Remarks: See §4.8, pages 198 till 203

Answers 4.91-1:

a. $\sigma_{(y=+100\text{mm};\, z)} = +2\ \text{N/mm}^2$ (left)

$\sigma_{(y=-100\text{mm};\, z)} = -4\ \text{N/mm}^2$ (right)

Equation of the neutral axis: $y = +33,33\ \text{mm}$

b. $\sigma_{(y=+100\text{mm};\, z=-50\text{mm})} = -1\ \text{N/mm}^2$ (top left)

$\sigma_{(y=+100\text{mm};\, z=+50\text{mm})} = +5\ \text{N/mm}^2$ (bottom left)

$\sigma_{(y=-100\text{mm};\, z=-50\text{mm})} = -7\ \text{N/mm}^2$ (top right)

$\sigma_{(y=-100\text{mm};\, z=+50\text{mm})} = -1\ \text{N/mm}^2$ (bottom right)

Equation of the neutral axis: $3y + 6z = 100\ \text{mm}$

Explanation 4.91-1:

Cross-sectional properties:

$A = 20\times10^3\ \text{mm}^2$

$I_{yy} = 66,667\times10^6\ \text{mm}^4$ and $I_{zz} = 16,667\times10^6\ \text{mm}^4$

a. $N = -20\ \text{kN}$

$M_y = Ne_y = (-20\ \text{kN})(\text{-}0,100\ \text{m}) = +2\ \text{kNm}$

$M_z = 0$

$$\sigma_{(y;z)} = \frac{-20\times10^3\ \text{N}}{20\times10^3\ \text{mm}^2} + \frac{(+2\times10^6\ \text{Nmm})\times y}{66,667\times10^6\ \text{mm}^4} + \frac{0\times z}{16,667\times10^6\ \text{mm}^4}$$

Equation of the neutral axis: $\sigma(y,z) = 0 \Rightarrow y = +(100\ \text{mm})/3$

b. $N = -20\ \text{kN}$

$M_y = Ne_y = (-20\ \text{kN})(\text{-}0,100\ \text{m}) = +2\ \text{kNm}$

$M_z = Ne_z = (-20\ \text{kN})(-0,050\ \text{m}) = +1\ \text{kNm}$

$$\sigma_{(y;z)} = \frac{-20\times10^3\ \text{N}}{20\times10^3\ \text{mm}^2} + \frac{(+2\times10^6\ \text{Nmm})\times y}{66,667\times10^6\ \text{mm}^4} + \frac{(+1\times10^6\ \text{Nmm})\times z}{16,667\times10^6\ \text{mm}^4}$$

Equation of the neutral axis: $\sigma(y,z) = 0 \Rightarrow 3y + 6z = 100\ \text{mm}$

Answers 4.91-2:

a. $\sigma_{(y=+105\text{mm};\, z)} = +3,64\ \text{N/mm}^2$ (left)

$\sigma_{(y=-45\text{mm};\, z)} = -7,27\ \text{N/mm}^2$ (right)

Equation of the neutral axis: $y = +55$ mm

b. $\sigma_{(y=+105\text{mm};\, z=-100\text{mm})} = -1,82\ \text{N/mm}^2$ (top left)

$\sigma_{(y=+105\text{mm};\, z=+100\text{mm})} = +9,09\ \text{N/mm}^2$ (bottom left)

$\sigma_{(y=-45\text{mm};\, z=-100\text{mm})} = -17,09\ \text{N/mm}^2$ (top right)

$\sigma_{(y=-45\text{mm};\, z=+100\text{mm})} = -6,18\ \text{N/mm}^2$ (bottom right)

Equation of the neutral axis: $4y + 3z = 220$ mm

Explanation 4.91-2:

Cross-sectional properties:

$A = 5\times10^3\ \text{mm}^2$

$I_{yy} = 12,375\times10^6\ \text{mm}^4$ and $I_{zz} = 36,667\times10^6\ \text{mm}^4$

a. $N = -20$ kN

$M_y = Ne_y = (-20\ \text{kN})(-0,045\ \text{m}) = +0,9\ \text{kNm}$

$M_z = 0$

$$\sigma_{(y;z)} = \frac{-20\times10^3\ \text{N}}{5\times10^3\ \text{mm}^2} + \frac{(+0,9\times10^6\ \text{Nmm})\times y}{12,375\times10^6\ \text{mm}^4} + \frac{0\times z}{36,667\times10^6\ \text{mm}^4}$$

Equation of the neutral axis: $\sigma(y,z) = 0 \Rightarrow y = +55$ mm

b. $N = -20$ kN

$M_y = Ne_y = (-20\ \text{kN})(-0,045\ \text{m}) = +0,9\ \text{kNm}$

$M_z = Ne_z = (-20\ \text{kN})(-0,100\ \text{m}) = +2\ \text{kNm}$

$$\sigma_{(y;z)} = \frac{-20\times10^3\ \text{N}}{5\times10^3\ \text{mm}^2} + \frac{(+0,9\times10^6\ \text{Nmm})\times y}{12,375\times10^6\ \text{mm}^4} + \frac{(+2\times10^6\ \text{Nmm})\times z}{36,667\times10^6\ \text{mm}^4}$$

Equation of the neutral axis $\sigma(y,z) = 0 \Rightarrow 4y + 3z = 220$ mm