Remarks: See §2.6.5, pages 42 till 44

Answers:

a. $\sigma = 25$ kN/m
b. $N = 60$ kN
c. $(0{,}119 \text{ mm}) \leq \Delta\ell \leq (0{,}476 \text{ mm})$
d. $\Delta\ell = 0{,}22$ mm

Explanation:

a. $\sigma = \dfrac{50 \text{ mm}}{200 \text{ mm}} \times (100 \text{ N/mm}^2) = 25 \text{ N/mm}^2$

b. $N = (50 \text{ mm})(12 \text{ mm})(100 \text{ N/mm}^2) = 60 \times 10^3 \text{ N}$

c. $\Delta\ell$ will be between that of a uniform plate of width 50 mm and that of 200 mm:

$$\frac{(60 \text{ kN})(1000 \text{ mm})}{(210 \text{ kN/mm}^2)(600 \text{ mm}^2)} \leq \Delta\ell \leq \frac{(60 \text{ kN})(1000 \text{ mm})}{(210 \text{ kN/mm}^2)(2400 \text{ mm}^2)}$$

d. $A(x) = (600 \text{ mm}^2)(1 + 3\dfrac{x}{\ell})$ where $\ell = 1000$ mm

$$\Delta\ell = \int_0^\ell \frac{N}{EA} \mathrm{d}x = \frac{N}{E} \int_0^\ell \frac{1}{A(x)} \mathrm{d}x =$$

$$= \frac{60 \times 10^3 \text{ N}}{210 \times 10^3 \text{ N}} \times \frac{1}{600 \text{ mm}^2} \times \underbrace{\int_0^\ell \frac{\mathrm{d}x}{(1 + 3\frac{x}{\ell})}}_{462{,}1 \text{ mm}} = 0{,}22 \text{ mm}$$

$$\int_0^\ell \frac{\mathrm{d}x}{(1 + 3\frac{x}{\ell})} = \int_0^\ell \frac{1}{(1 + 3\frac{x}{\ell})} \frac{\mathrm{d}(1 + 3\frac{x}{\ell})}{\frac{3}{\ell}} = \frac{\ell}{3} \times \int_1^4 \frac{\mathrm{d}\tilde{x}}{\tilde{x}} = \frac{\ell}{3} \times \ln 4 = 462{,}1 \text{ mm}$$