

ANSWERS – VOLUME2: STRESSES, STRAINS, DISPLACEMENTS

Chapter 8, Deformation Due to Flexure

problem 8.008, page 650

Remarks: See § 8.2, pages 557 till 575

Answers 8.8: 1:

$$\text{a) } w(x) = \frac{1}{EI} \left(T \frac{x^2}{2} - Tl \frac{x}{2} \right)$$

$$\text{b) } V(x) = 0 \quad M(x) = -T$$

$$\text{c) } \varphi(0) = \frac{Tl}{2EI} \quad \varphi(l) = -\frac{Tl}{2EI} \quad w\left(\frac{l}{2}\right) = -\frac{1}{8} \frac{Tl^2}{EI}$$

Answers 8.8: 3:

$$\text{a) } w(x) = \frac{1}{EI} \left(\frac{T}{l} \frac{x^3}{2} - T \frac{x^2}{2} \right)$$

$$\text{b) } V(x) = -\frac{3T}{l} \quad M(x) = T - \frac{3T}{l} x$$

$$\text{c) } \varphi(0) = 0 \quad \varphi(l) = -\frac{Tl}{2EI} \quad w\left(\frac{l}{2}\right) = -\frac{Tl^2}{16EI}$$

Answers 8.8: 2:

$$\text{a) } w(x) = \frac{1}{EI} \left(\frac{T}{l} \frac{x^3}{3} - T \frac{x^2}{2} + Tl \frac{x}{6} \right)$$

$$\text{b) } V(x) = -\frac{2T}{l} \quad M(x) = T - \frac{2T}{l} x$$

$$\text{c) } \varphi(0) = -\frac{Tl}{6EI} \quad \varphi(l) = -\frac{Tl}{6EI} \quad w\left(\frac{l}{2}\right) = 0$$

Answers 8.8: 4:

a) Hint: Due to the discontinuity, divide the beam into two parts with suitable boundary conditions:

$$w_1(0) = 0 \quad w_2\left(\frac{l}{2}\right) = 0$$

$$w_1\left(\frac{l}{2}\right) = w_2\left(\frac{l}{2}\right) \quad w_1'\left(\frac{l}{2}\right) = w_2'\left(\frac{l}{2}\right)$$

$$EIw_1''(0) = -M(0) = 0$$

$$EIw_2''(l) = -M(l) = 0$$

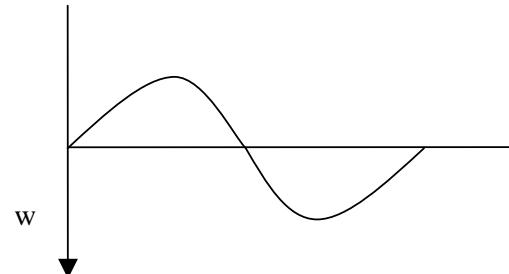
$$EIw_1''\left(\frac{l}{2}\right) = -M_1\left(\frac{l}{2}\right) = \frac{T}{2}$$

$$EIw_2''\left(\frac{l}{2}\right) = -M_2\left(\frac{l}{2}\right) = -\frac{T}{2}$$

Applying these boundary conditions one gets:

$$\frac{1}{EI} \left(\frac{T}{l} \frac{x^3}{6} - Tl \frac{x}{24} \right) \rightarrow \left\{ 0 < x < \frac{l}{2} \right\}$$

$$w(x) = \frac{1}{EI} \left(\frac{T}{l} \frac{x^3}{6} - T \frac{x^2}{2} + \frac{11}{24} Tlx - \frac{1}{8} Tl^2 \right) \rightarrow \left\{ \frac{l}{2} < x < l \right\}$$



b) $V(x) = -\frac{T}{l}$

$$M(x) = \begin{cases} -\frac{T}{l}x \rightarrow \left\{ 0 < x < \frac{l}{2} \right\} \\ -\frac{T}{l}x + T \rightarrow \left\{ \frac{l}{2} < x < l \right\} \end{cases}$$

c) $\varphi(0) = \frac{Tl}{24EI} \quad \varphi(l) = \frac{Tl}{24EI} \quad w\left(\frac{l}{2}\right) = 0$