Remarks: See §3.1.3 pages 77 till 82
See §3.1.4, example 5, pages 87 till 90

Answers 3.8-1:

a. $y_C = 521$ mm
b. $z_C = 521$ mm

Explanation:

Part 1: Large square. Part 2: (negative) Top-right triangle

$A_1 = 810\times10^3$ mm^2; $A_2 = -180\times10^3$ mm^2
$y_{C;1} = z_{C;1} = 450$ mm; $y_{C;2} = z_{C;2} = 200$ mm
$S_{y;1} = S_{z;1} = 364{,}5\times10^6$ mm^3; $S_{y;2} = S_{z;2} = -36\times10^6$ mm^3
$A = 630\times10^3$ mm^2
$S_y = S_z = 328{,}5\times10^6$ mm^3

$$y_C = z_C = \frac{328{,}5\times10^6 \text{ mm}^3}{630\times10^3 \text{ mm}^2} = 521 \text{ mm}$$

Hints:

Divide the figures into simple shapes (triangle, rectangle, parallelogram) and then calculate their static moments separately.

Answers 3.8-2:

a. $y_C = 280$ mm
b. $z_C = 600$ mm

Explanation:

Part 1: Two parallelograms. Part 2: Triangle. Separated from the parallelograms by the line $y = 200$ mm

$A_1 = 120\times10^3$ mm^2; $A_2 = 180\times10^3$ mm^2
$y_{C;1} = 100$ mm; $y_{C;2} = 400$ mm
$S_{y;1} = 12\times10^6$ mm^3; $S_{y;2} = 72\times10^6$ mm^3
$A = 300\times10^3$ mm^2
$S_y = 84\times10^6$ mm^3

$$y_C = \frac{84\times10^6 \text{ mm}^3}{300\times10^3 \text{ mm}^2} = 280 \text{ mm}$$

Answers 3.8-3:

a. $y_C = 38{,}16$ mm
b. $z_C = 26{,}05$ mm

Explanation:

Part 1: Left-side triangle. Part 2: Rectangle. Part 3: Right-side triangle
All parts contribute positively.

$A_1 = 337{,}5\ \text{mm}^2;\ A_2 = 1125\ \text{mm}^2;\ A_3 = 675\ \text{mm}^2$

$y_{C;1} = 60\ \text{mm};\ y_{C;2} = 42{,}5\ \text{mm};\ y_{C;3} = 20\ \text{mm}$
$S_{y;1} = 20250\ \text{mm}^3;\ S_{y;2} = 47812{,}5\ \text{mm}^3;\ S_{y;3} = 13500\ \text{mm}^3$

$$S_y = 81562{,}5\ \text{mm}^3;\quad y_C = \frac{81562{,}5\ \text{mm}^3}{(337{,}5+1125+675)\ \text{mm}^2} = 38{,}16\ \text{mm}$$

$z_{C;1} = 30\ \text{mm};\ z_{C;2} = 22{,}5\ \text{mm};\ z_{C;3} = 30\ \text{mm}$
$S_{z;1} = 10125\ \text{mm}^3;\ S_{z;2} = 25312{,}5\ \text{mm}^3;\ S_{z;3} = 20250\ \text{mm}^3$

$$S_z = 55687{,}5\ \text{mm}^3;\quad z_C = \frac{55687{,}5\ \text{mm}^3}{(337{,}5+1125+675)\ \text{mm}^2} = 26{,}05\ \text{mm}$$

Answers 3.8-4:

a. $y_C = 36{,}875$ mm
b. $z_C = 25$ mm

Explanation:

Part 1: Rectangle. Part 2: Center triangle. Part 3: Right-side triangle. Part 1 contributes positively, while the other part contribute negatively

$A_1 = 2700\ \text{mm}^2;\ A_2 = 225\ \text{mm}^2;\ A_3 = 675\ \text{mm}^2$

$y_{C;1} = 30\ \text{mm};\ y_{C;2} = 35\ \text{mm};\ y_{C;3} = 10\ \text{mm}$
$S_{y;1} = 81000\ \text{mm}^3;\ S_{y;2} = 7875\ \text{mm}^3;\ S_{y;3} = 6750\ \text{mm}^3$

$$S_y = 66375\ \text{mm}^3;\quad y_C = \frac{66375\ \text{mm}^3}{(2700-225-675)\ \text{mm}^2} = 36{,}875\ \text{mm}$$

$z_{C;1} = 22{,}5\ \text{mm};\ z_{C;2} = 25\ \text{mm};\ z_{C;3} = 15\ \text{mm}$
$S_{z;1} = 60750\ \text{mm}^3;\ S_{z;2} = 5625\ \text{mm}^3;\ S_{z;3} = 10125\ \text{mm}^3$

$$S_z = 45000\ \text{mm}^3;\quad z_C = \frac{45000\ \text{mm}^3}{(2700-225-675)\ \text{mm}^2} = 25\ \text{mm}$$