

a. $\Delta\ell = 175 \text{ mm}$

b. $\ell = 1656 \text{ m}$

Explanation:

Cable length: ℓ

Cable cross-sectional area: A

Acceleration due to gravity: $g = 10 \text{ N/kg}$

Define the x axis along the cable, with the origin at the hanging point.

Explanation continued:

a. Normal force in the cable:

$$N(x) = (7,85 \times 10^3 \text{ kg/m}^3)(10 \text{ N/kg})(\ell - x)A$$

$$\text{Taking } \ell = 633,5 \text{ m gives: } (\ell - x) = (633,5 \text{ m})(1 - \frac{x}{\ell})$$

$$N(x) = (49,73 \times 10^6 \text{ N/m}^2)(1 - \frac{x}{\ell})A$$

$$\varepsilon(x) = \frac{N(x)}{EA} = \frac{(49,73 \times 10^6 \text{ N/m}^2)}{90 \times 10^9 \text{ N/m}^2} (1 - \frac{x}{\ell}) = 0,552 \times 10^{-3} \times (1 - \frac{x}{\ell})$$

The strain varies linearly from $0,552\text{‰}$ in $x = 0$ to zero in $x = \ell$.

Draw the ε -diagram for the cable.

$$\Delta\ell = \int_0^{\ell} \varepsilon dx = \text{area under the } \varepsilon\text{-diagram} = \frac{1}{2} \times (0,552 \times 10^{-3})(633,5 \text{ m})$$

$$\text{b. } \sigma_{\max} = \frac{N_{\max}}{A} = \frac{(7,85 \times 10^3 \text{ kg/m}^3)(10 \text{ N/kg}) \cdot A \cdot \ell}{A} =$$

$$= (78,5 \times 10^3 \text{ N/m}^3) \cdot \ell \leq 130 \times 10^6 \text{ N/m}^2 \Rightarrow \ell = 1656 \text{ m}$$