Remarks: See §3.1.4, example 2, pages 84 till 85

Answers 3.12-1:

a. $A = \tfrac{4}{3}ab$

b. $y_\text{C} = 0$

c. $z_\text{C} = \tfrac{3}{5}a$

Explanation:

a. $z = \dfrac{a}{b^2}y^2$

$$A = \int_{-b}^{b}(a-z)\mathrm{d}y = (ay - \frac{a}{3b^2}y^3)\Big|_{-b}^{b} = \tfrac{4}{3}ab$$

b. $S_z = \displaystyle\int_{-b}^{b}\left(\frac{a+z}{2}\right)\mathrm{d}A = \int_{-b}^{b}\left(\frac{a+z}{2}\right)(a-z)\,\mathrm{d}y$ where: $z = \dfrac{a}{b^2}y^2$

$$S_z = \left(\frac{a^2}{2}y - \frac{a^2}{10b^4}y^5\right)\Bigg|_{-b}^{b} = \tfrac{4}{5}a^2b$$

$$z_\text{C} = \frac{S_z}{A} = \tfrac{3}{5}a$$

Answers 3.12-2:

a. $A = \tfrac{1}{3}ab$

b. $y_\text{C} = \tfrac{1}{4}b$

c. Isn't it a lot of work?? Or is there a better way?

Explanation:

a. $z = -\dfrac{a}{b^2}y^2 + \dfrac{2a}{b}y$

$$A = \int_{0}^{b}(a-z)\mathrm{d}y = (ay + \frac{a}{3b^2}y^3 - \frac{a}{b}y^2)\Big|_{0}^{b} = \tfrac{1}{3}ab$$

b. $S_y = \displaystyle\int_{0}^{b}y(a-z)\,\mathrm{d}y = \int_{0}^{b}y\left(a + \frac{a}{b^2}y^2 - \frac{2a}{b}y\right)\mathrm{d}y = \tfrac{1}{12}ab^2$

$$y_\text{C} = \frac{S_y}{A} = \tfrac{1}{4}b$$

c. $S_z = \displaystyle\int_{0}^{b}\left(\frac{a+z}{2}\right)\mathrm{d}A = \int_{-b}^{b}\left(\frac{a+z}{2}\right)(a-z)\,\mathrm{d}y$

$$S_z = \int\left(\tfrac{1}{2}a + \tfrac{1}{2}z\right)(a-z)\,\mathrm{d}y = \int\left(..y^2 + ..y + ..\right)\left(..y^2 + ..y + ..\right)\mathrm{d}y = ...$$